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SOME RESULTS IN WEAKLY RECIPROCALLY CONTINUOUS AND OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS ON PARTIAL METRIC SPACES

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Abstract

The aim of this paper is to generate two common fixed point (CFP) theorems on partial metric spaces using weakly reciprocally continuous maps and N-compatible and occasionally weakly compatible mappings. Further our results are justified with appropriate examples.

1. Introduction

The idea of partial metric space (shortly PMS) was introduced by Matthews [4] is an extension of metric space. In PMS the condition $d(a, a)$ is no necessarily zero and the condition $d(a, a) = 0$ is replaced by the condition $d(a,\ a) \leq d(a,\ b)$. Many results are found in partial metric spaces like [1], [2], [4] and [5].

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In the recent past, there are many studies on possible generalizations of the existing metric fixed point results to partial metric spaces. The notion of compatible mappings was introduced by Jungck in 1986. Further, Jungck and Rhoades [3] introduced occasionally weakly compatible maps which are weaker than weakly compatible maps.

R. P. Pant, R. K. Bisht and D. Arora [7] improved the concept of reciprocal continuity by introducing weak reciprocal continuity and proved many results. In this paper we generate two common fixed point theorems for four self maps using occasionally weakly compatible mappings, weakly reciprocally continuous and N-compatible mappings.

2. Preliminaries

Definition 2.1. Suppose X is a nonempty set and let $p : X \times X \to [0, \infty[$ satisfy

(P1) $\eta = \xi$ if and only if $p(\eta, \eta) = p(\xi, \xi) = p(\eta, \xi)$

(P2) $p(\eta, \eta) \leq p(\eta, \xi)$

(P3) $p(\eta, \xi) \leq p(\xi, \eta)$

(P4) $p(\eta, \xi) \leq p(\eta, \gamma) + p(\gamma, \xi) - p(\gamma, \gamma)$

for all η, ξ and $\gamma \in X$. Then (X, p) is said to be a partial metric space and p is said to be a metric on X.

Definition 2.2. Suppose (X, p) is a PMS, then a sequence $\{\eta_\theta\}$

(a) converges to $\eta \in X$ if and only if $p(\eta, \eta) = p(\eta, \eta_\theta)$ as $\theta \to \infty$.

(b) is known to be cauchy sequence if and only if $p(\eta_\theta, \eta_m) \to 0$ as $\theta, m \to \infty$.

(c) is known to be (X, p) complete if every cauchy sequence $\{\eta_\theta\}$ in it converges.

Remark 2.3. The following relations hold in PMS (X, p).

(i) If $p(\eta, \xi) = 0$, then $\eta = \xi$.

(ii) If $\eta \neq \xi$, then $p(\eta, \xi) > 0$.

Definition 2.4. The mappings K and M of a PMS defined as K-compatible if $\{KM\eta_\theta\} = \{MM\eta_\theta\}$ as $\theta \to \infty$ whenever $\{\eta_\theta\}$ in X such that $\{K\eta_\theta\}, \{M\eta_\theta\}$ converges to ω as $\theta \to \infty$ for some $\omega \in X$.

Definition 2.5. Two mappings K and M of a PMS are said to be K-weakly reciprocally continuous if $\{KM\eta_\theta\} = K\omega$ as $\theta \to \infty$ whenever $\{\eta_\theta\}$ in X such that $\{K\alpha_\theta\}, \{M\alpha_\theta\}$ converges to ω as $\theta \to \infty$ for some $\omega \in X$.

Definition 2.6. The mappings K and M of a PMS defined as occasionally weakly compatible mappings if they commute at some coincidence points. This means that the mappings K and M need not commute at all coincidence points.

Now we proceed for our main results which generalize and extend the existing theorem proved on compatible mappings in [4].

3. Main Results

Theorem 3.1. *Suppose K, L, M and N are self maps of a complete PMS (X, p) into itself with $K(X) \subset L(X)$ and $M(X) \subset N(X)$. If there exists $\lambda \in [0, 1]$ such that*

$$p(K\eta, M\xi) \leq \lambda\mu(\eta, \xi) \tag{1}$$

for any $\eta, \xi \in X$, where,

$$\mu(\eta, \xi) = \max\left\{p(K\eta, N\alpha), p(M\xi, L\xi), p(N\eta, L\xi), \frac{1}{2}[p(K\eta, L\xi)] + p(M\xi, N\eta)\right\}. \tag{2}$$

The couple $\{K, N\}$ is K- or N-weakly reciprocally continuous and K- or N-compatible and another couple $\{L, M\}$ is weakly compatible.

Then the mappings K, L, M and N have unique common fixed point.

Proof. Let η_0 be an arbitrary point in X, using the condition $K(X) \subset L(X)$ gives such that $K\eta_0 = L\eta_1$ for some $\eta_1 \in X$ and also from the condition $M(X) \subset N(X)$, for $\eta_1 \in X$ and $M\eta_1 \in N(X)$, there exists

$\eta_2 \in X$ such that $M\eta_1 = N\eta_2$. In general, $\eta_{2\theta+1} \in X$ is chosen such that $K\eta_{2\theta} = L\eta_{2\theta+1}$ and $\eta_{\theta+2} \in X$ such that $M\eta_{2\theta+1} = N\eta_{2\theta+2}$, we obtain a sequence $\{\xi_\theta\}$ in X such that

$$\xi_{2\theta} = K\eta_{2\theta} = L\eta_{2\theta+1},\ \xi_{2\theta+1} = M\eta_{2\theta+1} = N\eta_{2\theta+2}, \text{ for } \theta \geq 0. \tag{3}$$

Now we prove that $\{\xi_\theta\}$ is a cauchy sequence.

By (1) and using (3), we observe

$$\begin{aligned} p(\xi_{2\theta+1}, \xi_{2\theta+2}) &= p(L\eta_{2\theta+1}, N\eta_{2\theta+2}) \\ &= p(K\eta_{2\theta}, M\eta_{2\theta+1}) \\ &\leq \lambda\mu(\eta_{2\theta}, \eta_{2\theta+1}) \end{aligned} \tag{4}$$

where

$$\mu(\eta_{2\theta}, \eta_{2\theta+1}) = \max\{p(K\eta_{2\theta}, N\eta_{2\theta}), p(M\eta_{2\theta+1}, L\eta_{2\theta+1}), p(N\eta_{2\theta}, L\eta_{2\theta+1}),$$
$$\frac{1}{2}[p(K\eta_{2\theta}, L\eta_{2\theta+1}) + p(M\eta_{2\theta+1}, N\eta_{2\theta})]\}$$

using (3), we observe

$$\mu(\eta_{2\theta}, \eta_{2\theta+1}) = \max\{p(K\eta_{2\theta}, M\eta_{2\theta-1}), p(M\eta_{2\theta+1}, K\eta_{2\theta+1}), p(M\eta_{2\theta-1}, K\eta_{2\theta}),$$
$$\frac{1}{2}[p(K\eta_{2\theta}, K\eta_{2\theta}) + p(M\eta_{2\theta+1}, M\eta_{2\theta-1})]\} \tag{5}$$

From the definition (P4), we have

$$p(M\eta_{2\theta-1}, M\eta_{2\theta+1}) + p(K\eta_{2\theta}, K\eta_{2\theta}) \leq p(M\eta_{2\theta-1}, K\eta_{2\theta}) + p(M\eta_{2\theta+1}, K\eta_{2\theta}). \tag{6}$$

From (5) and (6), we observe

$$\mu(\eta_{2\theta}, \eta_{2\theta+1}) = \max\{p(K\eta_{2\theta}, M\eta_{2\theta-1}), p(M\eta_{2\theta+1}, K\eta_{2\theta})\}. \tag{7}$$

But if $\mu(\eta_{2\theta}, \eta_{2\theta+1}) = p(M\eta_{2\theta+1}, K\eta_{2\theta})$ then by (4), we observe

$$p(M\eta_{2\theta+1}, K\eta_{2\theta}) \leq \lambda p(M\eta_{2\theta+1}, K\eta_{2\theta}),\ \lambda \in [0, 1[\tag{8}$$

this implies that $p(M\eta_{2\theta+1}, K\eta_{2\theta}) = 0$. Thus, $\mu(\eta_{2\theta}, \eta_{2\theta+1}) = p(M\eta_{2\theta-1}, K\eta_{2\theta})$ and from (4), we get

$$p(M\eta_{2\theta+1},\ K\eta_{2\theta}) \leq \lambda p(M\eta_{2\theta-1},\ K\eta_{2\theta}), \tag{9}$$

which gives

$$p(\xi_{2\theta+2},\ \xi_{2\theta+1}) \leq \lambda p(\xi_{2\theta+1},\ \xi_{2\theta}) \text{ for all } \lambda \geq 0.$$

After simple computation, noting $0 < \lambda < 1$, we deduce that $\{\xi_\theta\}$ as a cauchy sequence. But (X, p) being complete, this gives $\{\xi_\theta\}$ converges to some point $\omega \in \lambda$. Consequently, the subsequences $\{K\eta_{2\theta}\}, \{L\eta_{2\theta+1}\}, \{M\eta_{2\theta+1}\}, \{N\eta_{2\theta+2}\}$ also converges to

$$\omega \in X. \tag{10}$$

Since the couple $\{K, N\}$ is N-weakly reciprocal continuous and N-compatible then the sequences $\{NK\eta_{2\theta}\} \to N\omega$ and $\{KN\eta_{2\theta}\} = \{KK\eta_{2\theta}\}$ as $\theta \to \infty$.

Therefore

$$\{KN\eta_{2\theta}\}, \{KK\eta_{2\theta}\} \to N\omega \text{ as } \theta \to \infty. \tag{11}$$

From the condition (1), on letting $\eta = K\eta_{2\theta}$, $\xi = \eta_{2\theta+1}$ we have

$$p(KK\eta_{2\theta},\ M\eta_{2\theta+1}) \leq \lambda\mu(K\eta_{2\theta},\ \eta_{2\theta+1}) \tag{12}$$

where

$$\mu(K\eta_{2\theta}, \eta_{2\theta+1}) = \max\{p(KK\eta_{2\theta}, NK\eta_\theta), p(M\eta_{2\theta+1}, L\eta_{2\theta+1}), p(NK\eta_{2\theta}, L\eta_{2\theta+1}),$$

$$\frac{1}{2}[p(KK\eta_{2\theta},\ L\eta_{2\theta+1})] + p(M\eta_{2\theta},\ NK\eta_{2\theta})\}$$

letting $\theta \to \infty$, using (10) and (11), we observe

$$\lim_{\theta\to\infty} \mu(K\eta_{2\theta}, \eta_{2\theta+1}) = \max\{p(N\omega, N\omega), p(\omega, \omega), p(N\omega, \omega),$$

$$\frac{1}{2}[p(N\omega,\ \omega)] + p(\omega,\ N\omega)\}$$

$$= p(\omega,\ N\omega). \tag{13}$$

From (12) and (13) together on letting $\theta \to \infty$ gives

$$\lim_{\theta\to\infty} p(KK\eta_{2\theta}, M\eta_{2\theta+1}) \le \lambda \lim_{\theta\to\infty} p(K\eta_{2\theta}, \eta_{2\theta+1})$$

this implies $p(N\omega, \omega) \le \lambda p(\omega, N\omega)$, which is impossible, since $\lambda \in [0, 1)$.

This implies that

$$N\omega = \omega. \tag{14}$$

From (1) on using

$$\eta = \omega, \xi = \eta_{2\theta+1}, p(K\omega, M\eta_{2\theta+1}) \le \lambda\mu(\omega, \eta_{2\theta+1}), \tag{15}$$

where

$$\mu = (\omega, \eta_{2\theta+1}) = \max\{p(K\omega, N\omega), p(M\eta_{2\theta+1}, L\eta_{2\theta+1}), p(N\omega, L\eta_{2\theta+1}),$$

$$\frac{1}{2}[p(K\eta_{2\theta+1}, L\eta_{2\theta+1})] + p(M\eta_{2\theta+1}, N\omega)\}$$

letting $\theta \to \infty$ using (10) and (14), we observe

$$\lim_{\theta\to\infty} \mu(\omega, \eta_{2\theta+1}) = \max\left\{p(K\omega, \omega), p(\omega, \omega), p(\omega, \omega), \frac{1}{2}[p(K\omega, \omega)] + p(\omega, \omega)\right\} \tag{16}$$

using (15), $\lim_{\theta\to\infty} p(K\omega, M\eta_{2\theta+1}) \le \lambda \lim_{\theta\to\infty} \mu(\omega, \eta_{2\theta+1})$ implies that $p(K\omega, \omega)$ $\le \lambda\mu p(K\omega, \omega)$, since $\lambda \in [0, 1)$.

This gives that $K\omega = \omega$.

Therefore $K\omega = N\omega = \omega$.

Again since $K(X) \subset L(X)$ implies $\exists\, \upsilon \in X$ such that

$$\omega = K\omega = L\omega. \tag{17}$$

To show $M\upsilon = \omega$, put $\eta = \omega, \xi = \upsilon$ in (1), we observe

$$p(K\omega, M\upsilon) \le \lambda\mu(\omega, \upsilon) \tag{18}$$

where

$$\mu(\omega, \upsilon) = \max\left\{p(K\omega, N\omega), p(M\upsilon, L\upsilon), (N\omega, L\upsilon), \frac{1}{2}[p(K\omega, L\upsilon)] + p(M\omega, N\omega)\right\}$$

$$= \max\left\{p(\omega, \omega), p(M\upsilon, \omega), p(\omega, \omega), \frac{1}{2}[p(\omega, \omega)] + p(M\upsilon, \omega)\right\}$$

$$= p(M\upsilon, \omega).$$

By (18), $p(\omega, M\upsilon) \leq \lambda p(M\upsilon, \omega)$ since $\lambda \in [0, 1[$ and gives that $M\upsilon = \omega$.

Therefore

$$K\omega = N\omega = M\upsilon = L\upsilon = \omega. \tag{19}$$

Since the couple $\{L, M\}$ is weakly compatible with $M\upsilon = L\upsilon = \omega$ and $LM\upsilon = ML\upsilon$. This gives

$$L\omega = M\omega. \tag{20}$$

Using (1),

$$p(K\eta_{2\theta}, M\omega) \leq \lambda\mu(\eta_{2\theta}, \omega) \tag{21}$$

$$\mu(\eta_{2\theta}, \omega) = \max\{p(K\eta_{2\theta}, N\eta_{2\theta}), p(M\omega, L\omega), p(N\eta_{2\theta}, L\omega),$$

$$\frac{1}{2}[p(K\eta_{2\theta}, L\omega)] + p(M\omega, N\eta_{2\theta})\}$$

and

$$\lim_{\theta\to\infty} \mu(\eta_{2\theta}, \omega) = \max\{p(K\omega, N\omega), p(M\omega, L\omega), p(N\omega, L\omega),$$

$$\frac{1}{2}[p(K\omega, L\omega)] + p(M\omega, \omega)\}$$

$$= p(M\omega, \omega).$$

By (21) $\lim_{\theta\to\infty} p(K\eta_{2\theta}, M\omega) \leq \lambda \lim_{\theta\to\infty} \mu(\eta_{2\theta}, \omega)$ implies that $p(\omega, M\omega) \leq \lambda(\omega, M\omega)$ which is impossible since $\lambda \in [0, 1[$. This gives $p(\omega, M\omega) = 0$ and implies that $M\omega = \omega$. Therefore $M\omega = L\omega = \omega$. Hence, $M\omega = L\omega = K\omega. = N\omega = \omega$.

To show ω is one and only one CFP, if possible assume that there is another CFP t of L, M, K and N. Then using (1), on letting $\eta = \omega, \xi = t$, we observe

$$p(\omega, t) = p(K\omega, Mt) \leq \lambda\mu(\omega, t)$$

where

$$\mu(\omega, t) = \max\left\{p(K\omega, N\omega), p(Mt, Lt), p(N\omega, L\omega), \frac{1}{2}[p(K\omega, Lt) + p(Mt, N\omega)]\right\}$$

$$= \max\left\{p(\omega, \omega), p(t, t), p(\omega, \omega), \frac{1}{2}[p(\omega, t) + p(t, \omega)]\right\}$$

$$= p(\omega, t).$$

Thus, $p(\dot{\omega}, t) \leq \lambda p(\dot{\omega}, t), \lambda \in [0, 1)$ and gives that $\omega = t$. So, ω is becoming unique CFP of K, L, M and N.

Example 3.2. Suppose $X = \left[0, \frac{1}{4}\right]$ and $p : X \times X \to [0, \infty[$ is defined by $p(\eta, \xi) = \max\{\eta, \xi\}$. Then (X, p) is a complete PMS. $K, L, M, N : X \to X$ are defined by

$$K\eta = M\eta = \begin{cases} \frac{1}{10} \; if \; 0 < \eta < \frac{1}{8} \\ \frac{1}{8} \; if \; \frac{1}{8} \leq \eta \leq \frac{1}{4} \end{cases} \text{ and } L\eta = N\eta = \begin{cases} \frac{1}{12} \; if \; 0 < \eta < \frac{1}{8} \\ \frac{1-4\eta}{4} \; if \; \frac{1}{8} \leq \eta \leq \frac{1}{4} \end{cases}.$$

We have $K(X) \subset L(X)$ and $M(X) \subset N(X)$. Let the sequence $\{\eta_\theta\}$ be defined as $\eta_\theta = \frac{1}{8} + \frac{1}{\theta}$ for $\theta \geq 0$.

Now $\lim_{\theta\to\infty} NK\eta_\theta = \lim_{\theta\to\infty} KK\eta_\theta = \frac{1}{8}$ and $\lim_{\theta\to\infty} NK\eta_\theta = S\left(\frac{1}{8}\right) = \frac{1}{8}$, which shows that $\{K, N\}$ is N-compatible and N weakly reciprocally continuous. We observe that $\frac{1}{8}$ is coincidence point of L and M and $ML\left(\frac{1}{8}\right) = LM\left(\frac{1}{8}\right)$. This shows the couple (L, M) is weakly compatible. Also the contractive condition (1) holds for the value $\lambda \in [0, 1[$. It can be observed that $\frac{1}{8}$ is becoming the unique CFP of maps K, L, M and N.

Now we generate another theorem on PMS using occasionally weakly compatible mappings.

Theorem 3.3. *Suppose* (X, p) *is a complete PMS and* K, L, M *and* N

are self mappings on X, *with* $K(X) \subset L(X)$ *and* $M(X) \subset N(X)$. *If there exists a* $\lambda \in [0, 1[$ *such that*

$$p(K\eta, M\xi) \leq \lambda\mu(\eta, \xi) \tag{22}$$

for any $\eta, \xi \in X$, *where*

$$\mu(\eta, \xi) = \max\left\{p(K\eta, N\eta), p(M\xi, L\xi), p(N\eta, L\xi), \frac{1}{2}[p(K\eta, L\xi) + p(M\xi, N\eta)]\right\}.$$

If (M, L) and (N, K) are occasionally weakly compatible and $L(X)$ or $N(X)$ is a complete subspace of X, then K, L, M and N have a common fixed point.

Proof. On using (10) of Theorem (3.1), the subsequences $\{K\eta_{2\theta}\}$, $\{L\eta_{2\theta+1}\}$, $\{M\eta_{2\theta+1}\}$, $\{N\eta_{2\theta+2}\}$ converge to ω as

$$\theta \to \infty. \tag{23}$$

Suppose that $N(X)$ is complete subspace of $X, \exists v \in X$ such that $\omega = Nv$. We shall show that $Kv = \omega$.

On using (22), we observe

$$p(Kv, M\eta_{2\theta+1}) \leq \lambda\mu(v, \eta_{2\theta+1}) \tag{24}$$

where

$$\eta = (v, \eta_{2\theta+1}) = \max\{p(Kv, Nv), p(M\eta_{2\theta+1}, L\eta_{2\theta+1}), p(Nv, L\eta_{2\theta+1}),$$

$$\frac{1}{2}[p(Kv, L\eta_{2\theta+1}) + p(M\eta_{2\theta+1}, Nv)]\}$$

letting $\theta \to \infty$ and using $M\eta_{2\theta+1}, L\eta_{2\theta+1} \to \omega$ and $Nv = \omega$, we observe

$$\lim_{\omega \to \infty} \mu(v, \eta_{2\theta+1}) = \max\left\{p(Kv, \omega), p(\omega, \omega), p(\omega, \omega), \frac{1}{2}[p(Kv, \omega) + p(\omega, \omega)]\right\}$$

$$\lim_{\theta \to \infty} \mu(v, \eta_{2\theta+1}) = p(Kv, \omega). \tag{25}$$

From (24) and (25), we observe

$$\lim_{\theta \to \infty} \mu(Kv, M\eta_{2\theta+1}) \leq \lambda\mu(v, \eta_{2\theta+1})$$

which gives $p(Kv, \omega) \leq \lambda p(Kv, \omega)$ which is impossible since $\lambda \in [0, 1[$.

This implies $p(Kv, \omega) = 0$ gives that $Kv = \omega$.

Therefore $Nv = Kv = \omega$ and this makes v is the coincidence point of N and K. (26)

Since $K(X) \subset L(X)$ and $\omega = Kv \in L(X)$ implies $\exists z \in X$ such that

$$Kv = Lz. \tag{27}$$

Put $\eta = v, \xi = z$ in (22), we obtain

$$p(\omega, Mz) = p(Kv, Mz) \leq \lambda\mu(v, z) \tag{28}$$

where

$$\mu(v, z) = \max\left\{p(Kv, Nv), p(Mz, Lz), p(Nv, Lz), \frac{1}{2}[p(Kv, Lz) + p(Mz, Nv)]\right\}$$

using (26) and (27), we observe

$$\mu(v, z) = \max\left\{p(\omega, \omega), p(Mz, \omega), p(\omega, Mz), \frac{1}{2}[p(\omega, Mz) + p(Mz, \omega)]\right\}$$

$$\mu(v, z) = p(Mz, \omega).$$

Thus, from (28), $p(Mz, \omega) \leq \lambda p(Mz, \omega)$ which is impossible since $\lambda \in [0, 1[$. Hence $p(Mz, \omega) = 0$ implies that $Mz = \omega$.

Therefore $\omega = Lz = Mz$ and z is becoming the coincidence point of L and M. On using the couple (L, M) and (N, K) as occasionally weakly compatible, we have $Lz = Mz$ and $Nv = Kv$. This in turn implies $LMz = MLz$ and $LLz = LMz = MLz = MMz$ and $NKv = KNv$ and

$$NKv = NKv = KNv = KKv. \tag{29}$$

On using (22), we have

$$p(KKv, Kv) = p(KKv, Mz) \leq \lambda\mu(Kv, z) \tag{30}$$

where

$$\mu(Kv, z) = \max\{p(KKv, NKv), p(Mz, Lz), p(NK, Lz),$$

$$\frac{1}{2}[p(KKv, Lz) + p(Mz, NKv)]\}$$

$$= \max\{p(KKv, KKv), p(Kv, Kv), p(KKv, Kv),$$

$$\frac{1}{2}[p(KKv, Kv) + p(Kv, KKv)]\}$$

$$= p(KKv, Kv). \tag{31}$$

From (30) and (31), we observe $p(KKv, Kv) \leq \lambda p(KKv, Kv)$ which is impossible, since $\lambda \in [0, 1)$ implies that $KKv = Kv$.

Therefore $KKv = Kv = NKv$ and this makes Kv CFP of K and N.

Likewise, we can prove that Mz is CFP of L and M.

Since $Kv = Mz$, we observe that Kv as a CFP of K, L, M and N. The uniqueness of the CFP can be proved easily. Now we justify the theorem with a suitable example.

Example 3.5. Let $X = [0, 4]$ and $p : X \times X \to [0, \infty)$ be defined by $p(\eta, \xi) = \max\{\eta, \xi\}$. Then (X, p) is a complete PMS. Define $K, L, M, N : X \to X$ by

$$L\eta = N\eta = \begin{cases} \frac{1}{2} & if\ 0 < \eta < 1 \\ \eta^2 & if\ 1 \leq \eta \leq 2 \end{cases} \text{ and } K\eta = M\eta = \begin{cases} \frac{2\eta + 1}{2} & if\ 0 < \eta < 1 \\ 1 & if\ 1 \leq \eta \leq 2 \end{cases}.$$

We have $L(X) = \left\{\frac{1}{2}\right\} \cup [1, 4]$ and $K(X) = \left[\frac{1}{2}, \frac{3}{2}\right)$ implies that the condition $K(X) \subset L(X)$ is satisfied.

Since 0 and 1 are coincidence points for maps L and M, $L0 = M0 = \frac{1}{2}$ and $LM1 = 1 = ML1$ but $LM0 \neq ML0$.

Therefore L and M are occasionally weakly compatible mappings but not weakly compatible as they do not commute at 0. It can be observed that '1' is the unique CFP of K, L, M and N.

4. Conclusion

In this paper, we generated two results. In the first result, one pair is assumed to be N-weakly reciprocally continuous and N-compatible and in the second result, two pairs are assumed to be occasionally weakly compatible mappings. Further, these two results are justified with suitable examples. Thus, we assert our results generalize and extend the results proved in [4].

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