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ANALYSIS OF MILD SOLUTION OF FRACTIONAL NEUTRAL STOCHASTIC FUNCTIONAL DIFFERENTIAL EQUATIONS WITH RANDOM IMPULSES

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Abstract

In this paper, we study the following fractional order neutral stochastic functional differential equations with random impulses which is given as,

$$\begin{cases} {}_0^cD_\gamma^\alpha[\vartheta^\alpha(\gamma) - \Upsilon(\gamma, \vartheta_\gamma)] = [\aleph\vartheta(\gamma) + \chi(\gamma, \vartheta_\gamma)]d\gamma + \sigma(\gamma, \vartheta_\gamma)d\mathcal{W}(\gamma), \ \gamma > \gamma_0, \ \gamma \neq \xi_k, \\ \vartheta(\xi_k) = b_k(\tau_k)\vartheta(\xi_k^-), \ \vartheta^\alpha(\xi_k) = b_k(\tau_k)\vartheta^\alpha(\xi_k^-), \ k = 1, 2, \ldots, \\ \vartheta_{\gamma_0} = \phi, \ \vartheta^\alpha(\gamma_0) = \psi, \end{cases} \tag{1}$$

where $\aleph : D(\aleph) \subset \mathfrak{H} \to \mathfrak{H}$ is the infinitesimal generator of a strongly continuous cosine family $\{\Psi(\gamma), \gamma \geq 0\}$. $\mathcal{W}(t)$ is a given Q-Winer process with a finite trace nuclear covariance operator $Q > 0$. τ_k is a random variable defined from Ω to $D_k \equiv (0, d_k)$ for $k = 1, 2, \ldots$. The aim of the present paper is to study existence and uniqueness of mild solutions of equation (1) by using the non compact measurement strategy and the Mönch fixed point theorem.

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1. Introduction

For our convenience and later use, we first recall definition of fractional derivative as follow.

Definition 1.1 Riemann-Liouville definition [6, 7, 8, 9]**.** For $\alpha \in [n-1, n)$ the α-derivative of f is

$$D_a^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n x}{dt^n} \int_a^{\alpha} \frac{f(x)}{(t-x)^{\alpha-n+1}} dx.$$

Definition 1.2. Caputo definition [6, 7, 8, 9]**.** For $\alpha \in (n-1, n)$ the α-derivative of f is

$${}_a^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha - n)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau.$$

The random impulsive fractional differential equations also have been discussed in [1, 2, 3, 4, 5], which is given by equation (1). As $i \neq j$, $(i, j = 1, 2, \ldots)$, and hence τ_i and τ_j are independent of each other. As $k = 1, 2, \ldots,$ then impulsive moments ξ_k are random variables and $\xi_k = \xi_{k-1} + \tau_k$. Obviously, $\{\xi_k\}$ is a processes with independent increments. $0 < t_0 = \xi_0 < \xi_1 < \xi_2 < \ldots < \xi_k < \ldots < \lim_{k \to \infty} \xi_k = \infty$, and $\vartheta(\xi_k^-) = \lim_{t \to \xi_{k-0}} \vartheta(\gamma)$. $b_k : D_k \to \mathfrak{H}$, for each $k = 1, 2, \ldots$. The time history $\vartheta_\gamma(\Theta) = \{\vartheta(\gamma + \Theta) : -\tau \leq \Theta \leq 0\}$ with some given $\tau > 0$. Moreover, Υ, χ, σ and ϕ, ψ will be specified later.

Let $\mathfrak{H}$ and $\mathcal{K}$ are two Hilbert spaces over real field with in norm and inner product are denoted by $\|\cdot\|$ and $\langle \cdot, \cdot \rangle$ respectively. Let $L(\mathcal{K}, \mathfrak{H}) = \{\phi : \mathcal{K} \to \mathfrak{H} \mid \phi$ is bounded linear operators$\}$ and $(\Omega, \mathcal{F}, P)$ denote a complete probability space equipped with a normal filtration $\{\mathcal{F}_\gamma\}_{\gamma \geq \gamma_0}$, $\mathcal{F}_{t_0}$ containing all P-null sets. Suppose that counting process $\{N(\gamma), \gamma \geq \gamma_0\}$ is generate by $\{\xi_k, k \geq 0\}$ and $\mathcal{F}_\gamma^{(1)}$ denote the minimal σ-algebra generated by $\{N(r), r \leq \gamma\}$. We suppose $\{\mathcal{W}(\gamma), \gamma \geq \gamma_0\}$ is a $\mathcal{K}$-valued wiener process and denote the $\mathcal{F}_\gamma^{(2)} = \sigma\{\mathcal{W}(r), r \leq \gamma\}$. Suppose that $\mathcal{F}_\infty^{(1)}, \mathcal{F}_\infty^{(2)}$, and $\mathcal{F}_{\gamma_0}$-are mutually independent, and $\mathcal{F}_\gamma = \mathcal{F}_\gamma^{(1)} V \mathcal{F}_\gamma^{(2)}$.

Let $\{\beta_n(\gamma)\}(n = 1, 2, \ldots)$ be a sequence of real valued Brownian motions mutually independent over $(\Omega, \mathcal{F}, P)$. Let $\aleph Q$-wiener process is defined as $W(\gamma) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\beta_n(\gamma)e_n, (\gamma \geq 0)$. As $\psi \in L(\mathcal{K}, \mathfrak{H})$, then we define

$$\| \psi \|_Q^2 = Tr(\psi Q \psi^*) = \sum_{n=1}^{\infty} \| \sqrt{\lambda_n}\psi e_n \|^2.$$

If $\| \psi \|_Q^2 < \infty$ then ψ is called a Q-Hilbert-Schmidt operator. Let, $\mathcal{T} \in (\gamma_0, +\infty)$, $J := [\gamma_0, \mathcal{T}]$, $J_k = [\xi_k, \xi_{k+1})$, $k = 0, 1, \ldots$, $\widetilde{J} = \{\gamma : \gamma \in J, \gamma \neq \xi_k, k = 1, 2, \ldots\}$. We denote $L_2(\Omega, \mathfrak{H})$ the collection of all square integrable, $\mathcal{F}_\gamma$-measurable, $\mathfrak{H}$-valued random variables, with the norm $\| \vartheta \|_{L_2} = (E \| \vartheta \|^2)^{\frac{1}{2}}$, where the expectation E is defined by $E \| \vartheta \|^2 = \int_\Omega \| \vartheta \|^2 dP$.

Definition 1.3. For a given $\mathcal{T} \in (\gamma_0, +\infty)$, a $\mathcal{F}_\gamma$-adapted process function $\{\vartheta \in \mathcal{B}, \gamma_0 - \tau \leq \gamma \leq \mathcal{T}\}$ is called a mild solution of system (1), if

(i) $\vartheta_{\gamma_0}(\nu) = \phi(\nu) \in L_2^0(\Omega, \mathcal{B})$ for $-\tau \leq \nu \leq 0$;

(ii) $\vartheta^\alpha(\gamma_0) = \psi(\gamma) \in L_2^0(\Omega, \mathfrak{H})$ for $\gamma \in J$;

(iii) the functions $\Upsilon(\nu, \vartheta_\gamma), \chi(\nu, \vartheta_\gamma), \sigma(\nu, \vartheta_\gamma)$ are integrable, and for a.e. $\gamma \in J$, the following integral equation is satisfied:

$$\vartheta(\gamma) = \sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k} b_i(\tau_i)\Psi(\gamma - \gamma_0)\phi(0) + \prod_{i}^{k} b_i(\tau_i)\mathcal{S}(\gamma - \gamma_0)[\varphi - \Upsilon(0, \phi)]\right.$$

$$+\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i}\Psi(\gamma-\nu)\frac{\Upsilon^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}+\frac{1}{\Gamma(-\alpha-n)}$$

$$\int_{\xi_k}^{\gamma}\Psi(\gamma-\nu)\frac{\Upsilon^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma,\nu)^{-\alpha+1-n}}+\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i}\nu(\gamma-\nu)\frac{\chi^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}\mathcal{S}(\gamma-\nu)\frac{\chi^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}+\sum_{i=1}^{k}\prod_{j=i}^{k}b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}$$

$$\left.\int_{\xi_{i-1}}^{\xi_i}\mathcal{S}(\gamma-\nu)\frac{\sigma^{(n)}(\nu,\vartheta_\nu)dW(\nu)}{(\gamma-\nu)^{-\alpha+1-n}}+\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}\mathcal{S}(\gamma-\nu)\frac{\sigma^{(n)}(\nu,\vartheta_\nu)dW(\nu)}{(\gamma-\nu)^{-\alpha+1-n}}\right]$$

$$I_{[\xi_k,\,\xi_{k+1})(\gamma)},\ \gamma\in[\gamma_0,\mathcal{T}],$$

where

$$\prod_{j=i}^{k}b_j(\tau_j)=b_k(\tau_k)b_{k-1}(\tau_{k-1})\ldots b_i(\tau_i),$$

and $I_{\aleph}(\cdot)$ is the index function, i.e.,

$$I_{\aleph}(\gamma)=\begin{cases}1, \text{ if } \gamma\in\aleph,\\ 0, \text{ if } \gamma\notin\aleph.\end{cases}$$

Lemma 1.4[10]. *If the set* $D\subset L^p(J;L_Q(\mathcal{K},\mathfrak{H}))$, $\mathcal{W}(\gamma)$ *is a Q-wiener process, then for any* $p\geq 2$, $\gamma\in[\gamma_0,\mathcal{T}]$, *Hausdorff non compactness measure* $\mho$ *then we have,*

$$\mho\left(\int_{\gamma_0}^{\gamma}D(\nu)d\mathcal{W}\nu\right)\leq\sqrt{(\mathcal{T}-\gamma_0)\frac{p}{2}}\alpha(D(\gamma)),$$

where

$$\int_{\gamma_0}^{\gamma}D(\nu)d\mathcal{W}\nu=\left\{\int_0^{\gamma}u(\nu)dW\nu : for\,all\,u\in D,\ \gamma\in[\gamma_0,\mathcal{T}]\right\}.$$

Lemma 1.5[11]. *Let D is a bounded convex subsets of $\mathfrak{H}$, with $0 \in D$. If a map $\mathcal{F} : D \to \mathfrak{H}$ is continuous, and if there exist a countable set $\mathcal{M} \subseteq D$, $\mathcal{M} \subseteq \bar{c}o(\{0\} \cup \mathcal{F}(\mathcal{M}))$, such that $\overline{\mathcal{M}}$ is a compact set then $\mathcal{F}$ has a fixed point in D.*

2. Main Result

To prove the existence of the mild solutions of (1), we set the following assumptions.

H_1 : As $\mathcal{S}(\gamma), \Psi(\gamma)(\gamma \in J)$ are equicontinuous and $\exists$ positive constants $N, \widetilde{N}$ such that

$$\sup_{\gamma \in J} \| \Psi(\gamma) \| \leq N, \sup_{\gamma \in J} \| \mathcal{S}(\gamma) \| \leq \widetilde{N}. \tag{2}$$

H_2 : Let $\chi : J \times \mathcal{C} \to \mathfrak{H}$ be the function such that.

(i) $\forall \gamma \in J, v \in \mathcal{C}$, the functions $\chi(\gamma, \cdot) : \mathcal{C} \to \mathfrak{H}$ and $\chi(\cdot, v) : J \to H$ are continuous and measurable respectively.

(ii) If $(\gamma, v) \in J \times \mathcal{C}$, then $\exists$ a continuous function $m(\gamma) \in L^{-1}(J, R^+)$, and $\Theta_\chi : R^+ \to R^+$, such that

$$E \| \chi^{(n)}(\gamma, v) \|^2 \leq m(\gamma) \Theta_\chi (E \| v \|_\gamma^2),$$

and Θ_χ satisfying

$$\lim_{r \to \infty} \inf \frac{\Theta_\chi(r)}{r} = 0.$$

(iii) For arbitrary bounded subset $Q \subset \mathcal{C}, \exists$ a positive function $\mathcal{K}_\chi(\gamma) \in L^1(J, R^+)$, and β satisfies

$$\beta(\chi^{(n)}(t, Q)) \leq K_\chi(\gamma) \sup_{-r \leq \theta \leq 0} \beta(Q(\theta)).$$

H_3 : Let $\Upsilon : J \times \mathcal{C} \to \mathfrak{H}$ be the function such that:

(i) $\forall \gamma \in J, \forall v \in \mathcal{C},$ the functions $\Upsilon(\gamma, \cdot) : \mathcal{C} \to \mathfrak{H}$ and $\Upsilon(\cdot, v) : J \to \mathfrak{H}$ are continuous and measurable respectively.

(ii) $\exists$ continuous functions $n(\gamma) \in L^{-1}(J, R^{+}),$ and $\Theta_{\Upsilon} : R^{+} \to R^{+},$ such that

$$E \| \Upsilon^{(n)}(\gamma, v) \|^2 \leq n(\gamma) \Theta_{\Upsilon}(E \| v \|_{\gamma}^2),$$

and the function Θ_g satisfying

$$\liminf_{r \to \infty} \frac{\Theta_{\Upsilon}(r)}{r} = 0.$$

(iii) for arbitrary bounded subset $Q \subset \mathcal{C}, \exists$ a positive function $\mathcal{K}_{\chi}(\gamma)$ $\in L^1(J, R^{+}),$ and β satisfies

$$\beta(\Upsilon^{(n)}(\gamma, Q)) \leq K_{\Upsilon}(\gamma) \sup_{-r \leq \theta \leq 0} \beta(Q(\theta)).$$

H_4 : Let $\sigma : J \times \mathcal{C} \to L_Q(\mathcal{K}, \mathfrak{H})$ be the function such that.

(i) If $\gamma \in J, v \in \mathcal{C}$ then the functions $\sigma(t, \cdot) : \mathcal{C} \to L_Q(\mathcal{K}, \mathfrak{H})$ and $\sigma(\cdot, v) : J \to L_Q(\mathcal{K}, \mathfrak{H})$ are continuous and measurable respectively.

(ii) $\exists$ a continuous function $\mu(\gamma) \in L^1(J, R^{+}),$ and a continuous positive nondecreasing function $\Theta_{\sigma} : R^{+} \to R^{+}$, such that

$$E \| \sigma^{(n)}(\gamma, v) \|^2 \leq n(\gamma) \Theta_{\sigma}(E \| v \|_{\gamma}^2),$$

for arbitrary $(\gamma, v) \in J \times \mathcal{C},$ and the function Θ_{σ} satisfying

$$\liminf_{r \to \infty} \frac{\Theta_{\sigma}(r)}{r} = 0.$$

(iii) For arbitrary bounded subset $Q \subset \mathcal{C}, \exists \mathcal{K}_{\chi}(\gamma) \in \mathcal{C}(J, R^{+}),$ the Hausdorff non-compact measure β satisfies

$$B(\sigma^{(n)}(\gamma, Q)) \leq \mathcal{K}_{\sigma}(\gamma) \sup_{-\tau \leq \theta \leq 0} \beta(Q(\theta)), \mathcal{K}_{\sigma}^{*} = \sup_{\gamma \in J} \mathcal{K}_{\sigma}(\gamma).$$

H_5 : If $\tau_j \in D_j (j = 1, 2, \ldots)$, then $\exists$ constants $\mathcal{M}$, such that

$$E\left\{\max_{i,k}\left\{\prod_{j=i}^{k}\| b_j(\tau_j) \|\right\}\right\} \leq \mathcal{M}.$$

H_6 : Let

$$H := 2N \max\{1, \mathcal{M}\}\| \mathcal{K}_\Upsilon \|_{L^1(J, R^+)} + 2\widetilde{N}\{1, \widetilde{M}\}\| \mathcal{K}_\Upsilon \|_{L^1(J, R^+)}$$

$$+ 2\widetilde{N} \max\{1, \mathcal{M}\}\mathcal{K}_\sigma^* \sqrt{(\mathcal{T} - \gamma_0)Tr(Q)} < 1.$$

Theorem 2.1. *If assumptions* $(H_1)-(H_6)$ *are satisfied, then there exists at least one mild solution of the system* (1).

Proof. We define the operator $\Phi : \mathcal{B} \to \mathcal{B}$ by Φ_x such that

$$(\Phi\vartheta)(\gamma) = \phi(\gamma),\ \gamma \in [\gamma_0 - \tau, \gamma_0],$$

$$(\Phi\vartheta)(\gamma) = \sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k} b_i(\tau_i)\Psi(\gamma - \gamma_0)\phi(0) + \prod_{i}^{k} b_i(\tau_i)\mathcal{S}(\gamma - \gamma_0)[\varphi - \Upsilon(0, \phi)]\right.$$

$$+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha - n)}\int_{\xi_{i-1}}^{\xi_i}\Psi(\gamma - \nu)\frac{\Upsilon^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma - \nu)^{-\alpha+1-n}} + \frac{1}{\Gamma(-\alpha - n)}$$

$$\int_{\xi_k}^{\gamma}\Psi(\gamma - \nu)\frac{\Upsilon^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma, \nu)^{-\alpha+1-n}} + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha - n)}\int_{\xi_{i-1}}^{\xi_i}\mathcal{S}(\gamma - \nu)\frac{\chi^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma - \nu)^{-\alpha+1-n}}$$

$$+ \frac{1}{\Gamma(-\alpha - n)}\int_{\xi_k}^{\gamma}\mathcal{S}(\gamma - \nu)\frac{\chi^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma - \nu)^{-\alpha+1-n}} + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha - n)}$$

$$\left.\int_{\xi_{i-1}}^{\xi_i}\mathcal{S}(\gamma - \nu)\frac{\sigma^{(n)}(\nu, \vartheta_\nu)dW(\nu)}{(\gamma - \nu)^{-\alpha+1-n}} + \frac{1}{\Gamma(-\alpha - n)}\int_{\xi_k}^{\gamma}\mathcal{S}(\gamma - \nu)\frac{\sigma^{(n)}(\nu, \vartheta_\nu)dW(\nu)}{(\gamma - \nu)^{-\alpha+1-n}}\right]$$

$$I_{[\xi_k, \xi_{k+1})(\gamma)},\ \gamma \in [\gamma_0, \mathcal{T}],$$

Now the problem of finding mild solutions of problem (1) is reduced to finding the fixed point of Φ. We divide the proof into several steps as follows:

Step I. Claim. There exits r such that Φ maps B_r into B_r.

For $\gamma \in [\gamma_0, \mathcal{T}]$, we have

$$E\|(\Phi\vartheta)(\gamma)\|^2 \leq 5E\left[\sum_{i=1}^{\infty}\prod_{i=1}^{k}\|b_i(\tau_i)\|\|\Psi(\gamma-\gamma_0)\|\|\phi(0)\|I_{[\xi_k,\xi_{k+1})}(\gamma)\right]^2$$

$$+5E\left[\sum_{i=1}^{\infty}\prod_{i=1}^{k}\|b_i(\tau_i)\|\|\varphi-\Upsilon(0,\phi)\|I_{[\xi_k,\xi_{k+1})}(\gamma)\right]^2$$

$$+5E\left[\sum_{i=1}^{\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|\Psi(\gamma-\nu)\|\frac{\|\Upsilon^{(n)}(\nu,\vartheta_\nu)\|d\nu}{\|\Gamma(-\alpha-n)(\gamma-\nu)^{-\alpha+1-n}\|}\right.\right.$$

$$\left.\left.+\int_{\xi_k}^{\gamma}\|\Psi(\gamma-\nu)\|\frac{\|\Upsilon^{(n)}(\nu,\vartheta_\nu)\|d\nu}{\|\Gamma(-\alpha-n)(\gamma-\nu)^{-\alpha+1-n}\|}\right]I_{[\xi_k,\xi_{k+1})}(\gamma)\right]^2$$

$$+5E\left[\sum_{i=1}^{\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|\mathcal{S}(\gamma-\nu)\|\frac{\|\chi^{(n)}(\nu,\vartheta_\nu)\|d\nu}{\|\Gamma(-\alpha-n)(\gamma-\nu)^{-\alpha+1-n}\|}\right.\right.$$

$$\left.\left.+\int_{\xi_k}^{\gamma}\|\mathcal{S}(\gamma-\nu)\|\frac{\|\chi^{(n)}(\nu,\vartheta_\nu)\|d\nu}{\|\Gamma(-\alpha-n)(\gamma-\nu)^{-\alpha+1-n}\|}\right]I_{[\xi_k,\xi_{k+1})}(\gamma)\right]^2$$

$$+5E\left[\sum_{i=1}^{\infty}\left[\sum_{i=1}^{k}\prod_{j=1}^{k}\|b_j(\tau_j)\|\int_{\xi_{i-1}}^{\xi_i}\|\mathcal{S}(\gamma-\nu)\|\frac{\|\sigma^{(n)}(\nu,\vartheta_\nu)\|d\mathcal{W}(\nu)}{\|\Gamma(-\alpha-n)(\gamma-\nu)^{-\alpha+1-n}\|}\right.\right.$$

$$\left.\left.+\int_{\xi_k}^{\gamma}\|\mathcal{S}(\gamma-\nu)\|\frac{\|\chi^{(n)}(\nu,\vartheta_\nu)\|d\nu}{\|\Gamma(-\alpha-n)(\gamma-\nu)^{-\alpha+1-n}\|}\right]I_{[\xi_k,\xi_{k+1})}(\gamma)\right]^2 := 5\sum_{i=1}^{5}R_i,$$

where

$$R_1 \leq N^2 E\left\{\max_k \left\{\prod_{i=1}^{k} \| b_i(\tau_i) \|\right\}\right\}^2 E\| \phi(0) \|^2 \leq N^2 \mathcal{M}^2 E\| \phi(0) \|^2,$$

$$R_2 \leq \tilde{N}^2 E\left\{\max_k \left\{\prod_{i=1}^{k} \| b_i(\tau_i) \|\right\}\right\}^2 E\| \varphi - \Upsilon(0, \phi) \|^2,$$

$$\leq \mathcal{M}^2 \tilde{N}^2 E\| \varphi - \Upsilon(0, \phi) \|^2,$$

$$R_3 \leq N^2 E\left\{\max_{i,k} \left\{1, \prod_{j=i}^{k} \| b_i(\tau_i) \|\right\}\right\}^2 (T - \gamma_0) \int_{\gamma_0}^{\gamma} E\| \Upsilon^{(n)}(\nu, \vartheta_\nu) \|^2 \frac{d\nu}{\zeta^*}$$

$$\leq N^2 \max\{1, \mathcal{M}^2\} \int_{\gamma_0}^{\gamma} n(\gamma) \Theta_\Upsilon (E\| \vartheta \|_\nu^2) \frac{d\nu}{\zeta^*},$$

$$R_4 \leq \tilde{N}^2 E\left\{\max_{i,k} \left\{1, \prod_{j=i}^{k} \| b_i(\tau_i) \|\right\}\right\}^2 \left[\int_{\gamma_0}^{\gamma} E\| \chi^{(n)}(\nu, \vartheta_\nu) \|^2 \frac{d\nu}{\zeta^*}\right]^2$$

$$\leq N^2 \max\{1, \mathcal{M}^2\} \int_{\gamma_0}^{\gamma} m(\gamma) \Theta_\chi (E\| \vartheta \|_\nu^2) \frac{d\nu}{\zeta^*},$$

$$R_5 \leq \tilde{N}^2 E\left\{\max_{i,k} \left\{1, \prod_{j=i}^{k} \| b_i(\tau_i) \|\right\}\right\}^2 E\left\| \int_{\gamma_0}^{\gamma} \sigma^{(n)}(\nu, \vartheta_\nu) \frac{d\mathcal{W}(\nu)}{\zeta^*} \right\|^2$$

$$\leq N^2 \max\{1, \mathcal{M}^2\} Tr(Q) \int_{\gamma_0}^{\gamma} E\| \sigma^{(n)}(\nu, \vartheta_\nu) \|^2 \frac{d\nu}{\zeta^*},$$

$$\leq N^2 \max\{1, \mathcal{M}^2\} Tr(Q) \int_{\gamma_0}^{\gamma} \mu(\nu) \Theta_\sigma (E\| \vartheta \|_\nu^2) \frac{d\nu}{\zeta^*}.$$

Also where $\Gamma(-\alpha - n)(\gamma - \nu)^{-\alpha+1-n} = \zeta$ and $\| \zeta \| = \zeta^*$.

If we assume that $\Phi(B_r) \not\subseteq B_r$, then for $r > 0$, $\exists$ a $\vartheta^r \in B_r$, such that $E\| (\Phi\vartheta^r) \|_{\mathcal{B}}^2 > r$. Therefore

$$r < \sup_{\gamma_0 \le \gamma \le \mathcal{T}} E \| (\Phi \vartheta^r) \|_\gamma^2 \le 5$$

$$\le 5[N^2 \mathcal{M}^2 E \| \phi(0) \|^2 + \mathcal{M}^2 \tilde{N}^2 E \| \varphi - \Upsilon(0, \phi) \|^2 + N^2 \max\{1, \mathcal{M}^2\} (\mathcal{T} - \gamma_0)$$

$$\| n \|_{L^{-1}(J, R)} \Theta_\Upsilon(r) + \tilde{N}^2 \max\{1, \mathcal{M}^2\} (\mathcal{T} - \gamma_0) \| m \|_{L^1(J, R)} \Theta_\chi(r)$$

$$+ \tilde{N}^2 \max\{1, \mathcal{M}^2\} Tr(Q) \| m \|_{L^1(J, R)} \Theta_\sigma(r)].$$

Since

$$\lim_{r \to \infty} \inf \frac{\Theta_\Upsilon(r)}{r} = 0, \ \lim_{r \to \infty} \inf \frac{\Theta_\chi(r)}{r} = 0, \ \lim_{r \to \infty} \inf \frac{\Theta_\sigma(r)}{r} = 0,$$

taking these limits and above equation, which gives that $1 \le 0$, a contradiction.

Thus $\exists r > 0$, $\Phi(B_r) \subseteq B_r$.

Step II. Claim. Φ is continuous in B_r.

Let $\{\vartheta_n \to \vartheta\}$ in B_r (as $n \to \infty$), then

$$E \| (\Phi \vartheta^n)(\gamma) - (\Phi \vartheta)(\gamma) \|^2$$

$$\le 3E \Bigg\| \sum_{k=0}^{+\infty} \Bigg[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \frac{1}{\Gamma(-\alpha - n)} \int_{\xi_{i-1}}^{\xi_i} \Psi(\gamma - \nu) \frac{(\Upsilon^{(n)}(\nu, \vartheta_\nu^n) - \Upsilon^{(n)}(\nu, \vartheta_\nu)) d\nu}{(\gamma - \nu)^{-\alpha + 1 - n}}$$

$$+ \frac{1}{\Gamma(-\alpha - n)} \int_{\xi_k}^{\gamma} \Psi(\gamma - \nu) \frac{(\Upsilon^{(n)}(\nu, \vartheta_\nu^n) - \Upsilon^{(n)}(\nu, \vartheta_\nu)) d\nu}{(\gamma - \nu)^{-\alpha + 1 - n}} \Bigg] I_{[\xi_k, \xi_{k+1})}(\gamma) \Bigg\|^2$$

$$+ 3E \Bigg\| \sum_{k=0}^{+\infty} \Bigg[\sum_{i=1}^{k} \prod_{j=i}^{k} b_j(\tau_j) \frac{1}{\Gamma(-\alpha - n)} \int_{\xi_{i-1}}^{\xi_i} \mathcal{S}(\gamma - \nu) \frac{(\chi^{(n)}(\nu, \vartheta_\nu^n) - \chi^{(n)}(\nu, \vartheta_\nu)) d\nu}{(\gamma - \nu)^{-\alpha + 1 - n}}$$

$$+ \frac{1}{\Gamma(-\alpha - n)} \int_{\xi_k}^{\gamma} \mathcal{S}(\gamma - \nu) \frac{(\chi^{(n)}(\nu, \vartheta_\nu^n) - \chi^{(n)}(\nu, \vartheta_\nu)) d\nu}{(\gamma - \nu)^{-\alpha + 1 - n}} \Bigg] I_{[\xi_k, \xi_{k+1})}(\gamma) \Bigg\|^2$$

$$+ 3E\| \sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i} \mathcal{S}(\gamma-\nu)\frac{(\sigma^{(n)}(\nu, \vartheta_\nu^n)-\sigma^{(n)}(\nu, \vartheta_\nu))d\nu}{(\gamma-\nu)^{-\alpha+1-n}}\right.$$

$$\left. + \frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma} \mathcal{S}(\gamma-\nu)\frac{(\sigma^{(n)}(\nu, \vartheta_\nu^n)-\sigma^{(n)}(\nu, \vartheta_\nu))d\nu}{(\gamma-\nu)^{-\alpha+1-n}}\right] I_{[\xi_k, \xi_{k+1})}(\gamma)\|^2.$$

By the continuity of functions Υ, χ, σ in the assumptions of $(H_2)-(H_4)$, and by Lebesgue dominated theorem, for $\gamma \in [\gamma_0, \mathcal{T}]$ we have

$$E\|(\Phi\vartheta^n)(\gamma)-(\Phi\vartheta)(\gamma)\|^2 \le \widetilde{N}^2 \max\{1, \mathcal{M}^2\}(\mathcal{T}-\gamma_0)^2 \sup_{\gamma_0\le\gamma\le T} E\|\Upsilon^{(n)}(\nu, \vartheta_\nu^n)$$

$$-\Upsilon^{(n)}(\nu, \vartheta_\nu)\|^2$$

$$\le \widetilde{N}^2 \max\{1, \mathcal{M}^2\}(\mathcal{T}-\gamma_0)^2 \sup_{\gamma_0\le\gamma\le\mathcal{T}} E\|\chi^{(n)}(\nu, \vartheta_\nu^n)-\chi^{(n)}(\nu, \vartheta_\nu)\|^2 \to 0,$$

$$(\text{as } n\to\infty).$$

Therefore, $\|(\Phi\vartheta^n)(\gamma)-(\Phi\vartheta)(\gamma)\|_{\mathcal{B}}^2 \to 0(\text{as } n\to\infty)$, which implies that Φ is continuous in B_r.

Step III. Claim. The operator $\Phi(B_r)$ is equicontinuous on every $[\xi_k, \xi_{k+1})$.

Let $\xi_k \le \gamma_1 < \gamma_2 < \xi_{k+1}, k = 0, 1, 2, \ldots$ and $\vartheta \in B_r$ then for any fixed $\vartheta \in B_r$, we have

$$E\|(\Phi\vartheta)(\gamma_1)-(\Phi\vartheta)(\gamma_2)\|^2 \le 5E\|\prod_{i=1}^{k} b_i(\tau_i)[\Psi(\gamma_1-\gamma_0)-\Psi(\gamma_2-\gamma_0)]\phi(0)\|^2$$

$$+ 5E\|\prod_{i=1}^{k} b_i(\tau_i)[\mathcal{S}(\gamma_1-\gamma_0)-\mathcal{S}(\gamma_2-\gamma_0)][\varphi-\Upsilon(0, \phi)]\|^2$$

$$+ 5E\| \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i}[\Psi(\gamma_2-\nu)-\Psi(\gamma_1-\nu)]\frac{\Upsilon^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}[\Psi(\gamma_2-\nu)-\Psi(\gamma_1-\nu)]\frac{\Upsilon^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\gamma_1}^{\gamma_2}\Psi(\gamma_2-\nu)\frac{\Upsilon^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}\|^2$$

$$+ 5E\| \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i}[\mathcal{S}(\gamma_2-\nu)-\mathcal{S}(\gamma_1-\nu)]\frac{\chi^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}[\mathcal{S}(\gamma_2-\nu)-\mathcal{S}(\gamma_1-\nu)]\frac{\chi^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\gamma_1}^{\gamma_2}\mathcal{S}(\gamma_2-\nu)\frac{\chi^{(n)}(\nu,\vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}\|^2$$

$$+ 5E\| \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i}[\mathcal{S}(\gamma_2-\nu)-\mathcal{S}(\gamma_1-\nu)]\frac{\sigma^{(n)}(\nu,\vartheta_\nu)d\mathcal{W}\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}[\mathcal{S}(\gamma_2-\nu)-\mathcal{S}(\gamma_1-\nu)]\frac{\sigma^{(n)}(\nu,\vartheta_\nu)d\mathcal{W}\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+\frac{1}{\Gamma(-\alpha-n)}\int_{\gamma_1}^{\gamma_2}\mathcal{S}(\gamma_2-\nu)\frac{\sigma^{(n)}(\nu,\vartheta_\nu)d\mathcal{W}\nu}{(\gamma-\nu)^{-\alpha+1-n}}\|^2.$$

By the equicontinuity of $\Psi(\gamma)$, $\mathcal{S}(\gamma)$ the assumption (H_1), $(H_2)-(H_5)$, and Lebesgue dominated theorem, as $\gamma_2 \to \gamma_1$, on every $[\xi_k, \xi_{k+1})$

$$E\|(\Phi x)(\gamma_1)-(\Phi x)(\gamma_2)\|^2 \to 0.$$

This proves that $(\Phi(B_r))$ is equicontinuous on J.

Step IV. Claim: The Mönch's condition holds.

Let $B = \bar{c}o(\{0\}\cup\Phi(B_r))$. For any $D \to B$, without loss of generality, we assume that $D = \{\vartheta^n\}_{n=1}^{\infty}$. Then clearly Φ maps D into itself and $D \subset \bar{c}o(\{0\} \cup \Phi(B_r))$ is equicontinuous on J_k.

Sub Claim. As β is the Hausdorff measure of non compactness then $\beta(D) = 0$.

Here, for convenience, we denote $\Phi = \Phi_1 + \Phi_2 + \Phi_3$ where

$$(\Phi_1\vartheta)(\gamma) = \sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k} b_i(\tau_i)\Psi(\gamma-\gamma_0)\phi(0) + \prod_{i=1}^{k} b_i(\tau_i)\mathcal{S}(\gamma-\gamma_0)[\varphi - \Upsilon(0, \phi)]$$

$$+ \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i} \Psi(\gamma-\nu)\frac{\Upsilon^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+ \frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}\Psi(\gamma-\nu)\frac{\Upsilon^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}\Bigg] I_{[\xi_k, \xi_{k+1})}(\gamma),$$

$$(\Phi_2\vartheta)(\gamma) = \sum_{k=0}^{+\infty}\Bigg[\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i} \mathcal{S}(\gamma-\nu)\frac{\chi^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+ \frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}\mathcal{S}(\gamma-\nu)\frac{\chi^{(n)}(\nu, \vartheta_\nu)d\nu}{(\gamma-\nu)^{-\alpha+1-n}}\Bigg] I_{[\xi_k, \xi_{k+1})}(\gamma),$$

$$(\Phi_3\vartheta)(\gamma) = \sum_{k=0}^{+\infty}\Bigg[\sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\frac{1}{\Gamma(-\alpha-n)}\int_{\xi_{i-1}}^{\xi_i} \mathcal{S}(\gamma-\nu)\frac{\sigma^{(n)}(\nu, \vartheta_\nu)d\mathcal{W}\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$+ \frac{1}{\Gamma(-\alpha-n)}\int_{\xi_k}^{\gamma}\mathcal{S}(\gamma-\nu)\frac{\sigma^{(n)}(\nu, \vartheta_\nu)d\mathcal{W}\nu}{(\gamma-\nu)^{-\alpha+1-n}}\Bigg] I_{[\xi_k, \xi_{k+1})}(\gamma).$$

By Lemma [4], Lemma 1.4 and the assumptions of $(H_1)-(H_5)$, we have

$$\beta(\{(\Phi_1\vartheta^n)(\gamma)\}_{n=1}^{\infty}) \leq 2\max\{1, \mathcal{M}\}N \frac{1}{\Gamma(-\alpha-n)} \int_{\gamma_0}^{\gamma} \frac{\beta(\{\Upsilon^{(n)}(\nu, \vartheta_\nu^n)\}_{n=1}^{\infty})}{(\gamma-\nu)^{-\alpha+1-n}} d\nu$$

$$\leq 2\max\{1, \mathcal{M}\} N \frac{1}{\Gamma(-\alpha-n)} \int_{\gamma_0}^{\gamma} \frac{\mathcal{K}_\Upsilon(\nu)\sup_{-\tau\leq\theta\leq 0}\beta(\{\vartheta_\nu^n(\theta)\}_{n=1}^{\infty})d\nu}{(\gamma-\nu)^{-\alpha+1-n}}$$

$$\leq 2\max\{1, \mathcal{M}\}\| \mathcal{K}_\Upsilon \|_{L^1(J, R^+)} \sup_{\gamma\in J}\beta(\{\vartheta^n(\gamma)\}_{n=1}^{\infty}),$$

$$\beta(\{(\Phi_2\vartheta^n)(\gamma)\}_{n=1}^{\infty}) \leq 2\max\{1, \mathcal{M}\}\tilde{N} \frac{1}{\Gamma(-\alpha-n)} \int_{\gamma_0}^{\gamma} \frac{\beta(\{\chi^{(n)}(\nu, \vartheta_\nu^n)\}_{n=1}^{\infty})}{(\gamma-\nu)^{-\alpha+1-n}} d\nu$$

$$\leq 2\max\{1, \mathcal{M}\}\| \mathcal{K}_\chi \|_{L^1(J, R^+)} \sup_{\gamma\in J}\beta(\{\vartheta^n(\gamma)\}_{n=1}^{\infty}),$$

$$\beta(\{(\Phi_3\vartheta^n)(\gamma)\}_{n=1}^{\infty}) \leq \max\{1, \mathcal{M}\}\tilde{N}\beta\left(\left\{\frac{1}{\Gamma(-\alpha-n)}\int_{\gamma_0}^{\gamma} \frac{\sigma(n)(\nu, \vartheta_\nu^n)d\mathcal{W}(\nu)_{n=1}^{\infty}}{(\gamma-\nu)^{-\alpha+1-n}}\right\}\right)$$

$$\leq 2\max\{1, \mathcal{M}\}\tilde{N}\sqrt{(\mathcal{T}-\gamma_0)Tr(Q)}\mathcal{K}_\sigma(\gamma) \sup_{-\tau\leq\theta\leq 0}\beta(\{\vartheta_\gamma^n(\theta)\}_{n=1}^{\infty})$$

$$\leq 2\tilde{N}\max\{1, \mathcal{M}\}\sqrt{(\mathcal{T}-\gamma_0)Tr(Q)}\mathcal{K}_\sigma^* \sup_{\gamma\in J}\beta(\{\vartheta^n(\gamma)\}_{n=1}^{\infty}).$$

Thus,

$$\beta(\{(\Phi\vartheta^n)(\gamma)\}_{n=1}^{\infty}) \leq \beta(\{(\Phi_1\vartheta^n)(\gamma)\}_{n=1}^{\infty}) + \beta(\{(\Phi_2\vartheta^n)(\gamma)\}_{n=1}^{\infty}) + \beta(\{(\Phi_3\vartheta^n)(\gamma)\}_{n=1}^{\infty})$$

$$\leq [2N\max\{1, \mathcal{M}\}\| \mathcal{K}_\Upsilon \|_{L^1(J, R^+)} + 2\tilde{N}\max\{1, \mathcal{M}\}\| \mathcal{K}_\chi \|_{L^1(J, R^+)}$$

$$+ 2\tilde{N}\max\{1, \mathcal{M}\}\sqrt{(\mathcal{T}-\gamma_0)Tr(Q)}\mathcal{K}_\sigma^*]\sup_{\gamma\in J}\beta(\{\vartheta^n(\gamma)\}_{n=1}^{\infty})$$

$$\leq \mathfrak{H}\sup_{\gamma\in J}\beta(D(\gamma)).$$

By Lemma [4] and assumption (H_6), we know

$$\beta(D) \leq \beta(\bar{c}o(\{0\}\cup\Phi(D))) = \beta(\Phi(D)) \leq \mathfrak{H}\sup_{t\in J}\beta(D(\gamma)) = \mathfrak{H}\beta(D) < \beta(D)$$

which implies $\beta(D) = 0,$ the set D is a relatively compact set. By Lemma 1.5, Φ has at least one fixed point ϑ in B_r. and hence the system (1) has at least a mild solution.

3. Application

In this section, we will discuss an example which illustrate the application and validity of our main results. For this, we consider the fractional neutral stochastic functional partial differential equation of the form,

$$\begin{cases} \dfrac{\partial^{\alpha+1} w(\gamma, \vartheta)}{\partial \gamma^{\alpha+1}} - \dfrac{\partial^{\alpha}}{\partial \gamma^{\alpha}}\left[\dfrac{a_1}{5}\displaystyle\int_{-\varrho}^{0} \lambda_1(\nu) w(\gamma+\nu, \vartheta) d\nu\right] \\ = \left[\dfrac{\partial^{\alpha+1} w(\gamma, \vartheta)}{\partial \gamma^{\alpha+1}} + \dfrac{a_2}{5}\displaystyle\int_{-\varrho}^{0} \lambda_2(\nu) w(\gamma, \vartheta) d\nu\right] d\gamma + \dfrac{a_3}{5}\displaystyle\int_{-\varrho}^{0} \lambda_3(\nu) w(\gamma, \vartheta) d\mathcal{W}(\gamma), \\ \gamma \geq \gamma_0,\ \gamma \neq \xi_k,\ \vartheta \in [0, \pi], \\ \omega(\xi_k, \vartheta) = \rho(k)\tau_k \omega(\overline{\xi_k}, \vartheta), \\ \dfrac{\partial^{\alpha} w(\xi_k, \vartheta)}{\partial \gamma^{\alpha}} = \rho(k)\tau_k \dfrac{\partial^{\alpha} w(\overline{\xi_k}, \vartheta)}{\delta \gamma^{\alpha}}, \\ w(\gamma_0, \vartheta) = y(\theta, \vartheta),\ \theta \in [-\varrho, 0],\ \vartheta \in [0, \pi],\ r > 0, \\ \dfrac{\partial^{\alpha} w(\gamma_0, \vartheta)}{\partial \gamma^{\alpha}} = \phi(\vartheta),\ \vartheta \in [0, \pi], \\ w(\gamma, 0) = w(\gamma, \pi) = 0. \end{cases} \tag{3}$$

Let τ_k be a random variable defined on $D_k = (0, d_k)$ where $0 < d_k < +\infty$ for $k = 1, 2, \ldots.$ Suppose τ_i and τ_j are independent of each other as $i \neq j$ for $i, j = 1, 2, \ldots, \xi_0 = \gamma_0 > 0$ and $\xi_k = \xi_{k-1} + \tau_k$ for $k = 1, 2, \ldots.$ Let $\mathcal{W}(\gamma)$ denotes a standard cylindrical wiener process in $\mathcal{L}^2([0, \pi])$. By suitable choices $\lambda, \lambda_i, \sigma,$ we firstly reduce the equations (3) into (1). Let σ be a function of k and $\eta_i : [-\varrho, 0] \to R$ are positive functions and $\lambda_i > 0$ for $i = 1, 2, 3.$ $\| \Psi(\gamma) \|, \| \mathcal{S}(\gamma) \|$ are bounded on R.

We assume that

(i) The function $\lambda(\theta)$ is continuous on $[-\varrho, 0], \int_{-\varrho}^{0} \lambda_i(\theta) d(\theta) < \infty (i = 1, 2, 3).$

(ii) $\max_{i,k}\{\prod_{j=i}^{k} E[\| \sigma(j)\tau_j \|^2]\} < \mathcal{M}.$

Under the above assumptions, and by choosing some suitable functions $\lambda_1, \lambda_2, \lambda_3, \rho,$ we can show that $\mathcal{L}_g = \frac{ra_1}{25}\int_{-\varrho}^{0} \gamma_1^2(\theta)d(\theta), \mathcal{L}_\chi = \frac{ra_2}{25}\int_{-\varrho}^{0} \gamma_2^2(\theta)d(\theta),$ $\mathcal{L}_\rho = \frac{ra_3}{25}\int_{-\varrho}^{0} \gamma_3^2(\theta)d(\theta).$ We assume that the functions $\lambda_1, \lambda_2, \lambda_3, \rho,$ in (3) satisfy all the assumptions of the Theorem 2.1 then the above problem (3) can be written as,

$$\begin{cases} {}_0^cD_\gamma^\alpha[\vartheta^\alpha(\gamma) - \Upsilon(\gamma, \vartheta_\gamma)] = [\aleph\vartheta(\gamma) + \chi(\gamma, \vartheta_\gamma)]d\gamma + \sigma(\gamma, \vartheta_\gamma)d\mathcal{W}(\gamma), \gamma > \gamma_0, \gamma \neq \xi_k, \\ \vartheta(\xi_k) = b_k(\tau_k)\vartheta(\xi_k^-), \vartheta^\alpha(\xi_k) = b_k(\tau_k)\vartheta^\alpha(\xi_k^-), k = 1, 2, \ldots, \\ \vartheta_{\gamma_0} = \phi, \vartheta^\alpha(\gamma_0) = \psi, \end{cases}$$

Hence by the Theorem 2.1, can be applied to guarantee the mild solution of the equation (3).

4. Conclusion

In this work, we discussed existence result of fractional order neutral stochastic functional systems with random impulses by using the non compact measurement and the Mönch fixed point theorem. Moreover, we obtained the mild solution of an equation (3), which demonstrate the validity and application of the main result.

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