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BIPOLAR FUZZY W-HAUSDORFF SPACE

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Abstract

In this paper the definition of bipolar fuzzy W-Hausdorff space is introduced by extending the definition of W-Hausdorff space introduced by Warren [4] and obtained results analogues to the results of W-Hausdorff space.

1. Introduction

In 1965, Zadeh [13] introduced the concept of fuzzy set. The notion of fuzzy topology was established by Chang [3] in 1968. In 1994, Zhang [14] introduced the notion of a bipolar fuzzy set. In 2018, Kim, et al. [8] defined bipolar fuzzy point and obtained some of its properties and introduced the concept of bipolar fuzzy topology in Changs sense [3]. In section 2 of this paper, preliminary definitions regarding fuzzy set and bipolar fuzzy set are given. In section 3 of this paper, the definition of bipolar fuzzy W-Hausdorff

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space is introduced by extending the definition of W-Hausdorff space introduced by Warren [4] and obtained result analogues to the results of W-Hausdorff space.

2. Preliminary Definitions

Definition 2.1 [13]. Let X be an arbitrary non empty set. Let $I = [0, 1]$. A *fuzzy set* in X is a mapping from X into I that is a fuzzy set is an element of I^X.

Definition 2.2 [3]. Let X be a non empty set. A subset $\tau \subset I^X$ is called a fuzzy topology on X iff τ satisfies the following requirements

(i) The constant fuzzy sets 0 and 1 belong to τ

(ii) $f_\lambda \in \tau$ for each $\lambda \in \wedge$ implies $\vee_{\lambda \in \Lambda} f_\lambda \in \tau$

(iii) $f, g \in \tau$ implies $f \wedge g \in \tau$.

the pair (X, τ) is called a *fuzzy topological space*.

Definition 2.3 [4]. A fuzzy topological space (X, τ) is said to be fuzzy W-Hausdorff if for every $x, y \in X, x \neq y,$ there exist $f, g \in \tau$ such that $f(x) = 1,\ g(y) = 1$ and $f \wedge g = 0$.

Definition 2.4 [8]. Let X be a non-empty set. Then a pair $A_{bp} = (A_{bp}{}^{+}, A_{bp}{}^{-})$ is called *bipolar-valued fuzzy set or bipolar fuzzy set* in X, where $A_{bp}{}^{+} : X \to [0, 1]$ and $A_{bp}{}^{-} : X \to [-1, 0]$.

The set of all bipolar fuzzy set in X is denoted as $BPF(X)$.

Definition 2.5 [8]. The *bipolar fuzzy null set* denoted by $0_{bp} = (0_{bp}{}^{+}, 0_{bp}{}^{-})$ is a bipolar fuzzy set in X defined as $0_{bp}{}^{+}(x) = 0$ and $0_{bp}{}^{-}(x) = 0,$ for each $x \in X$.

Definition 2.6 [8]. The *bipolar fuzzy whole set* denoted by $1_{bp} = (1_{bp}{}^{+}, 1_{bp}{}^{-})$ is a bipolar fuzzy set in X defined as $1_{bp}{}^{+}(x) = 1$ and $1_{bp}{}^{-}(x) = -1,$ for each $x \in X$.

Definition 2.7 [8]. Let X be a non-empty set and let $A_{bp} = (A_{bp}{}^{+}, A_{bp}{}^{-})$ $B_{bp} = (B_{bp}{}^{+}, B_{bp}{}^{-})$ be bipolar fuzzy sets in X, Then

(i) A_{bp} is a *subset* of B_{bp}, denoted by $A_{bp} \subset B_{bp}$ is defined as $A_{bp}{}^{+}(x) \leq B_{bp}{}^{+}(x)$ and $A_{bp}{}^{-}(x) \geq B_{bp}{}^{-}(x)$, for each $x \in X$.

(ii) The *complement* of A_{bp} is denoted by $A_{bp}{}^{c} = ((A_{bp}{}^{c})^{+}, (A_{bp}{}^{c})^{-})$ is a bipolar fuzzy set in X defined as $(A_{bp}{}^{c})^{+}(x) = 1 - A_{bp}{}^{+}(x)$ and $(A_{bp}{}^{c})^{-}(x) = -1 - A_{bp}{}^{-}(x)$, for each $x \in X$.

(iii) The *intersection* of A_{bp} and B_{bp}, denoted by $A_{bp} \cap B_{bp}$ is a bipolar fuzzy set in X defined as

$$(A_{bp} \cap B_{bp})(x) = (A_{bp}{}^{+}(x) \wedge B_{bp}{}^{+}(x), A_{bp}{}^{-}(x) \vee B_{bp}{}^{-}(x)), \text{ for each } x \in X.$$

(iv) The *union* of A_{bp} and B_{bp}, denoted by $A_{bp} \cup B_{bp}$, is a bipolar fuzzy set in X defined as

$$(A_{bp} \cup B_{bp})(x) = (A_{bp}{}^{+}(x) \vee B_{bp}{}^{+}(x), A_{bp}{}^{-}(x) \wedge B_{bp}{}^{-}(x)), \text{ for each } x \in X.$$

(v) The *intersection* of $((A_{bp})_{\lambda})_{\lambda \epsilon \Lambda}$, a collection of bipolar fuzzy subsets in X denoted by $\cap_{\lambda \epsilon \Lambda}(A_{bp})_{\lambda}$ is a bipolar fuzzy set in X defined as

$$(\cap_{\lambda \epsilon \Lambda}(A_{bp})_{\lambda})(x) = (\wedge_{\lambda \epsilon \Lambda}((A_{bp})_{\lambda}^{+})(x), \vee_{\lambda \epsilon \Lambda}((A_{bp})_{\lambda}^{-})(x)), \text{ for each } x \in X.$$

(vi) The *union* of $((A_{bp})_{\lambda})_{\lambda \epsilon \Lambda}$, a collection of bipolar fuzzy subsets in X denoted by $\cup_{\lambda \epsilon \Lambda}(A_{bp})_{\lambda}$ is a bipolar fuzzy set in X defined as

$$(\cup_{\lambda \epsilon \Lambda} (A_{bp})_{\lambda})(x) = (\vee_{\lambda \epsilon \Lambda} ((A_{bp})_{\lambda}^{+})(x), \Lambda_{\lambda \epsilon \Lambda}((A_{bp})_{\lambda}^{-})(x)), \text{ for each } x \in X.$$

Definition 2.8 [8]. Let X be a non-empty set and let $\mathfrak{B} \subset BPF(X)$. Then $\mathfrak{B}$ is called a *bipolar fuzzy topology on X*, if it satisfies the following axioms:

(i) $0_{bp}, 1_{bp} \epsilon \mathfrak{B}$.

(ii) $A_{bp} \cap B_{bp} \in \mathfrak{B}$, for any $A_{bp}, B_{bp} \subset \mathfrak{B}$.

(iii) $\cup_{\lambda \in \Lambda} (A_{bp})_{\lambda} \in \mathfrak{B}$, for any $((A_{bp})_{\lambda})_{\lambda \in \Lambda} \subset \mathfrak{B}$.

In this case the pair $(X, \mathfrak{B})$ is called a bipolar fuzzy topological space and each member of $\mathfrak{B}$ is called a bipolar fuzzy open set (BPFOS) in X.

$A_{bp} \in BPF(X)$ is said to be closed in X, if $A_{bp}{}^{c} \in \mathfrak{B}$.

The set of all bipolar fuzzy topologies on X is denoted as $BPFT(X)$.

3. Bipolar Fuzzy W-Hausdorff Space

Definition 3.1. A bipolar fuzzy topological space $(X, \mathfrak{B})$ is said to be *bipolar fuzzy W-Hausdorff or bipolar fuzzy* W - T_2, if for any two distinct points $x, y \in X$, there exists two bipolar fuzzy open sets

$A_{bp} = (A_{bp}{}^{+}, A_{bp}{}^{-})$ and $B_{bp} = (B_{bp}{}^{+}, B_{bp}{}^{-})$ such that

$A_{bp}{}^{+}(x) = 1, A_{bp}{}^{-}(x) = -1, B_{bp}{}^{+}(y) = 1, B_{bp}{}^{-}(y) = -1$ and $A_{bp} \cap B_{bp} = 0_{bp}$.

Definition 3.2. Let $(X, \mathfrak{B})$ be a bipolar fuzzy topological space. Let $Y \subseteq X$.

Let $A_{bp} = (A_{bp}{}^{+}, A_{bp}{}^{-}) \in \mathfrak{B}$. Define

$A_{bp}/Y = (A_{bp}{}^{+}/Y, A_{bp}{}^{-}/Y)$ such that

$(A_{bp}{}^{+}/Y)(z) = A_{bp}{}^{+}(z)$ and $(A_{bp}{}^{-}/Y)(z) = A_{bp}{}^{-}(z)$, for all $z \in Y$.

Define $(\mathfrak{B}/Y) = \{(A_{bp}/Y) \mid A_{bp} \in \mathfrak{B}\}$

Then $(\mathfrak{B}/Y)$ is called the *bipolar fuzzy subspace topology on* Y and $(Y, \mathfrak{B}/Y)$ is called a *bipolar fuzzy subspace of* $(X, \mathfrak{B})$ or simply Y is called a *bipolar fuzzy subspace* of X.

Theorem 3.3. *Subspace of a bipolar fuzzy W-Hausdorff space is bipolar fuzzy W-Hausdorff.*

Proof. Let $(X, \mathfrak{B})$ be a bipolar fuzzy W-Hausdorff space. Let Y be a non-empty subset of X.

To prove: $(Y, \mathfrak{B}/Y)$ is a bipolar fuzzy W-Hausdorff space

Consider $y_1, y_2 \epsilon Y$ such that $y_1 \neq y_2$. Then $y_1, y_2 \epsilon X$, there exists two bipolar fuzzy open sets

$U_{bp} = (U_{bp}{}^+, U_{bp}{}^-)$ and $V_{bp} = (V_{bp}{}^+, V_{bp}{}^-)$ such that

$U_{bp}{}^+(y_1) = 1, U_{bp}{}^-(y_1) = -1, V_{bp}{}^+(y_2) = 1, V_{bp}{}^-(y_2) = -1$ and

$U_{bp} \cap V_{bp} = 0_{bp}$. That is $U_{bp}{}^+ \wedge V_{bp}{}^+ = 0^+ = 0$ and $U_{bp}{}^- \wedge V_{bp}{}^- = 0^- = 0$.

Since Y is a subset of X, $U_{bp}/Y, V_{bp}/Y \in \mathfrak{B}/Y$ where

$U_{bp}/Y = (U_{bp}{}^+/Y, U_{bp}{}^-/Y)$ and $V_{bp}/Y = (V_{bp}{}^+/Y, V_{bp}{}^-/Y)$

Therefore $(U_{bp}{}^+/Y)(y_1) = U_{bp}{}^+(y_1) = 1$,

$$(U_{bp}{}^-/Y)(y_1) = U_{bp}{}^-(y_1) = -1,$$

$$(V_{bp}{}^+/Y)(y_2) = V_{bp}{}^+(y_2) = 1,$$

$$(V_{bp}{}^-/Y)(y_2) = V_{bp}{}^-(y_2) = 1,$$

Consider

$$(U_{bp}/Y) \cap (V_{bp}{}^+/Y) = ((U_{bp}{}^+/Y) \wedge (V_{bp}{}^+/Y), (U_{bp}{}^-/Y) \vee (V_{bp}{}^-/Y))$$

$((U_{bp}{}^+/Y) \wedge (V_{bp}{}^+/Y))(y) = (U_{bp}{}^+/Y)(y) \wedge (V_{bp}{}^+/Y)(y)$, for all $y \in Y \subseteq X$

$$= U_{bp}{}^+(y) \wedge V_{bp}{}^+(y), \text{ for all } y \in Y \subseteq X$$

$$= (U_{bp}{}^+ \wedge V_{bp}{}^+)(y), \text{ for all } y \in Y \subseteq X$$

$$= 0_{bp}{}^{+}(y), \text{ for all } y \in Y \subseteq X$$

$$(U_{bp}{}^{+} / Y) \wedge (V_{bp}{}^{+} / Y) = 0^{+}$$

$((U_{bp}{}^{-} / Y) \vee (V_{bp}{}^{-} / Y))(y) = (U_{bp}{}^{-} / Y)(y) \vee (V_{bp}{}^{-} / Y)(y),$ for all $y \in Y \subseteq X$

$$= U_{bp}{}^{-}(y) \vee V_{bp}{}^{-}(y), \text{ for all } y \in Y \subseteq X$$

$$= (U_{bp}{}^{-} \vee V_{bp}{}^{-})(y), \text{ for all } y \in Y \subseteq X$$

$$= 0_{bp}{}^{-}(y), \text{ for all } y \in Y \subseteq X$$

$$(U_{bp}{}^{-} / Y) \wedge (V_{bp}{}^{-} / Y) = 0^{-}$$

$$\Rightarrow (U_{bp} / Y) \cap (V_{bp} / Y) = (0^{+}, 0^{-}) = 0_{bp}$$

Therefore subspace of a bipolar fuzzy W-Hausdorff space is bipolar fuzzy W-Hausdorff.

Definition 3.4. Let $A_{bp} = (A_{bp}{}^{+}, A_{bp}{}^{-})$ and $B_{bp} = (B_{bp}{}^{+}, B_{bp}{}^{-})$ be two bipolar fuzzy sets in X and Y respectively. The *Cartesian product* of A_{bp} and B_{bp} is a bipolar fuzzy set in $X \times Y$ denoted as

$A_{bp} * B_{bp}$ and is defined as $A_{bp} * B_{bp} = (A_{bp}{}^{+} * B_{bp}{}^{+}, A_{bp}{}^{-} * B_{bp}{}^{-})$ where $(A_{bp}{}^{+} * B_{bp}{}^{+})(x, y) = \min\{A_{bp}{}^{+}(x), B_{bp}{}^{+}(y)\}$ and

$(A_{bp}{}^{-} * B_{bp}{}^{-})(x, y) = \max\{A_{bp}{}^{-}(x), B_{bp}{}^{-}(y)\}$ for every $(x, y) \in X \times Y$.

Definition 3.5. Let $(X, \mathfrak{B}_1)$ and $(Y, \mathfrak{B}_2)$ be two bipolar fuzzy topological spaces. Then the *product topology* $\mathfrak{B}_1 \times \mathfrak{B}_2$ on $X \times Y$ is the bipolar fuzzy topology having the collection $\{A_{bp} * B_{bp} / A_{bp} \epsilon \mathfrak{B}_1, B_{bp} \in \mathfrak{B}_2\}$ as a basis.

Theorem 3.6. *Product of two bipolar fuzzy W-Hausdorff spaces is bipolar fuzzy W-Hausdorff space.*

Proof. Let $(X, \mathfrak{B}_1)$ and $(Y, \mathfrak{B}_2)$ be two bipolar fuzzy W-Hausdorff spaces.

To prove: $(X \times Y, \mathfrak{B}_1 \times \mathfrak{B}_2)$ is a bipolar fuzzy W-Hausdorff space

Consider two distinct points $(x_1, y_1), (x_2, y_2) \in X \times Y$. Either $x_1 \neq x_2$ or $y_1 \neq y_2$.

Assume $x_1 \neq x_2$, therefore there exists two bipolar fuzzy open sets $A_{bp} = (A_{bp}{}^{+}, A_{bp}{}^{-})$ and $B_{bp} = (B_{bp}{}^{+}, B_{bp}{}^{-})$ such that

$A_{bp}{}^{+}(x_1) = 1,\ A_{bp}{}^{-}(x_1) = -1,\ B_{bp}{}^{+}(x_2) = 1,\ B_{bp}{}^{-}(x_2) = -1$ and

$A_{bp} \cap B_{bp} = 0_{bp}$ where 0_{bp} is a bipolar fuzzy null set in X.

$A_{bp} * 1_{bp} \in \mathfrak{B}_1 \times \mathfrak{B}_2$, since $A_{bp} \in \mathfrak{B}_1, 1_{bp} \in \mathfrak{B}_2$ and

$B_{bp} * 1_{bp} \in \mathfrak{B}_1 \times \mathfrak{B}_2$, since $B_{bp} \in \mathfrak{B}_1, 1_{bp} \in \mathfrak{B}_2$, where

$A_{bp} * 1_{bp} = (A_{bp}{}^{+} * 1_{bp}{}^{+}, A_{bp}{}^{-} * 1_{bp}{}^{-})$ and

$B_{bp} * 1_{bp} = (B_{bp}{}^{+} * 1_{bp}{}^{+}, B_{bp}{}^{-} * 1_{bp}{}^{-})$

Consider

$$\begin{aligned}(A_{bp}{}^{+} * 1_{bp}{}^{+})(x_1, y_1) &= \min\{A_{bp}{}^{+}(x_1), 1_{bp}{}^{+}(y_1)\}\\ &= \min\{1, 1\}\\ &= 1\end{aligned}$$

$$\begin{aligned}(A_{bp}{}^{-} * 1_{bp}{}^{-})(x_1, y_1) &= \min\{A_{bp}{}^{-}(x_1), 1_{bp}{}^{-}(y_1)\}\\ &= \min\{-1, -1\}\\ &= -1\end{aligned}$$

$$\begin{aligned}(B_{bp}{}^{+} * 1_{bp}{}^{+})(x_2, y_2) &= \min\{B_{bp}{}^{+}(x_2), 1_{bp}{}^{+}(y_2)\}\\ &= \min\{1, 1\}\\ &= 1\end{aligned}$$

$$(B_{bp}{}^{-} * 1_{bp}{}^{-})(x_2, y_2) = \min\{B_{bp}{}^{-}(x_2), 1_{bp}{}^{-}(y_2)\}$$
$$= \min\{-1, -1\}$$
$$= -1$$

Also

$A_{bp} \cap B_{bp} = 0_{bp}$

$\Rightarrow (A_{bp}{}^{+} \wedge B_{bp}{}^{+}, A_{bp}{}^{-} \vee B_{bp}{}^{-}) = 0_{bp}$

$\Rightarrow (A_{bp}{}^{+} \wedge B_{bp}{}^{+})(x) = 0^{+}(x)$ and $(A_{bp}{}^{-} \vee B_{bp}{}^{-})(x) = 0^{-}(x)$ for all $x \in X$

$\Rightarrow A_{bp}{}^{+}(x) \wedge B_{bp}{}^{+}(x) = 0^{+}(x)$ and $A_{bp}{}^{-}(x) \vee B_{bp}{}^{-}(x) = 0^{-}(x)$ for all $x \in X$

$\Rightarrow$ either $A_{bp}{}^{+}(x) = 0^{+}(x)$ or $B_{bp}{}^{+}(x) = 0^{+}(x)$ and either $A_{bp}{}^{-}(x) = 0^{-}(x)$ or $B_{bp}{}^{-}(x) = 0^{-}(x)$ for all $x \in X$

$\Rightarrow$ for a bipolar fuzzy whole set 1_{bp} in Y either $A_{bp}{}^{+}(x) \wedge 1_{bp}{}^{+}(y) = 0$ or $B_{bp}{}^{+}(x) \vee 1_{bp}{}^{+}(y) = 0$ and either

$A_{bp}{}^{-}(x) \vee 1_{bp}{}^{-}(y) = 0$ or $B_{bp}{}^{-}(x) \vee 1_{bp}{}^{-}(y) = 0$, for all $x \in X$ and $y \in Y$

$\Rightarrow$ either $(A_{bp}{}^{+} * 1_{bp}{}^{+})(x, y) = 0$ or $(B_{bp}{}^{+} * 1_{bp}{}^{+})(x, y) = 0$ and either

$(A_{bp}{}^{-} * 1_{bp}{}^{-})(x, y) = 0$ or $(B_{bp}{}^{-} * 1_{bp}{}^{-})(x, y) = 0$, for all $(x, y) \in X \times Y$

$\Rightarrow (A_{bp}{}^{+} * 1_{bp}{}^{+}) \wedge (B_{bp}{}^{+} * 1_{bp}{}^{+})(x, y) = 0$ and

$(A_{bp}{}^{-} * 1_{bp}{}^{-}) \wedge (B_{bp}{}^{-} * 1_{bp}{}^{-})(x, y) = 0$ for all $(x, y) \in X \times Y$

$\Rightarrow (A_{bp} * 1_{bp}) \cap (B_{bp} * 1_{bp}) = 0_{bp}$

Therefore product of two bipolar fuzzy W-Hausdorff spaces is bipolar fuzzy W-Hausdorff space.

Definition 3.7. Let $\{(X_{\lambda}, \mathfrak{B}_{\lambda}) \mid \lambda \in \Lambda\}$ be a family of bipolar fuzzy topological spaces and $X = \prod_{\lambda \epsilon \Lambda} X_\lambda$. Let $\{(A_{bp})_\lambda = ((A_{bp})_\lambda^+, (A_{bp})_\lambda^-) \mid \lambda \in \Lambda$ and $(A_{bp})_\lambda$ is a bipolar fuzzy set in $X_\lambda\}$. Then their *product* $\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda$ is a bipolar fuzzy set in $\prod_{\lambda \epsilon \Lambda} X_\lambda$ defined as

$$\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda = (\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^+, \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^-) \text{ where}$$

$$\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^+(x_\lambda) = \min \{(A_{bp})_\lambda^+(x_\lambda)\}, \text{ for all } x \in \prod x_\lambda \in X$$

$$\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^-(x_\lambda) = \max \{(A_{bp})_\lambda^-(x_\lambda)\}, \text{ for all } x \in \prod x_\lambda \in X$$

The product topology on X is the one with basic bipolar fuzzy open sets of the form $\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda$ where $(A_{bp})_\lambda \in \mathfrak{B}_\lambda$ and $(A_{bp})_\lambda = 1_{bp}$ except for finitely many λ's.

Theorem 3.8. *Arbitrary product of bipolar fuzzy W-Hausdorff spaces is bipolar fuzzy W-Hausdorff space.*

Proof. Let $\{(X_\lambda, \mathfrak{B}_\lambda) \mid \lambda \epsilon \Lambda\}$ be a collection of bipolar fuzzy W-Hausdorff spaces

Let $X = \prod_{\lambda \epsilon \Lambda} X_\lambda$.

Consider two distinct points $(x_\lambda)_{\lambda \epsilon \Lambda}, (y_\lambda)_{\lambda \epsilon \Lambda} \in \prod_{\lambda \epsilon \Lambda} X_\lambda$.

Assume $x_\mu \neq y_\mu$ for some $\mu \in \Lambda$. Therefore there exists two bipolar fuzzy open sets,

$(A_{bp})_\mu = ((A_{bp})_\mu^+, (A_{bp})_\mu^-)$ and $(B_{bp})_\mu = ((B_{bp})_\mu^+, (B_{bp})_\mu^-) \in \mathfrak{B}_\mu$ such that $(A_{bp})_\mu^+(x_\mu) = 1, (A_{bp})_\mu^-(x_\mu) = -1, (B_{bp})_\mu^+(y_\mu) = 1, (B_{bp})_\mu^-(y_\mu) = -1$ and $(A_{bp})_\mu \cap (B_{bp})_\mu = (0_{bp})_\mu$

Let $A_{bp} = \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda$, where $(A_{bp})_\lambda = (1_{bp})_\lambda$ for $\lambda \neq \mu$ and

$B_{bp} = \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda$, where $(B_{bp})_\lambda = (1_{bp})_\lambda$ for $\lambda \neq \mu$.

Then $A_{bp}, B_{bp} \in \prod_{\lambda \epsilon \Lambda} \mathfrak{B}_\lambda$

$A_{bp} = \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda = (\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{+}, \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{-})$ and

$B_{bp} = \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda = (\prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{+}, \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{-})$

$$\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{+}(x_\lambda) = \min\{(A_{bp})_\lambda^{+}(x_\lambda)\}, \text{ for all } \lambda \in \Lambda$$

$$= (A_{bp})_\mu^{+}(x_\mu), \text{ for some } \mu \in \Lambda$$

$$= 1$$

$$\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{-}(x_\lambda) = \max\{(A_{bp})_\lambda^{-}(x_\lambda)\}, \text{ for all } \lambda \in \Lambda$$

$$= (A_{bp})_\mu^{-}(x_\mu), \text{ for some } \mu \in \Lambda$$

$$= -1$$

$$\prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{+}(x_\lambda) = \min\{(B_{bp})_\lambda^{+}(x_\lambda)\}, \text{ for all } \lambda \in \Lambda$$

$$= (B_{bp})_\mu^{+}(x_\mu), \text{ for some } \mu \in \Lambda$$

$$= 1$$

$$\prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{-}(x_\lambda) = \max\{(B_{bp})_\lambda^{-}(x_\lambda)\}, \text{ for all } \lambda \in \Lambda$$

$$= (B_{bp})_\mu^{-}(x_\mu), \text{ for some } \mu \in \Lambda$$

$$= -1$$

Consider $A_{bp} \cap B_{bp} = \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda \cap \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda$

$$= (\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{+}, \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{-}) \cap (\prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{+}, \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{-})$$

$$= (\prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{+} \wedge \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{+}, \prod_{\lambda \epsilon \Lambda} (A_{bp})_\lambda^{-} \vee \prod_{\lambda \epsilon \Lambda} (B_{bp})_\lambda^{-})$$

Then

$(\prod_{\lambda\epsilon\Lambda}(A_{bp})_\lambda^{+} \wedge \prod_{\lambda\epsilon\Lambda}(B_{bp})_\lambda^{+})(x_\lambda)$

$= (\prod_{\lambda\epsilon\Lambda}(A_{bp})_\lambda^{+}(x_\lambda)) \wedge (\prod_{\lambda\epsilon\Lambda}(B_{bp})_\lambda^{+}(x_\lambda))$, for all $\lambda \in \Lambda$

$= (\min\{(A_{bp})_\lambda^{+}(x_\lambda)\}) \wedge (\min\{(B_{bp})_\lambda^{+}(x_\lambda)\})$, for all $\lambda \in \Lambda$

$= (A_{bp})_\mu^{+}(x_\mu) \wedge (B_{bp})_\mu^{+}(x_\mu)$

$= ((A_{bp})_\mu^{+} \wedge (B_{bp})_\mu^{+})(x_\mu)$

$= (0_{bp}{}^{+})_\mu(x_\mu)$

$= (0_{bp}{}^{+})_\lambda(x_\lambda)$

Therefore $(\prod_{\lambda\epsilon\Lambda}(A_{bp})_\lambda^{+} \wedge \prod_{\lambda\epsilon\Lambda}(B_{bp})_\lambda^{+}) = (0_{bp}{}^{+})_\lambda = 0^{+}$

$(\prod_{\lambda\epsilon\Lambda}(A_{bp})_\lambda^{-} \wedge \prod_{\lambda\epsilon\Lambda}(B_{bp})_\lambda^{-})(y_\lambda)$

$= (\prod_{\lambda\epsilon\Lambda}(A_{bp})_\lambda^{-}(y_\lambda)) \vee (\prod_{\lambda\epsilon\Lambda}(B_{bp})_\lambda^{-}(y_\lambda))$, for all $\lambda \in \Lambda$

$= (\min\{(A_{bp})_\lambda^{-}(y_\lambda)\}) \vee (\min\{(B_{bp})_\lambda^{-}(y_\lambda)\})$, for all $\lambda \in \Lambda$

$= (A_{bp})_\mu^{-}(y_\mu) \vee (B_{bp})_\mu^{-}(y_\mu)$

$= ((A_{bp})_\mu^{-} \vee (B_{bp})_\mu^{-})(y_\mu)$

$= (0_{bp}{}^{-})_\mu(y_\mu)$

$= (0_{bp}{}^{-})_\lambda(y_\lambda)$

Therefore $(\prod_{\lambda\epsilon\Lambda}(A_{bp})_\lambda^{-} \vee \prod_{\lambda\epsilon\Lambda}(B_{bp})_\lambda^{-}) = (0_{bp}{}^{-})_\lambda = 0^{-}$

$A_{bp} \cap B_{bp} = (0^{+}, 0^{-}) = 0_{bp}$

Therefore arbitrary product of bipolar fuzzy W-Hausdorff spaces is a bipolar fuzzy W-Hausdorff space.

4. Conclusion

In this paper, the concept of bipolar fuzzy W-Hausdorff space is introduced and some basic properties are proved.

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