Advances and Applications in Mathematical Sciences
Volume 21, Issue 2, December 2021, Pages 539-553

MINIMUM TOTAL DOMINATING HYPERENERGETIC GRAPHS

K. MALATHY[1] and S. MEENAKSHI[2]

[1]Research Scholar
[2]Associate Professor
Department of Mathematics
VISTAS, Chennai, India
E-mail: malu14cog@gmail.com
meenakshikarthikeyan@yahoo.co.in

Abstract

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. A set S of vertices in a graph $G(V, E)$ is called a total dominating set if every vertex $v \in V$ is adjacent to an element of S. The minimum cardinality of a total dominating set of G is called the total domination number of G which is denoted by $\gamma_t(G)$. The energy of the graph is defined as the sum of the absolute values of the eigenvalues of the adjacency matrix. The graphs whose energy is greater than that of complete graph are called hyperenergetic, i.e. $E(G) > 2n - 2$. In this paper, we computed minimum total dominating hyperenergetic of some standard graph such as Friendship graph, Wheel graph and Star Graph and compared the energy by plotting the graph.

1. Introduction

In 1960, the mathematical study of Domination Theory in graphs was started. I. Gutman introduced the concept of energy of a graph in the year 1978. The roots go back to 1862 when C. F. De Jaenisch studied the problem of determining the minimum number of queens necessary to cover a $n \times n$ chess board in such way that every square is attacked by one of the queens.

The graph invariant is closely connected to a chemical quantity known as the total π electron energy of conjugated hydrocarbon molecules. The study of Graph Energy first arouses in the field of chemistry. Chemists used Huckel's

2020 Mathematics Subject Classification: 05C65.
Keywords: Dominating set, Total dominating set, Total dominating Energy, Minimum Total dominating Energy, Hyperenergetic.
Received May 17, 2021; Accepted June 7, 2021

method to approximate energies associated with Π-electron orbitals in a special class of molecules called conjugated hydrocarbons. Gutman first introduced the concept of 'energy of graph' for a simple graph. At first very few mathematicians seemed to be interested in this concept. However, over the years graph energy has become an interesting area of research for mathematicians and several variations have been introduced.

Let G be a graph with x-vertices and y-edges. Let $A = (a_{ij})$ be the adjacency matrix of a graph. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of adjacency matrix of a graph G. The values are in non-decreasing order, that is $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Since $A(G)$ is real and symmetric, its eigenvalues are real number. The energy $E(G)$ of graph is defined as sum of the absolute values of its eigenvalues of graph G, i.e., $E(G) = \sum_{i=1}^{n} | \lambda_i |$ [10] [11] [12] [13].

Some time ago it was conjectured that among all graphs on n vertices, the complete graph K_n has the greatest energy; $E(K_n) = 2 - 2$. After that it was recognized that the conjecture is false and that there exist graphs for which $E(G) > 2n - 2$. Graphs on n vertices, the energy of which exceeds $2n - 2$ are referred to as hyperenergetic graphs.

In this paper, we computed minimum total dominating Hyperenergetic of some standard graph such as Friendship graph, Wheel graph and Star Graph and compared the energy by plotting the graph.

2. Preliminaries

Definition 2.1. Dominating set:

Let G be a simple graph with vertex set $V(G)$. Let $S \subseteq V(G)$. A set S of vertices of G is a dominating set if every vertex in $V(G) - S$ is adjacent to at least one vertex in S. The minimum cardinality of a dominating set of G is called the domination number of G which is denoted by $\gamma(G)$ [2] [5].

Definition 2.2. Total Dominating set:

A set S of vertices in a graph $G(V, E)$ is called a total dominating set if every vertex $v \in V$ is adjacent to an element of S. The minimum cardinality of a total dominating set of G is called the total domination number of G which is denoted by $\gamma_t(G)$ [1].

Definition 2.3. Energy:

Energy of a simple graph $G = (V, E)$ with adjacency matrix A is defined as the sum of absolute values of eigenvalues of A denoted by $E(G)$, i.e., $E(G) = \sum_{i=1}^{n} |\lambda_i|$ where λ_i is an eigenvalues of A, $i = 1, 2, \ldots, n$. [3] [4] [10] [11].

Definition 2.4. Line Graph:

Line graph of G is obtained by creating a vertex per edge in G and linking two vertices in $L(G)$ if, and only if, the corresponding edges in G have an end in common.

Example

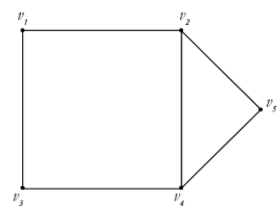

Figure (i) Graph G.

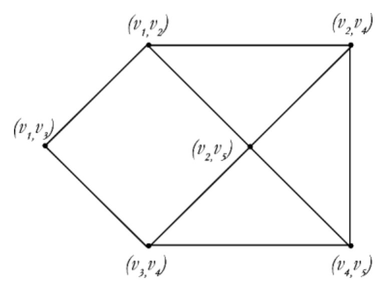

Figure (ii) Graph $L(G)$.

Graph G contains six edges, which means that $L(G)$ contains six vertices. The vertices $[v_1, v_2]$ and $[v_1, v_3]$ are linked by an edge in $L(G)$ because the corresponding edges in G have a vertex in common. However, there is no edge linking the vertices $[v_1, v_3]$ and $[v_2, v_4]$ in $L(G)$ because those two edges in G have no ends in common.

3. Minimum Total Dominating Hyperenergetic of Some Standard Graphs

3.1. Minimum total dominating energy of Friendship graph

Definition 3.1. Friendship graph

The friendship graph F_n can be constructed by joining n copies of the cycle graph C_n with a common vertex.

The minimum total dominating energy of Friendship graph is given below.

Minimum Total dominating energy of Friendship graph			
Value of n	Required energy	The value of $2(n-1)$	Difference between $2(n-1)$ and energy
5	6.9624	8	1.0376
7	10.8762	12	1.1238
9	13.6917	16	2.3083
11	18.5687	20	1.4313
13	21.6980	24	2.3020

From the above table we can say that minimum total dominating energy of Friendship graph is non-hyperenergetic for n vertices.

Line Graph of F_5

Let G be a Friendship graph with 5 vertices $= \{v_1, v_2, v_3, v_4, v_5\}$.

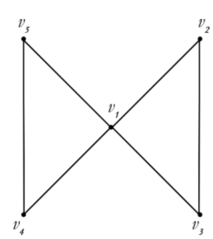

Figure (i) Friendship Graph F_5.

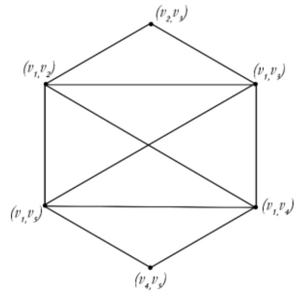

Figure (ii) $L(F_5)$.

The minimum total dominating set, $L(F_5) = [(v_1, v_2), (v_1, v_4)\}$.

The minimum cardinality of a total dominating set $\gamma_t(L(F_5)) = 2$.

The minimum total dominating matrix,

$$A_{TD}(L(F_5)) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

The characteristic polynomial is $x^6 - 2x^5 - 9x^4 + 0x^3 + 10x^2 + 6x + 1$.

The eigenvalues are $\lambda_1 = -1.8019$ $\lambda_2 = -0.4450$, $\lambda_4 = -0.3569$ $\lambda_5 = 1.2470$ $\lambda_6 = 4.0489$.

Therefore, the minimum total dominating energy, $E_{TD}(L(F_5)) = 10.4767$.

Minimum Total dominating energy of $L(F_5)$			
Value of n	Required energy	The value of $2(n-1)$	Difference between $2(n-1)$ and energy
5	8.5917	8	– 0.5917
7	12. 9890	12	– 0.9890
9	16.7628	16	– 0.7628
11	20.5241	20	– 0.5241
13	24.7618	24	– 0.7618

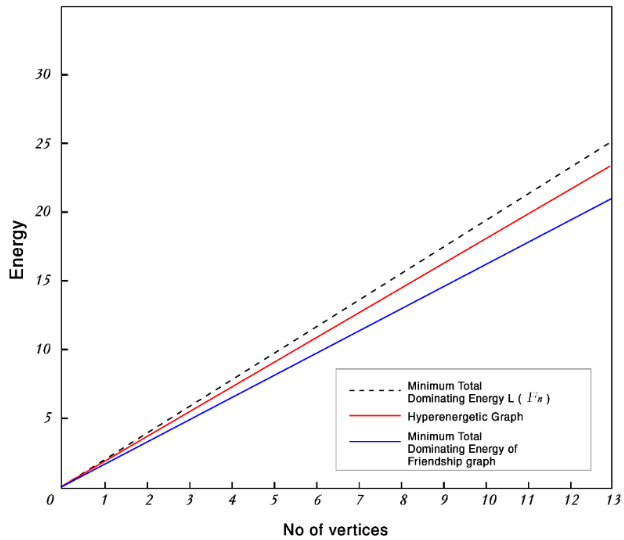

Figure (iii) Comparison of energies of Friendship graph.

Hence, we can say that minimum total dominating energy of $L(F_n)$ is hyperenergetic for $n \geq 5$.

Minimum total dominating energy of Friendship graph and minimum total dominating energy of Friendship graph of F_n plotted in the below graph to compare against the value of $2(n-1)$.

From the above graph we can say that minimum total dominating energy of Friendship graph is non-hyper energetic for n vertices and Line Graph of F_n is hyperenergetic for $n \geq 5$.

3.2. Minimum total dominating energy of Wheel graph

Definition 3.2. Wheel graph

Wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. The wheel graph W_n can be defined as the graph $K_1 + C_{n-1}$, where K_1 is the singleton graph and C_{n-1} is the cycle graph.

The minimum total dominating energy of Wheel graph is given below

Minimum total dominating energy of Wheel graph			
Value of n	Required energy	The value of $2(n-1)$	Difference between $2(n-1)$ and energy
4	5.1232	6	0.8768
5	6.6582	8	1.3418
6	9.2977	10	0.7023
7	11.0636	12	0.9364
8	13.0234	14	0.9766
9	15.7890	16	0.2110
10	17.6740	18	0.3260
11	19.0678	20	0.9322
12	21.7896	22	0.2104
13	23.3285	24	0.6715

From the above table we can say that minimum total dominating energy of Wheel graph is non-hyperenergetic for n vertices.

Line Graph of W_4.

Let G be a wheel graph with 4 vertices $= \{v_1, v_2, v_3, v_4\}$.

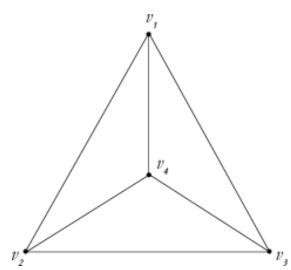

Figure (iv) Wheel Graph W_4.

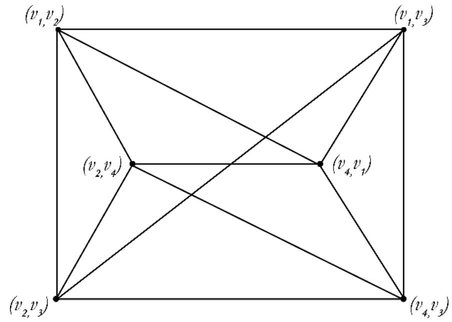

Figure (v) $L(W_4)$.

The minimum total dominating set, $L(W_4) = \{(v_4, v_1), (v_2, v_3)\}$.

The minimum cardinality of a total dominating set $\gamma_t(L(W_4)) = 2$.

The minimum total dominating matrix,

$$A_{TD}(L(W_4)) = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

The characteristic polynomial is $x^6 - 2x^5 - 11x^4 - 0x^3 + 12x^2 + 0x + 0$.

The eigenvalues are $\lambda_1 = -2$ λ_2 -1.3723 $\lambda_3 = -1.7321$ $\lambda_4 = 0$ $\lambda_5 = 0$ $\lambda_6 = 1$ $\lambda_7 = 4.3723$.

Therefore, the minimum total dominating energy, $E_{TD}(L(W_4))$ $= 10.4767$.

Minimum total dominating energy of $L(W_4)$			
Value of n	Required energy	The value of $2(n-1)$	Difference between $2(n-1)$ and energy
4	10.4767	6	− 4.4767
5	12.6870	8	− 4.6870
6	14.2977	10	− 4.2977

7	15.0636	12	– 3.0636
8	16.0234	14	– 2.0234
9	18.7890	16	– 2.7890
10	19.6740	18	– 1.6740
11	21.0678	20	– 1.0678
12	24.7896	22	– 2.7896
13	26.3285	24	– 2.3285

Hence, we can say that minimum total dominating energy of Line graph of W_n is hyperenergetic for $n \geq 4$.

Minimum total dominating energy of Wheel graph and minimum total dominating energy of Line graph of W_n plotted in the below graph to compare against the value of $2(n-1)$.

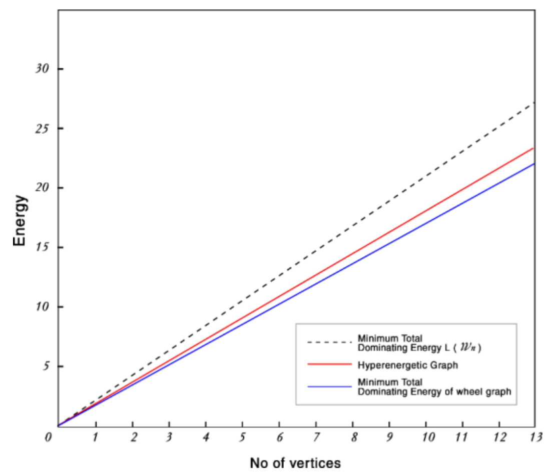

Figure (vi) Comparison of energies of Wheel graph.

From the above graph we can say that minimum total dominating energy Wheel graph is non-hyperenergetic for n vertices and Line graph of W_n is hyperenergetic for $n \geq 4$.

3.3. Minimum total dominating energy of Star graph

Definition 3.3. Star graph

A complete bipartite graph of the form $K_{1,n-1}$ is a star graph with n-vertices. A star graph is a complete bipartite graph if a single vertex belongs to one set and all the remaining vertices belong to the other set.

The minimum total dominating energy of Star graph is given below.

Minimum total dominating energy of Star graph			
Value of n	Required energy	The value of $2(n-1)$	Difference between $2(n-1)$ and energy
5	3.4523	8	4.5477
6	5.4186	10	4.5814
7	5.9748	12	6.252
8	6.3991	14	7.6009
9	6.6289	16	9.3711
10	6.9801	18	11.0199
11	7.0123	20	12.9877
12	7.6296	22	14.3704
13	9.0104	24	14.9896
14	11.8767	26	14.1233
15	15.6370	28	12.363
16	17.9817	30	12.0183
17	18.7271	32	13.2729

From the above table we can say that minimum total dominating energy of Star graph is non-hyperenergetic for n vertices.

Line graph of S_4.

Let G be a Star graph with 4 vertices $= \{v_1, v_2, v_3, v_4\}$.

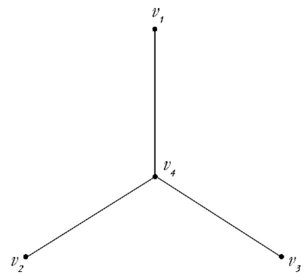

Figure (vii) Star Graph S_4.

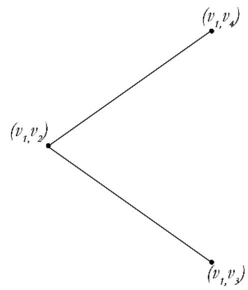

Figure (viii) $L(S_4)$.

The minimum total dominating set, $L(S_4) = \{(v_1, v_2), (v_1, v_4)\}$.

The minimum cardinality of a total dominating set $\gamma_t(L(S_4)) = 2$.

The minimum total dominating matrix,

$$A_{TD}(L(S_4)) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

The characteristic polynomial is $x^3 - 2x^2 - x + 1$.

The eigenvalues are $\lambda_1 = -0.8019$ $\lambda_2 = 0.5550$ $\lambda_3 = 2.2470$.

Therefore, the minimum total dominating energy, $E_{TD}(L(S_4)) = 3.6079$.

Line graph of S_5.

Let G be a Star graph with 5 vertices $= \{v_1, v_2, v_3, v_4, v_5\}$.

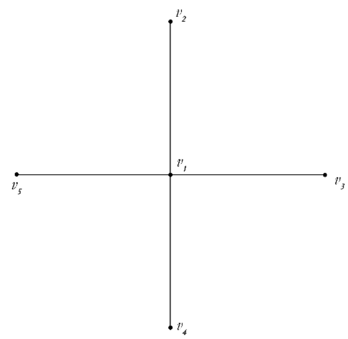

Figure (ix) Star graph S_5.

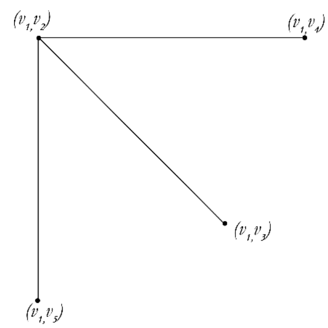

Figure (x) $L(S_5)$.

The minimum total dominating set, $L(S_5) = \{(v_1, v_2), (v_1, v_4)\}$.

The minimum cardinality of a total dominating set $\gamma_t(L(S_5)) = 2$.

The minimum total dominating matrix,

$$A_{TD}(L(S_5)) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

The characteristic polynomial is $x^4 - 2x^3 - 2x^2 + 2x + 0$.

The eigenvalues are $\lambda_1 = -1.1701$ $\lambda_2 = 0$ $\lambda_3 = 0.6889$ $\lambda_4 = 2.4812$.

Therefore, the minimum total dominating energy, $E_{TD}(L(S_5)) = 4.3402$.

Minimum total dominating energy of $L(S_n)$			
Value of n	Required energy	The value of $2(n-1)$	Difference between $2(n-1)$ and energy
4	3.6079	6	2.3921
5	4.3402	8	3.6598
6	5.8793	10	4.1207
7	6.2810	12	5.7190
8	8.7291	14	5.2709
9	9.9181	16	6.0819
10	11.0234	18	6.9766
11	12.8920	20	7.1080
12	14.0191	22	7.9809
13	16.9204	24	7.0796

Hence, we can say that minimum total dominating energy of Line graph of S_n is non-hyperenergetic for n vertices.

Minimum total dominating energy of Star graph and minimum total dominating energy of Line graph of S_n plotted in the below graph to compare against the value of $2(n-1)$.

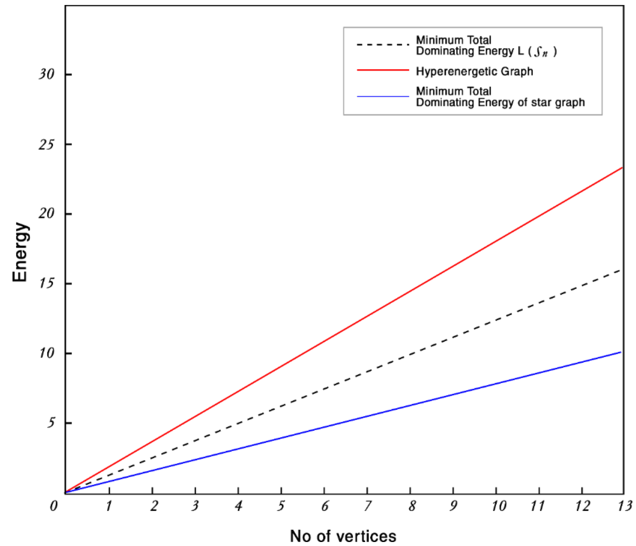

Figure (xi) Comparison of energies of Star graph.

From the above graph we can say that minimum total dominating energy Star graph and Line graph of S_n is non-hyperenergetic for n vertices.

Conclusion

In this paper, minimum total dominating energy and hyperenergetic of graph is defined and has been found that Friendship graph is non-hyper energetic for n vertices. Line graph of Friendship graph is hyperenergetic for $n \geq 5$. Wheel graph is non-hyperenergetic for n vertices. Line graph of Wheel graph is hyperenergetic for $n \geq 4$. Star graph and Line graph of star graph is non-hyperenergetic for n vertices. For future research, the minimum total dominating hyperenergetic can be compared for other standard graphs.

References

[1] E. J. Cockayne, R. M. Dawes and S. T. Hedetniemi, Total dominations in graphs, Networks 10 (1980), 211-219.

[2] E. J. Cockayne and S. T. Hedetniemi, Towards a theory of domination in graphs, Networks (1977), 247-261.

[3] I. Gutman, The energy of a graph, Ber. Math. Statist. Sket. For Schungsz, Graz 103 (1978), 1-22.

[4] I. Gutman, The energy of a graph: Old and new results (Eds: A. Betten, A. Kohnert, R. Laue, A. Wassermann), Algebraic Combinatorics and Applications, Springer, Berlin (2001), 196-211.

[5] V. R. Kulli, Theory of domination in graphs, Vishwa International Publications, Gulbarga, India, 2010.

[6] S. Meenakshi and S. Lavanya, A survey on energy of graphs, Annals of Pure and Applied Mathematics ISSN:2279-087X(P) 8(2) (2014), 183-191.

[7] M. R. Rajesh Kanna, B. N. Dharmndra and G. Sridhara, The minimum dominating energy of a graph, International Journal of Pure and Applied Mathematics 85(4) (2013), 707-718.

[8] I. Gutman, Y. Hou, H. B. Walikar, H. S. Ramane and P. R. Hampiholi, No Huckel graph is hyperenergetic, Journal of the Serbian Chemical Society 65(11) (2000), 799-801.

[9] G. Indulal, I. Gutman and Vijaykumar, On distance energy of graph, MATCH Commun. Math. Comput. Chem. 60 (2008), 461-472.

[10] C. Adiga, A. Bayad, J. Gutman and S. A. Srinivas, The minimum covering energy of a graph, Kragujevac J. Sci. 34 (2012), 39-56.

[11] I. Gutman and B. Zhou, Laplacian energy of a graph, Lin. Algebra Appl. 414 (2006), 29-37.

[12] M. R. Joo, D. Yandeh and M. Kiani Mirzakhah, Incidence energy of a graph MATCH, Commun. Math. Comput. Chem. 62 (2009), 561-572.

[13] M. R. Rajesh Kanna, B. N. Dharmndra and G. Sridhara, The minimum dominating energy of a graph, International Journal of Pure and Applied Mathematics 85(4) (2013), 707-718.