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TWO MODULO THREE GRACEFUL LABELING OF BIPARITITE GRAPHS

A. SASIKALA and V. POOVILA

Associate Professor and Research Scholar
Department of Mathematics
Periyar Maniammai Institute
of Science and Technology
Periyar Nagar, Vallam, Thanjavur
613403, Tamilnadu, India
E-mail: sasikala@pmu.edu
pudurpoovila@gmail.com

Abstract

A function f is called a two modulo three graceful labeling of a graph G if $f : V(G) \to \{2, 5, 8, 11, 14, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, 12, \ldots, 3q\}$ defined as $f^*(uv) = |f(u) - f(v)|$ is bijective. A graph which admits two modulo three graceful labeling is called two modulo three graceful graph. In this paper, we have proved two modulo three graceful labeling of bipartite graphs $K_{2,n}$, $K_{3,n}$, $K_{4,n}$, $K_{m,n}$.

1. Introduction

All graphs considered here are simple, finite, connected and undirected. The symbols $V(G)$ and $E(G)$ denote the vertex set and the edge set of a graph G. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Several types of graph labeling and a detailed survey is available in [6].

For standard terminology and notations we follow Harary [2]. V.

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Swaminathan and C. Sekar introduced the concept of one modulo three graceful labeling in [3]. Velmurgan and Ramachandran [2019] defined the M modulo N graceful labeling of path and star [5].

In this paper, we investigate the two modulo three graceful labeling of bipartite graphs $K_{2,n}$, $K_{3,n}$, $K_{4,n}$, $K_{m,n}$.

2. Basic Definitions

Definition 2.1. A function f is called a *graceful labeling* of a graph G if $f : V(G) \to \{0, 1, 2, 3, \ldots, q\}$ is injective and the induced function $f^* : E(G) \to \{1, 2, 3, \ldots, q\}$ defined as $f^*(e = uv) = | f(u) - f(v) |$ is bijective. This type of graph labeling first introduced by Rosa in 1967 as a β-valuation[1], later on Solomon W. Golomb called as graceful labeling [1].

Definition 2.2. A graph G is said to be *M modulo N graceful labeling* [where N is a positive integer and M is defined to be 1to N] if there is a function f from the vertex set of G to $\{0, M, N, (N + M), 2N, (2N + M), \ldots, N(q - 1), N(q - 1) + M\}$ in such a way that

(i) f is $1 - 1$.

(ii) f induces a bijection f^* from the edge set of G to $\{M, N + M, 2N + M, \ldots, N(q - 1) + M\}$, where $f^*(uv) = | f(u) - f(v) |$ $Gu, v \in \forall$.

Definition 2.3. A function f is called a *two modulo three graceful labeling* of a graph G if $f : V(G) \to \{2, 5, 8, 11, 14, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, 12, \ldots, 3q\}$ defined as $f^*(uv) = | f(u) - f(v) |$ is bijective. A graph which admits two modulo three graceful labeling is called two modulo three graceful graph.

Definition 2.4. A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y so that each edge has one end in X and the other end in Y, such a partition of the graph. In a bipartite graph, no two vertices in X are adjacent. And no two vertices in Y are adjacent.

Definition 2.5. A complete bipartite graph is a simple bipartite graph in which each vertex of X is joined to each vertex of Y. If $|x| = m$, $|y| = n$, such a graph is denoted by $k_{m,n}$.

3. Main Results

Theorem 3.1. *The bipartite graph $K_{2,n}$ is two modulo three graceful labeling.*

Proof. The bipartite graph has $n + 2$ vertices denoted by $\{v_1, v_2, \ldots, v_{n+2}\}$ and $2n$ edges denoted by $\{e_1, e_2, \ldots, e_{2n}\}$.

We define the vertex labeling

$$f : V(K_{2,n}) \to \{2, 5, 8, \ldots, 3q + 8\}$$

as

$$f(v_i) = 3i - 1,\ i = 1, 2.$$

$$f(v_{j+2}) = 6j + 2,\ j = 1, 2, 3, \ldots, n.$$

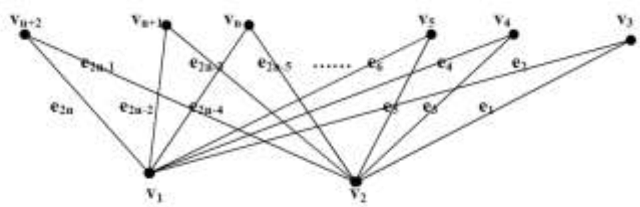

Figure 1. Two modulo three graceful labeling of $K_{2,n}$.

Hence the induced edge labeling

$$f^* : E(K_{2,n}) \to \{3, 6, 9, \ldots, 3q\}$$

will be defined as

$$f(e_i) \to 3i;\ i = 1, 2, 3, \ldots, 2n.$$

Hence, the graph G if $f : V(G) \to \{2, 5, 8, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, 12, \ldots, 3q\}$ defined as $f^*(u) = |f(u) - f(v)|$ is bijective.

Hence the graph $K_{2,n}$ admits two modulo three graceful labeling. Hence the graph $K_{2,n}$ is two modulo three graceful graph.

Example 3.1. The bipartite graph $K_{2,6}$ is two modulo three graceful labeling.

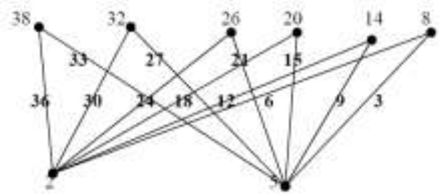

Figure 2. Two modulo three graceful labeling of $K_{2,6}$.

Theorem 3.2. *The bipartite graph* $K_{3,n}$ *is two modulo three graceful labeling.*

Proof. The bipartite graph has $n+3$ vertices denoted by $\{v_1, v_2, \ldots, v_{n+3}\}$ and $3n$ edges denoted by $\{e_1, e_2, \ldots, e_{3n}\}$.

We define the vertex labeling

$$f : V(K_{3,n}) \to \{2, 5, 8, \ldots, 3q+8\}$$

as

$$f(v_i) = 3i - 1,\ i = 1, 2, 3.$$

$$f(v_{j+3}) = 9j + 2,\ j = 1, 2, 3, \ldots, n.$$

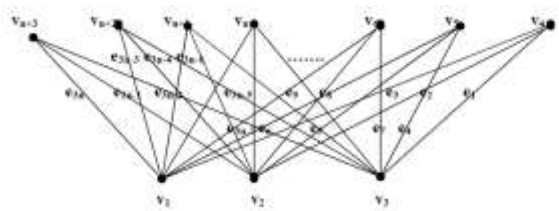

Figure 3. Two modulo three graceful labeling of $K_{3,n}$.

Hence the induced edge labeling

$$f^* : E(K_{3,n}) \to \{3, 6, 9, \ldots, 3q\}$$

will be defined as

$$f(e_i) \to 3i;\ i = 1, 2, 3, \ldots, 3n.$$

Hence, the graph G if $f : V(G) \to \{2, 5, 8, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, 12, \ldots, 3q\}$ defined as $f^*(u) = |f(u) - f(v)|$ is bijective.

Hence the graph $K_{3,n}$ admits two modulo three graceful labeling. Hence the graph $K_{3,n}$ is two modulo three graceful graph.

Example 3.2. The bipartite graph $K_{3,7}$ is two modulo three graceful labeling.

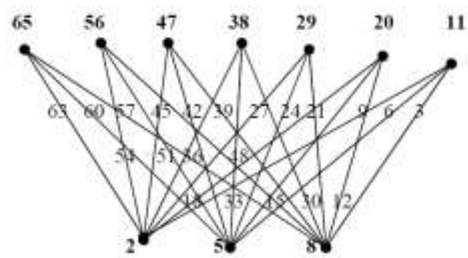

Figure 4. Two modulo three graceful labeling of $K_{3,7}$.

Theorem 3.3. *The bipartite graph $K_{4,n}$ is two modulo three graceful labeling.*

Proof. The bipartite graph has $n + 4$ vertices denoted by $\{v_1, v_2, \ldots, v_{n+4}\}$ and $4n$ edges denoted by $\{e_1, e_2, \ldots, e_{4n}\}$.

We define the vertex labeling

$$f : V(K_{4,n}) \to \{2, 5, 8, \ldots, 3q + 8\}$$

as

$$f(v_i) = 3i - 1;\ i = 1, 2, 3, 4.$$

$$f(v_{j+4}) = 12j + 2;\ j = 1, 2, 3, \ldots, n.$$

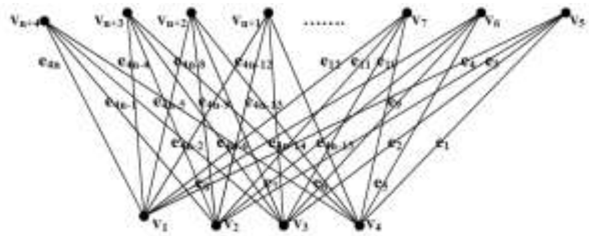

Figure 5. Two modulo three graceful labeling of $K_{4,n}$.

Hence the induced edge labeling

$$f^* : E(K_{4,n}) \to \{3, 6, 9, \ldots, 3q\}$$

will be defined as

$$f(e_i) \to 3i;\ i = 1, 2, 3, \ldots, 4n.$$

Hence, the graph G if $f : V(G) \to \{2, 5, 8, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, 12, \ldots, 3q\}$ defined as $f^*(u) = |f(u) - f(v)|$ is bijective.

Hence the graph $K_{4,n}$ admits two modulo three graceful labeling. Hence the graph $K_{4,n}$ is two modulo three graceful graph.

Example 3.3. The bipartite graph $K_{4,5}$ is two modulo three graceful labeling.

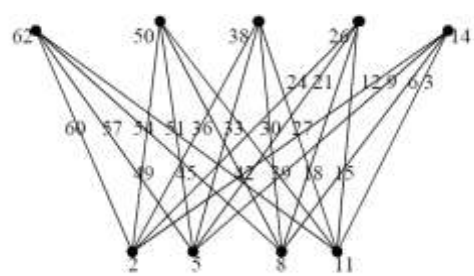

Figure 6. Two modulo three graceful labeling of $K_{4,5}$.

Theorem 3.4. *The bipartite graph* $K_{m,n}$ *is two modulo three graceful labeling.*

Proof. The bipartite graph has $m+n$ vertices denoted by $\{v_1, v_2, \ldots, v_{m+n}\}$ and mn edges denoted by $\{e_1, e_2, \ldots, e_{mn}\}$.

We define the vertex labeling

$$f : V(K_{m,n}) \to \{2, 5, 8, \ldots, 3q + 8\}$$

as

$$f(v_i) = 3i - 1;\ i = 1, 2, 3, \ldots, n.$$

$$f(v_{j+m}) = 3mj + 2;\ j = 1, 2, 3, \ldots, n.$$

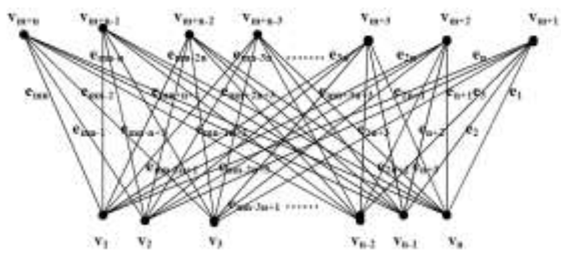

Figure 7. Two modulo three graceful labeling of $K_{m,n}$.

Hence the induced edge labeling

$$f^* : E(K_{m,n}) \to \{3, 6, 9, \ldots, 3q\}$$

will be defined as

$$f(e_i) \to 3i;\ i = 1, 2, 3, \ldots, mn.$$

Hence, the graph G if $f : V(G) \to \{2, 5, 8, \ldots, 3q + 8\}$ is injective and the induced function $f^* : E(G) \to \{3, 6, 9, 12, \ldots, 3q\}$ defined as $f^*(u) = |f(u) - f(v)|$ is bijective.

Hence the graph $K_{m,n}$ admits two modulo three graceful labeling. Hence the graph $K_{m,n}$ is two modulo three graceful graph.

4. Conclusion

In this paper, we have investigated the Two modulo three graceful labeling of bipartite graphs.

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