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A REVIEW ON NUMERICAL AND WAVELET METHOD BASED ALGORITHM FOR THE SOLUTION OF COMPUTATIONAL PROBLEM IN THE AREA OF ELECTROMAGNETICS

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Abstract

A lot of papers based on numerical techniques used in the area of computational electromagnetic have been studied. Based on the study, a flow chart of the algorithm used by Euler's method, runga-kutta method and taylor series in the solution of problems has been presented in this paper. Besides these, wavelet methods for solving electromagnetic problems have also been discussed in this paper. Because now a days a very fast method i.e. wavelet is being used in solving computational problems based on differential/integral/integro-differential equation.

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1. Introduction

Differential equations (DEs) are often used to model electromagnetic field phenomena. These are useful approaches for describing the natural occurrences in scientific and engineering models. DEs are used in physics to describe heat transfer and wave propagation, for instance. As a result of this, many engineering applications are mathematically modelled as DEs with beginning and limit conditions. In nuclear reactor dynamics, population models, dispersion of a chemically reactive material, and many other physical phenomena, DEs are used [1-8].

A wide range of engineering issues were analyzed in order to develop DEs. In the second part of the 20th century, linear and nonlinear differential equations (DEs) had their foundations well established. The study of DE issues is being conducted by scientists and mathematicians who are actively interested in their research. In particular, it plays a major role in applied mathematics, mathematics, and electromagnetics in the 21st century. Determining the solution to linear/non-linear issues has been done using a number of methods, including the system of characteristics, spectrum methods, and disruption techniques. For linear and nonlinear DEs, however, there is no universal approach. The solution of these equations requires novel numerical methods. As a result, all traditional approaches for solving DEs and the application of these methods are becoming more and more essential [9-19].

In electromagnetism, differentiable forms are closely related to geometric images and give a more spatial perspective. Electromagnetic theory is combining physical, statistical, and geometrical ideas. In today's fast-paced environment, geometric model's visualization capabilities encourage new ideas and concepts. After the development of differential forms notation or external calculus, the relationship between mathematical and geometric knowledge of field and source quantities was well described in electromagnetic issues literature [20-29].

Calculus, trigonometry and other classical techniques should be used to determine the behavior of differential equations. Different circumstances can be used to understand the differential equations activity in relation to these solutions. Empirical approaches are employed to determine the precise answer. Even still, it's limited to the most fundamental computations. And therefore it is exceedingly difficult to obtain exact solutions to differential equations of higher order with complex coefficients. Numerical methods are required to solve these equations. For ordinary differential equations, there exist unique initial-limit value issues in a number of different fields. As a result, many original and constrained value issues cannot be solved only by computational methods. Even while collocating approaches based on wavelets are highly successful numerically, collocating is one of the most commonly utilised numerical schemes for solving differential equations in these circumstances [30-42].

On the one hand, the Euler's technique, Runge-Kutta methods and Taylor Expansions are discussed, as well as the Haar Wavelet and Legendre Wavelet methods, the Chebyshev Wavelet method, Hermite Wavelet method and the Petrov-Galerkin method. A total of two parts comprise the whole document. First, we looked at numerical approaches' algorithms, and then we looked at wavelet methods. Organization of this paper is such a way that section 2 discusses about different numerical method algorithm for solving computational problems. Section 3 tells about wavelet methods for solving computational problems. Discussion and conclusion is given section 4.

2. Numerical Methods

Numeral methods are mathematical tools used in numerical analysis. As a computer language, numerical algorithm refers to the implementation of an acceptable numerical technique with a convergence check. Methods for approximating mathematical operations (example of a mathematical procedure is an integral). As a result of inability to solve the technique analytically (such as the standard normal cumulative distribution function) or intractability of the analytical method, we must make approximations (example is solving a set of a thousand simultaneous linear equations for a thousand unknowns for finding forces in a truss) [42-44].

In mathematics and computer science, numerical analysis is the process of creating, analyzing and implementing methods for numerically solving problems in continuous mathematics. As a result of real-world algebra, geometry, and calculus applications in the actual world, these issues feature variables that are constantly changing. Problems of this nature can be found in all fields of scientific and social sciences, as well as in medical, engineering and business. A dramatic increase in the capacity of digital computers in science, health, engineering, and business began in the 1940s, and numerical analysis of increasing sophistication has been required to solve these increasingly precise and sophisticated mathematical models. The formal academic subject of numerical analysis ranges from highly theoretical mathematical research to computer science concerns relating to the influence of computer hardware and software on the execution of certain algorithms, among other things. Figure 1 illustrates the types of numerical approaches that are available in the literature for solving mathematical problems [45-47].

2.1. Euler's Method

Euler's Method is a technique for analyzing differential equations that relies on the notion of local linearity or linear approximation. The Euler method is a first-order numerical approach for solving ordinary differential equations (ODEs) with a given starting value in mathematics and computer science. As seen in Figure 2, the algorithm is described [48-49].

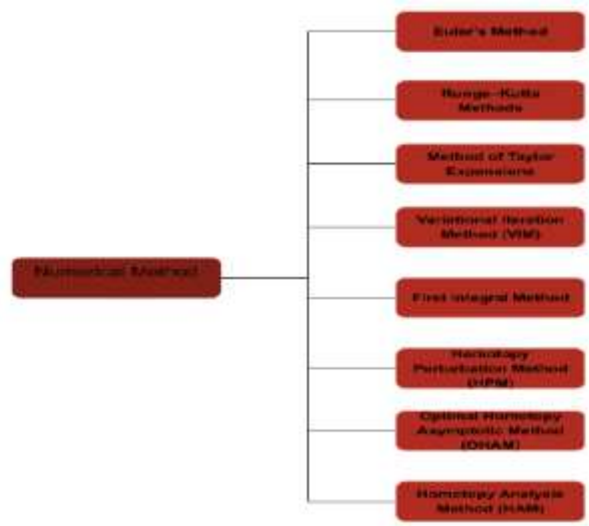

Figure 1. Types of Numerical methods.

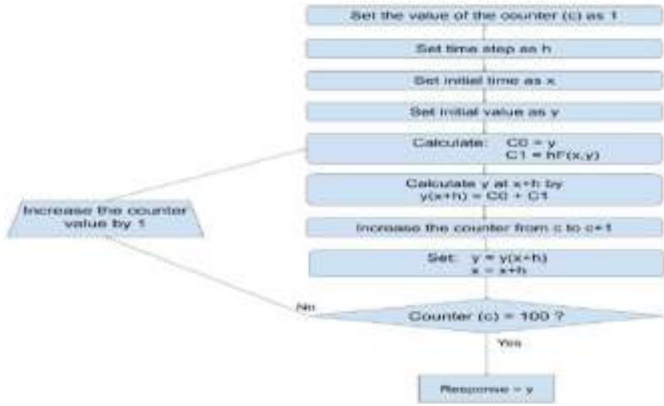

Figure 2. Euler's methods [50-51].

2.2. Runge-Kutta Methods

In differential equations, the Runge-Kutta technique is an effective and commonly used approach for addressing initial-value issues. A high order numerical technique may be constructed using the Runge-Kutta method without the necessity for high order derivatives. Figure 3 and 4 discusses the Runge-Kutta algorithm for 1st and 2nd order differential equation based computational problem [52-53].

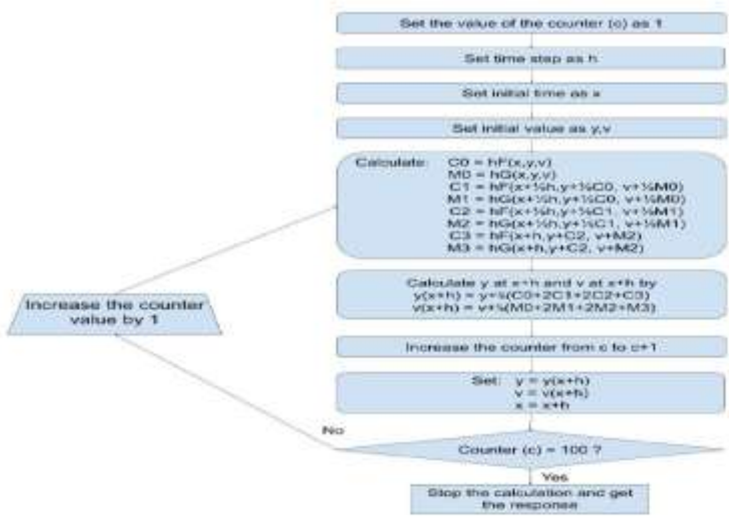

Figure 3. Algorithm for 1st order equation (Runge – Kutta Method) [53-54].

2.3. Taylor Expansion Method

An approximation Taylor series uses simpler polynomial functions to approximate a complex function. To represent complex (yet well-behaved) functions, a sequence of rising powers is used. Figure 5 and 6 tells about the taylor expansion method based algorithm for computational problem [55-58].

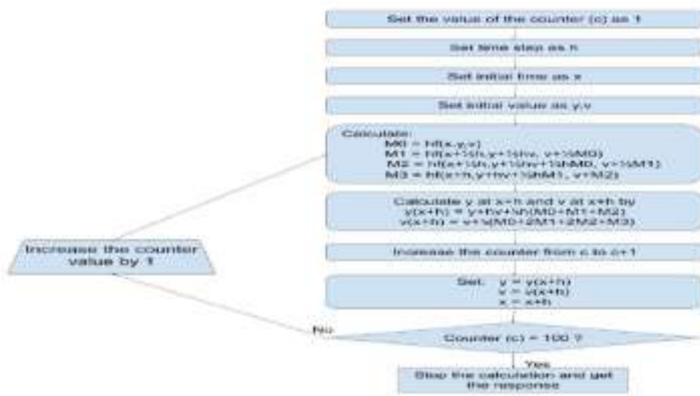

Figure 4. Algorithm for 2nd order equation (Runge – Kutta Method) [51-56].

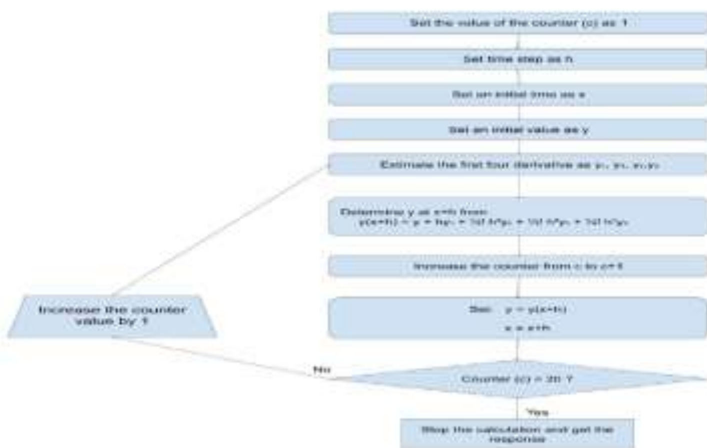

Figure 5. Taylor Program [51-56].

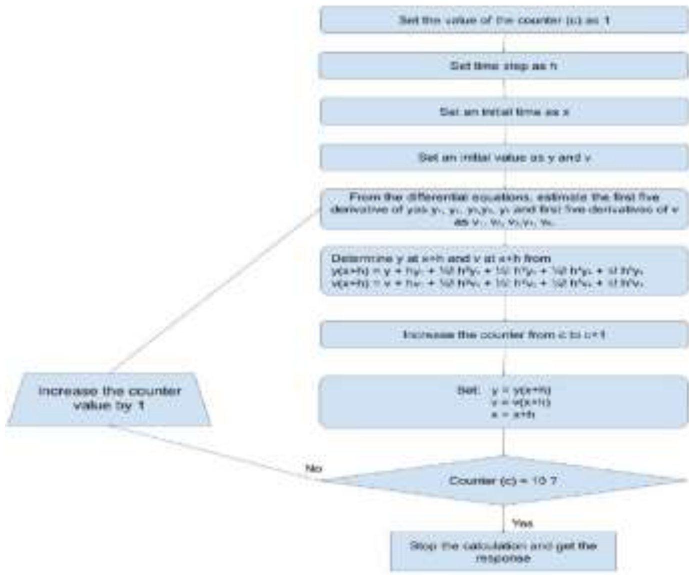

Figure 6. Taylor system Program [51-59].

3. Wavelets

Localized waves or tiny waves are called wavelets. Instead of continuing to oscillate indefinitely, they come to a halt. As a novel area of mathematical research, wavelets theory is gaining popularity. Applications include signal analysis for wave form representation and segmentation as well as temporal frequency analysis and harmonic analysis, among other technical fields. Many functions and operators may be accurately represented with wavelets. Wavelets are considered to be continuous-time basis functions $\psi_{i,j}(x)$. Basis is a collection of linearly independent functions that may be used to create all acceptable functions, such as $f(t)$, by combining them. The wavelet basis has the unique property that all functions $\psi_{i,j}(x)$ are built from a single function termed the mother wavelet $\psi(x)$ which is a tiny pulse. Translation and dilation of a mother wavelet generate a collection of linearly independent functions (basis). For the solution of singular initial value and boundary value issues, many approaches were devised. Wavelet analysis technique, an efficient wavelet-based spectrum approach, Wavelet Galerkin, Haar wavelet collocation, Legendre wavelet method, Adomian decomposition method, Lagurre wavelets method, etc. are few examples. Figure 7 describes the types of wavelet method for computational problem [60-64].

Figure 7. Wavelet Techniques.

4. Discussion and Conclusions

In this paper, we have studied conventional numerical methods and wavelet methods for solving differential equation based computational electromagnetic problems. Algorithm related to Euler's method, runge-kutta method and taylor series expansion has been discussed in flow chart manner. Types of wavelet methods has been presented in this paper for application in the solution of CEM problems.

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