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ON SOFT GENERALIZED ** CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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Abstract

In this paper we introduced a new class of soft sets called soft generalized** closed (briefly soft g^{**}-Closed) set and soft generalized** open (briefly soft g^{**}-Open) set in soft topological spaces. This new class is defined over an initial universe and with a fixed set of parameters. Moreover we discussed the relationship among soft g^{**}-closed set and some existing soft closed sets. And finally some basic properties of soft g^{**}-closed sets and soft g^{**}-open sets are investigated and studied.

I. Introduction

Soft set theory is a generalization of fuzzy set theory that was proposed by Molodtsov [1] in 1999 to deal with uncertainity in a parametric manner. A soft set is a parameterised family of sets intuitively this is "soft" because the boundary of the set depends on the parameters. The notion of topological space for soft sets was introduced by Muhammad Shabir and Munazza Naz in

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2011 [2]. And later the soft sets was extended and defined by Cagman et al. [3] in 2015. N. Levine introduced generalized closed sets in general topology [4]. Kannan [5] introduced soft generalized closed and open sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. In 2000 M. K. R. S. Veerakumar [6] defined Between closed and g-closed sets. M. Pauline Mary Helen, Veronica Vijayan and Ponnuthai selvarani defined and introduced g^{**}-closed sets in topological spaces [7]. A. Devika [8] defined soft g^{*}-closed sets in soft topological spaces. In 2016 A. Kalavathi and G. Sai Sundara Krishnan [9] introduced soft g^{*}-closed sets and soft g^{*}-open sets in soft topological spaces. In this paper we introduce soft g^{**}-closed sets and soft g^{**}-open sets in soft topological spaces and studied their properties. Also we discussed some of the applications.

II. Preliminaries

In this section, we have presented basic definitions of soft set theory which may be found in earlier. Let X be an initial universe and E be the set of parameters, $P(X)$ denote the power set of X and (F_A) denotes a soft set in a soft topological space. Throughout this paper $(F_A, \tilde{\tau})$ represents non-empty soft topological spaces on which no separation axioms are assumed.

Definition 2.1. Let X be a non-empty set. Let τ be a non-empty collection of subsets of X satisfying the following axioms

(i) X and $\emptyset \in \tau$.

(ii) The arbitrary union of sets in τ is in τ.

(iii) The finite intersection of sets in τ is in τ.

Then τ is called a topology on X and (X, τ) is called a topological space.

Definition 2.2. A set X for which a topology τ has been specified is called a topological space (X, τ). A topological space is an ordered pair (X, τ) consisting of a set X and a topology $\tilde{\tau}$ on X.

Definition 2.3. For a subset A of a topological space (X, τ) closure of A is defined as the intersection of all closed sets containing A. It is denoted by $cl(A)$ or $\overline{A}$.

Definition 2.4. For a subset A of a topological space (X, τ) interior of A is defined as the union of all open sets contained in A. It is denoted by $\text{int}(A)$ or A°.

Definition 2.5. A soft set F_A on the universe U is defined by the set of ordered pairs, E be the set of parameters and $A \subseteq E$, then $F_A = \{(x, f_A(x)) : x \in E\}$ where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here the value of $f_A(x)$ may be aribitary. Some of them may be empty some may have non-empty intersection.

Note that the set of all soft sets with the parameter set E over U will be denoted by $S(U)$.

Definition 2.6. Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in A$ then F_A called an empty soft set, denoted by $F_{\emptyset}$.

Definition 2.7. Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$ then F_A is called a A-universal soft set, denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition 2.8. Let $F_A, F_B \in S(U)$. Then soft union $F_A \mathbin{\tilde{\cup}} F_B$, Soft intersection $F_A \mathbin{\tilde{\cap}} F_B$, and soft difference $F_A \setminus F_B$ of F_A and F_B are defined by respectively.

$$f_{A \tilde{\cup} B}(x) = f_A(x) \cup f_B(x),\ f_{A \tilde{\cap} B}(x) = f_A(x) \cap f_B(x),\ f_{A \tilde{\setminus} B}(x) = f_A(x) \setminus f_B(x),$$

and the soft complement $F_A^{\tilde{c}}$ of F_A is defined by $f_A^{\tilde{c}}(x) = f_A^c$ where $f_A^c(x)$ is complement of the set $f_A(x)$, that is $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

Definition 2.9. Let $F_A \in S(U)$. The relative complement of F_A is denoted by F'_A and is defined by $(F_A)' = (F'_A)$ where $F'_A : A \to P(U)$ is a mapping given by $F'_{\propto} = U \setminus F_{\propto}$ for all $\propto \in A$.

Definition 2.10. Let $F_A \in S(U)$. A soft topology on F_A denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having following conditions.

(i) $F_A, F_\emptyset \in \tilde{\tau}$.

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

Then the pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Definition 2.11. Let $(F_A, \tilde{\tau})$ be a soft topological space, then every element of $\tilde{\tau}$ is called a soft open sets in $\tilde{\tau}$.

Definition 2.12. Let $(F_A, \tilde{\tau})$ be a soft topological space. A soft set F_A is said to be a soft closed set, if its relative complement F'_A belongs to $\tilde{\tau}$.

Definition 2.13. Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft interior of soft set F_A is defined as the union of all soft open sets contained in F_A. It is denoted by $\text{int}(F_A)$.

Definition 2.14. Let $(F_A, \tilde{\tau})$ be a soft topological space, then soft closure of soft set F_A is defined as the intersection of all soft closed super sets containing in F_A. It is denoted by $cl(F_A)$.

Definition 2.15. A Subset A of a topological space (X, τ) is called a

(1) Semi open set if $A \subseteq cl(\text{int}(A))$ and a semi closed set if $\text{int}(cl(A)) \subseteq A$.

(2) Pre open set if $A \subseteq \text{int}(cl(A))$ and a pre closed set if $cl(\text{int}(A)) \subseteq A$.

(3) α-open set if $A \subseteq \text{int}(cl(\text{int}(A)))$ and a α-closed set if $cl(\text{int}(cl(A))) \subseteq A$.

(4) β-open set if $A \subseteq cl(\text{int}(cl(A)))$ and a β-closed set if $\text{int}(cl(\text{int}(A))) \subseteq A$.

(5) Regular open set if $A = \text{int}(cl(A))$ and a regular closed set if $A = cl(\text{int}(A))$.

(6) Generalized closed (briefly g-closed) set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

(7) Semi generalized closed (briefly g-closed) set if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U_A is semi open in (X, τ).

(8) Generalized ^ closed (briefly $\hat{g}$-closed) set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U_A is semi open in (X, τ).

(9) Generalized * closed (briefly g^*-closed) set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U_A is g-open in (X, τ).

(10) Regular generalized closed (briefly rg-closed) set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open in (X, τ).

(11) α-generalized closed (briefly αg-closed) set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

(12) Weekly generalized closed (briefly wg-closed) set if $cl(\text{int}(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

(13) Generalized semi pre closed (briefly gsp-closed) set if $spcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

(14) Generalized pre regular closed (briefly gpr-closed) set if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open in (X, τ).

(15) Generalized **-closed (briefly g^{**}-closed) set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g^*-open in (X, τ).

The relative complement of the above closed sets is called open sets respectively.

Definition 2.16. Let $(F_A, \tilde{\tau})$ be a soft topological space, a soft set F_A over U is called a

(1) Soft semi open set if $F_A \subseteq cl(\text{int}(F_A))$ and a semi closed set if $\text{int}(cl(F_A)) \subseteq F_A$.

(2) Soft pre open set if $F_A \subseteq \text{int}(cl(F_A))$ and a pre closed set if $cl(\text{int}(F_A)) \subseteq F_A$.

(3) Soft α-open set if $F_A \subseteq \text{int}(cl(\text{int}(F_A)))$ and a α-closed set if $cl(\text{int}(cl(F_A))) \subseteq F_A$.

(4) Soft β-open set if $F_A \subseteq cl(\text{int}(cl(F_A)))$ and a β-closed set if $\text{int}(cl(\text{int}(F_A))) \subseteq F_A$.

(5) Soft regular open set if $F_A = \text{int}(cl(F_A))$ and a regular closed set if $F_A = cl(\text{int}(F_A))$.

(6) Soft generalized closed (briefly soft g-closed) set if $cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

(7) Soft semi generalized closed (briefly soft g-closed) set if $scl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft semi open in $(F_A, \tilde{\tau})$.

(8) Soft generalized ^ closed (briefly soft $\hat{g}$-closed) set if $cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft semi open in $(F_A, \tilde{\tau})$.

(9) Soft generalized * closed (briefly soft g^*-closed) set if $cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft g-open in $(F_A, \tilde{\tau})$.

(10) Soft regular generalized closed (briefly soft rg-closed) set if $cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft regular open in $(F_A, \tilde{\tau})$.

(11) Soft α-generalized closed (briefly α soft g-closed) set if $\alpha cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

(12) Soft weekly generalized closed (briefly soft wg-closed) set if $cl(\text{int}(F_A)) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

(13) Soft generalized semi pre closed (briefly soft gsp-closed) set if $spcl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft open in $(F_A, \tilde{\tau})$.

(14) Soft generalized pre regular closed (briefly soft gpr-closed) set if $pcl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft regular open in $(F_A, \tilde{\tau})$.

The relative complement of the above soft closed sets are called Soft open sets respectively.

III. Soft g^{**} Closed Sets

Definition 3.1. A Soft set F_A is called a soft generalized ** closed (briefly soft g^{**}-closed) set if $cl(F_A) \subseteq U_A$, whenever $F_A \subseteq U_A$ and U_A is soft g^*-open in $(F_A, \widetilde{\tau})$.

Example 3.2. Let $U = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$

$F_A = \{(e_1, \{a, b\}), (e_2, \{b, c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_2, \{b\})\}$

$F_{A_5} = \{(e_2, \{c\})\}$

$F_{A_6} = \{(e_2, \{b, c\})\}$

$F_{A_7} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{A_8} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_9} = \{(e_1, \{a\}), (e_2, \{b, c\})\}$

$F_{A_{10}} = \{(e_1, \{b\}), (e_2, \{b\})\}$

$F_{A_{11}} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{A_{11}} = \{(e_1, \{b\}), (e_2, \{c\})\}$

$F_{A_{12}} = \{(e_1, \{b\}), (e_2, \{b, c\})\}$

$F_{A_{13}} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$

$F_{A_{14}} = \{(e_1, \{a, b\}), (e_2, \{c\})\}$

$F_{A_{15}} = F_A$

$F_{A_{16}} = F_\emptyset$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_9}, F_{A_6}, F_{A_7}, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}, F_{A_6}, F_{A_7}, F_{A_8}, F_{A_9}\}$

Theorem 3.3. *Every soft closed set is soft g^{**}-closed set.*

Proof. Let F_A be a soft closed set in $(F_A, \tilde{\tau}), \Rightarrow cl(F_A) = F_A$ and let U_A be a soft g^*-open set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since F_A is soft closed set, $\Rightarrow cl(F_A) = F_A \subseteq U_A$. Therefore $cl(F_A) \subseteq U_A$ and U_A is soft g^*-open set in $(F_A, \tilde{\tau})$. Hence F_A is soft g^{**}-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.4. Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_1, \{a\}), (e_2, \{d\})\}$

$F_{A_5} = \{(e_1, \{b\}), (e_2, \{d\})\}$

$F_{A_6} = \{(e_2, \{d\})\}$

$F_{A_7} = F_A$

$F_{A_8} = F_{\emptyset}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_{\emptyset}, F_{A_5}, F_{A_2}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_{\emptyset}, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_{\emptyset}, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_{\emptyset}, F_{A_2}, F_{A_5}, F_{A_6}\}$

Here $F_{A_3} = \{(e_1, \{a, b\})\}$ is soft g^{**}-closed set but not a soft closed set.

Theorem 3.5. *Every soft g^{**}-closed set is soft rg-closed set.*

Proof. Let F_A be a soft g^{**}-closed set in $(F_A, \tilde{\tau}), \Rightarrow cl(F_A) = F_A$ and let U_A be a soft regular open set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since every regular open set is g^*-open set, we have $cl(F_A) \subseteq U_A$ and U_A is soft regular open set in $(F_A, \tilde{\tau})$. Hence F_A is soft *rg*-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a\}), (e_2, \{b, c\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{a\}), (e_2, \{b\})\}$

$F_{A_3} = \{(e_1, \{a\}), (e_2, \{c\})\}$

$F_{A_4} = F_A$

$F_{A_5} = \{(e_1, \{b\})\}$

$F_{A_6} = \{(e_2, \{c\})\}$

$F_{A_7} = \{(e_2, \{b, c\})\}$

$F_{A_8} = F_{\emptyset}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_{\emptyset}, F_{A_2}, F_{A_7}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_{\emptyset}, F_{A_1}, F_{A_3}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $(\tilde{\tau}) = \{F_A, F_{\emptyset}, F_{A_1}, F_{A_3}, F_{A_6}\}$

Soft g^{**}-open sets are $(\tilde{\tau}) = \{F_A, F_{\emptyset}, F_{A_2}, F_{A_5}, F_{A_7}\}$

Let $F_A = F_{A_2}$ be a soft *rg*-closed set but it is not a soft g^{**}-closed set.

Since $F_{A_2} = \{(e_1, \{a\}), (e_2, \{b\})\}$, $cl(F_{A_2}) = F_A \Rightarrow F_A \not\subset F_{A_2}$. Therefore F_{A_2} is not a soft g^{**} closed set.

Theorem 3.7. *Every soft g^{**}-closed set is weakly soft g-closed set.*

Proof. Let F_A be a soft g^{**}-closed set in $(F_A, \tilde{\tau}) \Rightarrow cl(F_A) = F_A$ and let U_A be a softopen set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since every open set is g^*-open set, we have $cl(F_A) \subseteq U_A$. But $cl(\text{int}(F_A)) \subseteq cl(F_A) \subseteq U_A \Rightarrow$ $cl(\text{int}(F_A)) \subseteq U_A$ and U_A is soft open set in $(F_A, \tilde{\tau})$. Hence F_A is soft weakly soft gclosed set.

Remark 3.8. The converse of the above theorem need not be true as seen from the above example 3.6

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{b, c\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_7}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_6}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_7}\}$

Let $F_A = F_{A_5}$ be a weakly soft g-closed set but it is not a soft g^{**}-closed set.

Since $F_{A_5} = \{(e_2, \{b\})\}$, $cl(F_{A_5}) = F_A \Rightarrow F_A \not\subset F_{A_5}$. Therefore F_{A_5} is not a soft g^{**} closed set.

Theorem 3.9. *Every soft g^{**}-closed set is soft gpr-closed set.*

Proof. Let F_A be a soft g^{**}-closed set in $(F_A, \tilde{\tau}) \Rightarrow cl(F_A) = F_A$ and let U_A be a soft regular open set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since every regular open set is g^*-open set, we have $cl(F_A) \subseteq U_A$. But $pcl(F_A) \subseteq cl(F_A) \subseteq U_A \Rightarrow pcl(F_A) \subseteq U_A$ and U_A is soft regular open set in $(F_A, \tilde{\tau})$. Hence F_A is soft *gpr*-closed set.

Remark 3.10. The converse of the above theorem need not be true as seen from the above example 3.6

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{b, c\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_7}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_6}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_7}\}$

Let $F_A = F_{A_2}$ be a soft gpr-closed set but it is not a soft g^{**}-closed set.

Since $F_{A_7} = \{(e_2, \{b, c\})\}, cl(F_{A_7}) = F_A \Rightarrow F_A \not\subset F_{A_7}$. Therefore F_{A_7} is not a soft g^{**} closed set.

Theorem 3.11. *Every soft g^{**}-closed set is soft αg-closed set*

Proof. Let F_A be a soft g^{**}-closed set in $(F_A, \tilde{\tau}) \Rightarrow cl(F_A) = F_A$ and let U_A be a soft open set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since ever open set is g^*-open set, we have $cl(F_A) \subseteq U_A$. But $\alpha cl(F_A) \subseteq cl(F_A) \subseteq U_A \Rightarrow \alpha cl(F_A) \subseteq U_A$ and U_A is soft open set in $(F_A, \tilde{\tau})$. Hence F_A is soft αg-closed set.

Remark 3.12. The converse of the above theorem need not be true as seen from the above example 3.4

Let $U = \{a, b, c, d\}, E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_6}\}$

Let $F_A = F_{A_6}$ be a soft αg-closed set but it is not a soft g^{**}-closed set.

Since $F_{A_6} = \{(e_2, \{d\})\}, cl(F_{A_6}) = F_{A_4} \Rightarrow F_{A_4} \not\subset F_{A_6}$. Therefore F_{A_6} is not a soft g^{**}-closed set.

Theorem 3.13. *Every soft g^{**}-closed set is soft gsp-closed set*

Proof. Let F_A be a soft g^{**}-closed set in $(F_A, \tilde{\tau}) \Rightarrow cl(F_A) = F_A$ and let U_A be a soft open set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since ever open set is g^*-open set, we have $cl(F_A) \subseteq U_A$. But $spcl(F_A) \subseteq cl(F_A) \subseteq U_A \Rightarrow spcl(F_A) \subseteq U_A$ and U_A is soft open set in $(F_A, \tilde{\tau})$. Hence F_A is soft gsp-closed set.

Remark 3.14. The converse of the above theorem need not be true as seen from the above example 3.4

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_6}\}$

Let $F_A = F_{A_6}$ be a soft gsp-closed set but it is not a soft g^{**}-closed set.

Since $F_{A_6} = \{(e_2, \{d\})\}$, $cl(F_{A_6}) = F_{A_4} \Rightarrow F_{A_4} \not\subset F_{A_6}$. Therefore F_{A_6} is not a soft g^{**}-closed set.

Theorem 3.15. *Every soft g^{**}-closed set is soft gp-closed set.*

Proof. Let F_A be a soft g^{**}-closed set in $(F_A, \tilde{\tau}) \Rightarrow cl(F_A) = F_A$ and let U_A be a soft open set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since ever open set is g^*-open set, we have $cl(F_A) \subseteq U_A$. But $pcl(F_A) \subseteq cl(F_A) \subseteq U_A \Rightarrow pcl(F_A) \subseteq U_A$ and U_A is soft open set in $(F_A, \tilde{\tau})$. Hence F_A is soft gp-closed set.

Remark 3.16. The converse of the above theorem need not be true as seen from the above example 3.4

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_6}\}$

Let $F_A = F_{A_5}$ be a soft *gp*-closed set but it is not a soft g^{**}-closed set.

Since $F_{A_5} = \{(e_1, \{b\}), (e_2, \{d\})\}$, $cl(F_{A_5}) = F_{A_5} \Rightarrow F_{A_5} \not\subset F_{A_6}$. Therefore F_{A_5} is not a soft g^{**} closed set.

Theorem 3.17. *If F_A is soft g^{**}-closed set in $(F_A, \tilde{\tau})$ and $F_A \subseteq G_A \subseteq cl(F_A)$ then G_A is soft g^{**}-closed set.*

Proof. Suppose that F_A is soft g^{**}-closed set in $(F_A, \tilde{\tau})$ and $F_A \subseteq G_A \subseteq cl(F_A)$. Let $G_A \subseteq U_A$ and U_A is soft g^{**}-open set in X. Since $F_A \subseteq G_A$ and $G_A \subseteq U_A \Rightarrow F_A \subseteq U_A$ Hence $cl(F_A) \subseteq (U_A)$ [Since F_A is soft g^{**}-closed]. $\Rightarrow G_A \subseteq cl(F_A)$. Taking soft closure on both sides, $cl(G_A) \subseteq cl(cl(F_A))$, $\Rightarrow cl(G_A) \subseteq F_A$ [since $F_A \subseteq U_A$] $\Rightarrow G_A \subseteq U_A$ and U_A is soft g^{*} -open set in $(F_A, \tilde{\tau})$. Therefore G_A is soft g^{**}-closed set.

Theorem 3.18. *If F_A and G_A are soft g^{**}-closed set, then $F_A \cup G_A$ is soft g^{**}-closed set.*

Proof. Suppose that F_A and G_A are soft g^{**}-closed sets. Let $F_A \cup G_A \subseteq U_A$ is soft g^{*}-open set. $\Rightarrow F_A \subseteq U_A$ and $G_A \subseteq U_A$. We know that U_A is soft g^* -open set in $(F_A, \tilde{\tau})$ and F_A and G_A are soft g^{**}-closed sets $\Rightarrow cl(F_A) \subseteq U_A$ and $cl(F_A) \subseteq U_A$. Therefore $cl\{(F_A \cup G_A)\} = cl(F_A) \cup cl(G_A) \subseteq cl(F_A) \cup cl(G_A) \subseteq U_A$.

Theorem 3.19. *If a set F_A is soft g^{**}-closed set in $(F_A, \tilde{\tau})$ if and only if $cl(F_A) \setminus (F_A)$ contains only null soft closed set.*

Proof. Suppose that F_A is soft g^{**}-closed set in $(F_A, \tilde{\tau})$. Let G_A be soft closed set and $G_A \subseteq cl(F_A) \setminus (F_A)$. Since G_A is soft closed and its relative complement G'_A is soft open. Now $G_A \subseteq cl(F_A) \setminus (F_A) \Rightarrow G_A \subseteq cl(F_A)$ and $G_A \not\subset F_A \Rightarrow G_A \subseteq cl(F_A)$ and $G_A \subseteq F'_A$. Hence $F_A \subseteq G'_A$ consequently $cl(F_A) \subseteq G'_A h$. [Since F_A is soft g^{**}-closed set]. Therefore $G_A \subseteq cl(F'_A) \Rightarrow G_A = cl(F_A) \cap (F'_A) = \emptyset. \Rightarrow G_A = \emptyset$. Therefore $cl(F_A) \setminus (F_A)$ contains only null soft closed set.

Conversely,

$cl(F_A) \setminus (F_A)$ contains only null soft closed set.

Since $cl(F_A) \setminus (F_A) = \emptyset \Rightarrow cl(F_A) = F_A \Rightarrow F_A$ is soft closed set. We know that every soft closed set is soft g^{**}-closed set. Therefore F_A is soft g^{**}-closed set.

Theorem 3.20. *A soft g^{**}-closed set F_A is soft closed if and only if $cl(F_A) \setminus (F_A)$ is soft closed.*

Proof. If F_A is soft closed. $\Rightarrow cl(F_A) = F_A$. Therefore $cl(F_A) \setminus (F_A) = \emptyset$ and is soft closed set.

Conversely, Suppose that $cl(F_A) \setminus (F_A)$ is soft closed, Now by theorem 3.19 since F_A is soft g^{**}-closed. $cl(F_A) \setminus (F_A) = \emptyset$. Therefore F_A is soft closed.

Remark 3.21. The intersection of any two g^{**}-closed sets in a soft topological space $(F_A, \tilde{\tau})$ is a soft g^{**}-closed set.

The above remark was proved by the previous example 3.2

$F_{A_4} = \{(e_2, \{b\})\}$ is a soft g^{**}-closed set and $F_{A_6} = \{(e_2, \{b, c\})\}$ is a soft g^{**}-closed set, then $F_{A_4} \cap F_{A_6} = \{(e_2, \{b\})\}$ is also a soft g^{**}-closed set.

IV - Soft g^{**}-Open Sets

Definition 4.1. A soft set F_A is called a soft generalized** open (briefly soft g^{**}-open) set in a soft topological space $(F_A, \tilde{\tau})$, if the relative complement F'_A is soft g^{**}-closed set in $(F_A, \tilde{\tau})$.

Equivalently, A soft set F_A is g^{**}-open set, if $U_A \subseteq Int(F_A)$ whenever $U_A \subseteq F_A$ and U_A is soft g^*-closed set.

Example 4.2.

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_6}\}$

Theorem 4.3. *Every soft open set is soft g^{**}-open set.*

Proof. Let F_A be a soft open set in $(F_A, \tilde{\tau})$ and let U_A be a soft g^*-closed set in $(F_A, \tilde{\tau})$ such that $F_A \subseteq U_A$. Since F_A is soft open set, $\text{int}(F_A) = F_A \Rightarrow \text{int}(F_A) = F_A \subseteq U_A$. Therefore $\text{int}(F_A) \subseteq U_A$ and U_A is soft g^*-closed set in $(F_A, \tilde{\tau})$. Hence (F_A) is soft g^{**}-open set.

Remark 4.4. The converse of the above theorem need not be true as seen from the previous example 3.2

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{b, c\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_9}, F_{A_6}, F_{A_7}, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Let $(F_{A_5}) = \{(e_2, \{c\})\}$ is soft g^{**}-open set but not a soft open set.

Theorem 4.5. *If F_A is soft g^{**}-open set in $(F_A, \tilde{\tau})$ and $\text{int}(F_A) \subseteq G_A \subseteq F_A$, then G_A is soft g^{**}-open set.*

Proof. Suppose that F_A is soft g^{**}-open set in $(F_A, \tilde{\tau})$ and $\text{int}(F_A) \subseteq G_A \subseteq F_A$. Let $H_A \subseteq G_A$ and H_A is soft g^*-closed set in $(F_A, \tilde{\tau})$. Since $G_A \subseteq F_A$ and $H_A \subseteq G_A$, we have $H_A \subseteq F_A$. since F_A is soft g^{**}- open. $\Rightarrow H_A \subseteq \text{int}(F_A)$, also since $\text{int}(F_A) \subseteq G_A \Rightarrow H_A \subseteq \text{int}(F_A) \subseteq \text{int}(G_A) \Rightarrow H_A \subseteq \text{int}(G_A)$.

Therefore G_A is soft g^{**}-open set.

Theorem 4.6. *If F_A and G_A are soft g^{**}-open set, then $(F_A \cap G_A)$ is soft g^{**}-open set.*

Proof. Suppose that F_A and G_A are soft g^{**}-open sets. Let $U_A \subseteq F_A \cap G_A$ and U_A is soft g^*-closed set in $(F_A, \tilde{\tau})$. Since $U_A \subseteq F_A \cap G_A, \Rightarrow U_A \subseteq F_A$ and $U_G \Rightarrow G_A$. We know that U_A is soft g^*-closed set in $(F_A, \tilde{\tau})$ and F_A and G_A are soft g^{**}-open sets $\Rightarrow U_A \subseteq \text{int}(F_A)$ and $U_A \subseteq \text{int}(G_A)$. Therefore $U_A \subseteq \text{int}(F_A) \cap \text{int}(G_A)$.

Definition 4.7. Let A $(F_A, \tilde{\tau})$ be a soft topological space, $F_B \subseteq F_A$ and $\alpha \in F_B$. If there exists a soft g^{**}- open set F_C such that $\alpha \in F_C \subseteq F_B$ then

α is called a soft g^{**}-interior point of F_B and the soft union of all soft g^{**}-interior points of F_B is denoted by $\text{int}(F_B)$.

Definition 4.8. Let A $(F_A, \tilde{\tau})$ be a soft topological space, $F_B \subseteq F_A$ and $\alpha \in F_A$. If there exists a soft g^{**}-open set F_C such that $\alpha \in F_C \subseteq F_B$ then F_B is called a soft g^{**}-neighbourhood of α. Set of all soft g^{**}-neighbourhoods of α is denoted by $N(\alpha)$, is called family of soft g^{**}-neighbourhoods of α that is $N(\alpha) = \{F_B : F_C \in \tilde{\tau}\}$ and $\alpha \in F_C \subseteq F_B$. In particular, $V(\alpha) = \{F_C \in \tilde{\tau} : \alpha \in F_C\}$.

Example 4.9.

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

$F_{A_1} = \{(e_1, \{a\})\}$

$F_{A_2} = \{(e_1, \{b\})\}$

$F_{A_3} = \{(e_1, \{a, b\})\}$

$F_{A_4} = \{(e_1, \{a\}), (e_2, \{d\})\}$

$F_{A_5} = \{(e_1, \{b\}), (e_2, \{d\})\}$

$F_{A_6} = \{(e_2, \{d\})\}$

$F_{A_7} = F_A$

$F_{A_8} = F_\emptyset$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_5}, F_{A_2}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_6}\}$

Let (i) $\alpha_1 = \{(e_1, \{a\})\}$. Then soft g^{**}-neighbourhoods $\tilde{N}(\alpha_1) = \{F_A\}$ and soft g^{**}-open neighbourhoods $\tilde{V}(\alpha_1) = \{F_A\}$.

(ii) $\alpha_2 = \{(e_1, \{b\})\}$. Then soft g^{**}-neighbourhoods $\tilde{N}(\alpha_2) = \{F_A, F_{A_3}, F_{A_5}\}$ and soft g^{**}-open neighbourhoods $\tilde{V}(\alpha_2) = \{F_A\}$.

(iii) $\alpha_3 = \{(e_1, \{a, b\})\}$. Then soft g^{**}-neighbourhoods $\tilde{N}(\alpha_3) = \{F_A\}$ and soft g^{**}-open neighbourhoods $\tilde{V}(\alpha_3) = \{F_A\}$.

(iv) $\alpha_4 = \{(e_1, \{a\}), (e_2, \{d\})\}$. Then soft g^{**}-neighbourhoods $\tilde{N}(\alpha_4) = \{F_A\}$ and soft g^{**}-open neighbourhoods $\tilde{V}(\alpha_4) = \{F_A\}$.

(v) $\alpha_5 = \{(e_1, \{b\}), (e_2, \{d\})\}$. Then soft g^{**}-neighbourhoods $\tilde{N}(\alpha_5) = \{F_A\}$ and soft g^{**}-open neighbourhoods $\tilde{V}(\alpha_5) = \{F_A\}$.

(vi) $\alpha_6 = \{(e_2, \{d\})\}$. Then soft g^{**}-neighbourhoods $\tilde{N}(\alpha_6) = \{F_A, F_{A_5}\}$ and soft g^{**}-open neighbourhoods $\tilde{V}(\alpha_6) = \{F_A, F_{A_5}\}$.

Remark 4.10. A soft g^{**}-neighbourhood generally need not be soft g^{**}-open set as shown in the above example (4.9)

Let $U = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subseteq E$.

$F_A = \{(e_1, \{a, b\}), (e_2, \{d\})\}$

Soft open sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_5}, F_{A_2}\}$

Soft closed sets $(\tilde{\tau}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_4}\}$

Then $(F_A, \tilde{\tau})$ is a soft topological space, and

Soft g^{**}-closed sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}\}$

Soft g^{**}-open sets are $\tilde{\tau} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_6}\}$

$\alpha = F_{A_2}\{(e_1, \{b\})\}$. Then soft g^{**}-neighbourhoods $N(\alpha) = \{F_A, F_{A_3}, F_{A_5}\}$ but it is not a soft g^{**}-open set because F_{A_3} is not a soft g^{**}-open set.

Theorem 4.11. *Every soft neighbourhood is a soft g^{**}-neighbourhood.*

Proof. Let α be a soft neighbourhood of a soft set $F_B \subseteq (F_A, \tilde{\tau})$, then there exists a soft open set F_C such that $\Rightarrow \alpha \in F_C \subseteq F_B$. As we know that Every soft open set is soft g^{**}-open set such that $\alpha \in F_C \subseteq F_B$. Hence α is soft g^{**}-neighbourhood of F_A.

V - Conclusion

In the present paper, we have introduced the notions of soft g^{**}-Closed sets, soft g^{**} -open sets, soft g^{**}-interior, soft g^{**}-neighbourhood and their basic properties are investigated. In extension of this work, using this new soft set we will introduce soft continuous function, soft homeomorphism, soft separation axioms. Also in future, we would try to use this new soft set in decision making problems as an application.

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