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E- GRAPHS

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Abstract

A E-Graph has the vertex set as the elements of the group G and two edges xy are adjacent if x is the inverse of y and vice versa and all the vertices are adjacent to the identity element of the group. In this paper, for different types of groups we are going to construct the E-Graph and discuss about the properties of E-Graph.

1. Introduction

A Graph is a pair $G = (V, E)$ where V is the set of vertices and E is the set of edges. Here we consider only nontrivial, finite, connected and simple Graphs $|V(G)| = p$ and $|E(G)| = q$. [1].

Finite graph. A graph is finite if both of its vertex set and edge set are finite; otherwise it is an infinite graph.

Adjacent vertices. Any pair of vertices that are connected by an edge in a graph is called adjacent vertices. [2]

Adjacent edges. Any pair of edges that are connected by a common

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vertex in a graph is called adjacent edges.

Complete graph. A graph G is said to be complete if there exists an edge between every pair of vertices of G.

Simple graph. A graph without any parallel edges and self loops is called a simple graph.

Connected graph. A graph G is said to be connected if there exist at least one path between every pair of vertices in G, otherwise G is disconnected. [3]

Degree of a vertex. The degree of the vertex v is the number of edges incident to that vertex v.

Hamiltonian circuit. A circuit which covers all the vertices of the graph exactly once is called Hamilton circuit.

Hamiltonian graph. A graph which has Hamiltonian circuit is called Hamiltonian graph.

Euler graph. A graph G is said to be Euler graph if it has a Euler circuit. Euler circuit (Euler Tour) is a circuit in G covering all the edges of G exactly once. [1]

Group. A group is a set G along with some operation $*$ that takes in two elements and outputs another element of our group, such that we satisfy the following properties:

- **Identity.** There is a unique identity element $e \in G$ such that for any other $g \in G$, we have $e \cdot g = g \cdot e = g$.
- **Inverses.** for any $g \in G$, there is a unique $g - 1$ such that $g \cdot g - 1 = g - 1\ g = e$.
- **Associativity.** for any three $a, b, c \in G$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c.$ [2]

Definition. A subgroup H of a group, is any subset H of G such that H is also a group with respect to the same operation in G.

Definition. A group $(G, \cdot)$ is called abelian, or commutative, if it satisfies the following additional property.

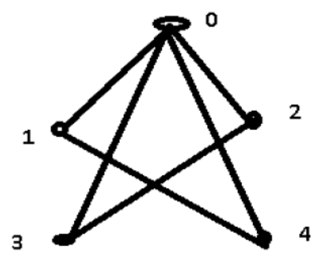
- **Commutativity.** for any $a, b \in G, a \cdot b = b \cdot a$ [4]

2. E-Graphs

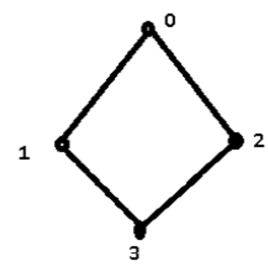
Definition 2.1. A E-Graph has the vertex set as the elements of the group G and two edges xy are adjacent if x is the inverse of y and vice versa and all the vertices are adjacent to the identity element of the group. The E-Graph is denoted by E_G.

Example.

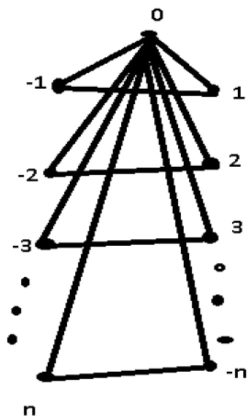
$G = Z_5 = \{0, 1, 2, 3, 4\}$. The E-Graph with respect to G is as follows.



$G = Z_4 = \{0, 1, 2, 3\}$. The E-Graph with respect to G is as follows.



$G = (Z, +)$. The E-Graph with respect to G is as follows.



Definition 2.2. The vertex set of E-Graph is denoted by $V(E(G))$ and edge set is denoted by $E(E(G))$.

The number of vertices of E-Graph is denoted by $|V(E(G))|$ and edge set is denoted by $|E(E(G))|$.

3. Main Results

Theorem 3.1. *Let the graph be E-Graph then the following are Equivalent.*

1. *E-Graph is either Eulerian or Hamiltonian.*

2. *All the vertices in E-Graphs has degree* 2 *except the identity vertex.*

3. $|E(E(G))| = n + 1$ *where n is thenumber of vertices.*

Proof. Let $G =$ E-graph

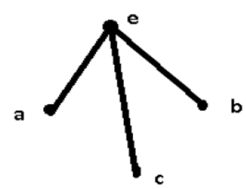
Vertex set $V(E(G)) =$ the elements of group G.

Edge set $E(E(G)) =$ the identity element of the group is adjacent with all the other vertices and inverse elements are adjacent.

This implies that the inverse elements have degree two and the identity element has degree $n - 1$. E-Graph consist of circuit which covers either all the vertices or all the edges. Therefore E-Graphs become either Eulerian or Hamiltonian. Also observe that the number of edges is equal to the number vertices plus one.

Theorem 3.2. *Let $G = \{e, a, b, c\}$, then E-Graph of G is a tree.*

Proof. Let $ee = e$, $ea = ae = a$, $eb = be = b$, $ec = ce = c$, $ab = ba = c$, $bc = cb = a$, $ca = ac = b$, $aa = e$, $bb = e$, $cc = e, ee = e$, $ea = ae = a$, $eb = be = b$, $ec = ce = c$, $ab = ba = c$, $bc = cb = a$, $ca = ac = b$, $aa = e$, $cc = e$. Then, ee is a unit element of GG and $e-1 = e$, $a-1 = a$, $b-1 = b$, $c-1 = c \cdot e-1 = e$, $a-1 = a$, $b-1 = b$, $c-1 = c$. Vertex set $V(E(G)) =$ the elements of group G. Edge set $E(E(G)) =$ the identity element of the group is adjacent with all the other vertices and inverse elements are adjacent.



Conclusion

E-Graphs can be constructed for infinite groups also.

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