Advances and Applications in Mathematical Sciences
Volume 21, Issue 3, January 2022, Pages 1491-1508

A STUDY ON FUZZY CRITICAL PATH WITH QUADRILATERAL FUZZY NUMBERS

N. RAMESHAN[1] and D. STEPHEN DINAGAR[2]

[1]Research Scholar, [2]Associated Professor
PG and Research, Department of Mathematics
T. B. M. L. College
(Affiliated to Bharathidasan University)
Porayar-609 603, Tamil Nadu, India
E-mail: nrameshan14@gmail.com
dsdina@rediffmail.com

Abstract

We represent a way to identify the critical path utilizing critical activities of a quadrilateral fuzzy number (QDFN), which is a special instance of an octagonal fuzzy number, in this research (OCFN). Left skewed QDFN and right skewed QDFN membership functions have been defined, and both QDFN's arithmetic operations have been described. A numerical representation was presented separately for both left and right skewed QDFN to help grasp the concept.

In this paper, we use critical activities to determine the critical path of a quadrilateral fuzzy number (QDFN), which is a special case of the octagonal fuzzy number (OCFN). The left skewed QDFN and the right skewed QDFN membership functions have been defined, and the arithmetic operations of both QDFNs have been examined.

1. Introduction

The network diagram is critical in understanding the project completion time. Generally, a project will include a number of activities. Some activities are self-contained, while others may be dependent on others. Network analysis is a technique for determining the various sequences of activities associated with a project as well as its completion time. To solve decision

2020 Mathematics Subject Classification: Primary 03E72; Secondary 90B99.
Keywords: Octagonal fuzzy number, Quadratic fuzzy number, Critical path method, Fuzzy project network, Fuzzy ranking.
Received October 30, 2021; Accepted November 10, 2021

making problems, the critical path method, a network technique can be used. Because the network activities can be carried out concurrently, the shortest time to complete the project is the length of the longest path from the beginning to the end. The ambiguity of the problem's time parameters, on the other hand, has resulted in the development of fuzzy CPM. T. Pathinathan and S. Santhosh Kumar [3] introduced arithmetic operations of quadrilateral fuzzy numbers. The unidentified problem that could occur in a specific field can be very well controlled using this fuzzy CPM. J. Johnson Savarimuthu and P. Abirami [2] proposed fundamental concepts of fuzzy numbers. D. Stephen Dinagar and N. Rameshan [1, 4] discussed the critical path by octagonal fuzzy number and developed the procedure for finding the critical path using the TOPSIS method. D. Stephen Dinagar and D. Abirami [5] studied the method for fuzzy critical path in another view point.

In this paper, organized as: In section 2, some basic concepts of fuzzy numbers were recalled. In section 3, basic definitions of quadrilateral fuzzy numbers were introduced. In section 4, the arithmetic operation of QDFN was proposed. In section 5, ranking function, property and algorithms were discussed. In section 6, a numerical example was illustrated. In section 7, the conclusion of this research is also included.

2. Preliminaries

Some basic definitions have been proposed, which will be important in this paper.

Definition 2.1 [**Fuzzy Number**]**.** The fuzzy number $\widetilde{A}$ is fuzzy set if membership function satisfies (i) A fuzzy set of the universe of discourse X is convex (ii) $\widetilde{A}$ is normal if $\exists\, x_i \in X, \mu_{\widetilde{A}}(x_i) = 1$ (iii) $\mu_{\widetilde{A}}(x)$ is piecewise continuous.

Definition 2.2 [Octagonal fuzzy number]**.** A fuzzy number $\widetilde{A}$ is a normal octagonal fuzzy number denoted by $(o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8)$, whose membership function is

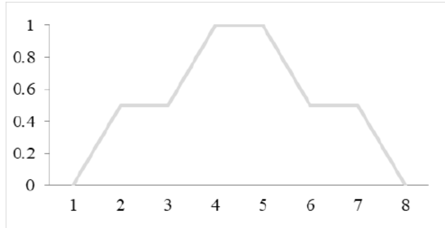

Figure 1. Membership Function of OCFN.

$$\mu_{\tilde{o}}(x) = \begin{cases} 0, & x \le o_1 \\ \lambda\left(\dfrac{x - o_1}{o_2 - o_1}\right), & o_1 \le x \le o_2 \\ \lambda, & o_2 \le x \le o_3 \\ \lambda + (1-\lambda)\left(\dfrac{x - o_3}{o_4 - o_3}\right), & o_3 \le x \le o_4 \\ 1, & o_4 \le x \le o_5 \\ \lambda + (1-\lambda)\left(\dfrac{o_6 - x}{o_6 - o_5}\right) & o_5 \le x \le o_6 \\ \lambda, & o_6 \le x \le o_7 \\ \lambda\left(\dfrac{o_8 - x}{o_8 - o_7}\right) & o_7 \le x \le o_8 \\ 0, & x \ge o_8. \end{cases} \quad \text{where } 0 < \lambda < 1$$

If $\lambda = 0$ and $\lambda = 1$ then OCFN reduces to trapezoidal fuzzy number (q_3, q_4, q_5, q_6) and trapezoidal fuzzy number (q_1, q_4, q_5, q_8) respectively and its follows some certain conditions

(i) $\mu_{\tilde{o}}(x)$ is a continuous function in the interval $(0, 1)$

(ii) $\mu_{\tilde{o}}(x)$, x is strictly increasing and continuous functions on (q_1, q_2, q_3, q_4)

(iii) $\mu_{\tilde{o}}(x)$, x is strictly decreasing and continuous functions on (q_5, q_6, q_7, q_8)

3. Quadratic Fuzzy Number

In this part, we defined the notion of QDFN and their arithmetic operations.

Definition 3.1. Considering the slopes of line segment $[q_1, q_2]$, $[q_2, q_3]$,

$\ldots, [q_7, q_8]$ are $m_1, m_2, \ldots, m_7$ respectively. If the slope of the line segments then the given fuzzy number is a complete OCFN.

(i) In an OCFN, if either $m_1 \neq m_2, m_2 = m_3, m_4 = m_5 = m_6 = m_7$ (or) $m_1 \neq m_2, m_2 = m_3, m_4 = m_5 = m_6 = m_7$ then Quadrilateral Fuzzy number (Left Skewed).

(ii) In an OCFN, if either $m_1 = m_2 = m_3, m_4 = m_5 = m_6, m_6 \neq m_7$ (or) $m_1 = m_2 = m_3, m_4 = m_5 = m_6, m_6 \neq m_7$ then Quadrilateral Fuzzy number (Right Skewed).

Definition 3.2. A fuzzy number $\tilde{Q}$ is a normal QDFN denoted by (q_1, q_2, q_4, q_8) (or) (q_1, q_2, q_5, q_8) which real numbers, the membership function of QDFN (Left Skewed) [LSQDFN] is

$$\mu_{\tilde{Q}_L}(x) = \begin{cases} \dfrac{x - q_1}{q_2 - q_1}, & q_1 \leq x \leq q_2 \\ \dfrac{q_2 - x}{q_2 - q_4}, & q_2 \leq x \leq q_3 \\ 1, & q_4 = 1 \\ \dfrac{x - q_4}{q_4 - q_8}, & q_4 \leq x \leq q_8 \\ 0, & \text{otherwise} \end{cases} \quad \text{(OR)}$$

$$\mu_{\tilde{Q}_L}(x) = \begin{cases} \dfrac{x - q_1}{q_2 - q_1}, & q_1 \leq x \leq q_2 \\ \dfrac{q_2 - x}{q_2 - q_5}, & q_2 \leq x \leq q_5 \\ 1, & q_5 = 1 \\ \dfrac{x - q_5}{q_5 - q_8}, & q_5 \leq x \leq q_8 \\ 0, & \text{otherwise} \end{cases}$$

and its satisfies

(i) $\mu_{\tilde{Q}_L}(x)$ is a continuous functions in the interval $(0, 1)$

(ii) $\mu_{\tilde{Q}_L}(x)$, x is continuous and increasing functions on $[q_1, q_2], [q_2, q_4]$ (or) $[q_1, q_2], [q_2, q_5]$

(iii) $\mu_{\tilde{Q}_L}(x)$, x is strictly decreasing and continuous functions on $[q_4, q_8]$ (or) $[q_5, q_8]$.

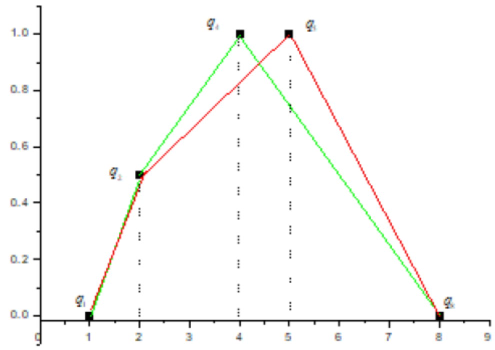

Figure 2. Left Skewed Quadrilateral Fuzzy Number.

Definition 3.3. A fuzzy number $\tilde{Q}$ is a normal QDFN denoted by (q_1, q_4, q_7, q_8) (or) (q_1, q_5, q_7, q_8) which real numbers, the membership function of QDFN (Right Skewed) [RSQDFN] is

$$\mu_{\tilde{Q}_R}(x) = \begin{cases} \dfrac{x - q_1}{q_4 - q_1}, & q_1 \leq x \leq q_4 \\ 1, & q_4 = 1 \\ \dfrac{q_4 - x}{q_7 - q_4}, & q_4 \leq x \leq q_7 \\ \dfrac{q_7 - x}{q_8 - q_7}, & q_7 \leq x \leq q_8 \\ 0, & \text{otherwise} \end{cases} \quad \text{(OR)}$$

$$\mu_{\tilde{Q}_R}(x) = \begin{cases} \dfrac{x - q_1}{q_5 - q_1}, & q_1 \leq x \leq q_5 \\ 1, & q_5 = 1 \\ \dfrac{q_5 - x}{q_7 - q_5}, & q_5 \leq x \leq q_7 \\ \dfrac{q_7 - x}{q_8 - q_7}, & q_4 \leq x \leq q_8 \\ 0, & \text{otherwise} \end{cases}$$

and its satisfies

(i) $\mu_{\tilde{Q}_L}(x)$ is a continuous functions in the interval $(0, 1)$

(ii) $\mu_{\tilde{Q}_L}(x)$, x is continuous and increasing function on $[q_1, q_4]$ (or) $[q_1, q_5]$

(iii) $\mu_{\tilde{Q}_L}(x)$, x is strictly decreasing and continuous functions on $[q_4, q_7], [q_7, q_8]$ (or) $[q_1, q_5], [q_7, q_8]$.

4. Arithmetic Operations using QDFN

If $P_{\tilde{Q}} = (p_1, p_2, p_3, p_4)$ and $Q_{\tilde{Q}} = (q_1, q_2, q_3, q_4)$ be any two QDFNs then

(i) **Addition of QDFN**

$$P_{\tilde{Q}} + Q_{\tilde{Q}} = (p_1 + q_1,\ p_2 + q_2,\ p_3 + q_3,\ p_4 + q_4)$$

(ii) **Subtraction of QDFN**

$$P_{\tilde{Q}} - Q_{\tilde{Q}} = (p_1 - q_1,\ p_2 - q_2,\ p_3 - q_3,\ p_4 - q_4)$$

(iii) **Multiplication of QDFN**

If $P_{\tilde{Q}}$ and $Q_{\tilde{Q}}$ are LSQDFN then

$$P_{\tilde{Q}} * Q_{\tilde{Q}} = \begin{pmatrix} \frac{p_1(q_1 + q_2 + q_3 + q_4)}{4}, \frac{p_2(q_1 + q_2 + q_3 + q_4)}{4}, \\ \frac{p_3(q_1 + q_2 + q_3 + q_4)}{4}, \frac{p_4(q_1 + q_2 + q_3 + q_4)}{4} \end{pmatrix}$$

If $P_{\tilde{Q}}$ and $Q_{\tilde{Q}}$ are RSQDFN then

$$P_{\tilde{Q}} * Q_{\tilde{Q}} = \begin{pmatrix} \frac{(p_1 + p_2 + p_3 + p_4)q_1}{4}, \frac{(p_1 + p_2 + p_3 + p_4)q_2}{4}, \\ \frac{(p_1 + p_2 + p_3 + p_4)q_3}{4}, \frac{(p_1 + p_2 + p_3 + p_4)q_4}{4} \end{pmatrix}$$

(iv) **Division of QDFN**

If $P_{\tilde{Q}}$ and $Q_{\tilde{Q}}$ are LSQDFN then

$$\frac{P_{\tilde{Q}}}{Q_{\tilde{Q}}} = \begin{pmatrix} \dfrac{p_4}{4(q_1 + q_2 + q_3 + q_4)}, \dfrac{p_3}{4(q_1 + q_2 + q_3 + q_4)}, \\ \dfrac{p_3}{4(q_1 + q_2 + q_3 + q_4)}, \dfrac{p_4}{4(q_1 + q_2 + q_3 + q_4)} \end{pmatrix}$$

If $P_{\tilde{Q}}$ and $Q_{\tilde{Q}}$ are RSQDFN then

$$\frac{P_{\tilde{Q}}}{Q_{\tilde{Q}}} = \begin{pmatrix} \dfrac{q_4}{4(p_1 + p_2 + p_3 + p_4)}, \dfrac{q_2}{4(p_1 + p_2 + p_3 + p_4)}, \\ \dfrac{q_3}{4(p_1 + p_2 + p_3 + p_4)}, \dfrac{q_4}{4(p_1 + p_2 + p_3 + p_4)} \end{pmatrix}$$

5. Ranking function, Properties and Algorithm

5.1 Ranking function

(i) Let $\tilde{Q} = (q_1, q_2, q_3, q_4)$ be the QDFN maps into set of real numbers. The ranking function of QDFN is defined as

$$R(\tilde{Q}) = \frac{2(q_1 + q_4) + q_2 + q_3}{4}$$

(ii) Let $\tilde{Q} = (q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8)$ be the OCFN maps into set of real numbers. The ranking function used as

$$R(\tilde{Q}) = \frac{2(q_1 + q_8) + 3(q_2 + q_7) + 4(q_3 + q_6) + 5(q_4 + q_5)}{28}$$

5.2 Symbolic Representation

For Basic computations, Let us use the following notations,

$A_{FN}(ij) =$ Activity b/w event i (tail) and event j (head)

$E_{FN}(i) =$ Earliest occurrence event time i

$L_{FN}(j) =$ Latest occurrence event time j

$ES_{FN}(ij) =$ Earliest starting time from activity i to j

$EF_{FN}(ij) =$ Earliest finishing time from activity i to j

$LS_{FN}(ij) =$ Latest starting time from activity i to j

$LF_{FN}(ij) =$ Earliest finishing time from activity i to j

$TF_{FN}(ij) =$ Total Float time of $A_{FN}(ij)$

$D_{FN}(ij) =$ Estimated completion time.

5.3 Properties

Fix the starting node as zero [i.e., $ES_{FN}(ij) = (0, 0, 0, 0)$]

(i) $ES_{FN}(ij) = Max\{ES_{FN}(ij) + AF_{FN}(ij)\}$

(ii) $LS_{FN}(ij) = Min\{LS_{FN}(ij) - AF_{FN}(ij)\}$

(iii) $TF_{FN}(ij) = [LF_{FN}(ij) - LS_{FN}(ij)] - AF_{FN}(ij)$

5.4 Algorithm

Step 1. Construct a project network diagram and identify each single node of the OCFN.

Step 2. Calculate the slopes between each node's subsequent points in order to determine the respective QDFN (Ref. Definition 3.1).

Step 3. Calculate the distance between nodes in the specified project network using the distance formula.

Step 4. Determine the predicted length of all activities using the fuzzy ranking technique.

Step 5. Find the earliest fuzzy event time and latest fuzzy event time.

Step 6. By the property (iii), to find the total float of each activity.

Step 7. The is critical activity, if.

Step 8. To determine the fuzzy critical path from the start to finish node of a fuzzy project.

5.5 Basic Formulae

If $(x_1, y_1), (x_2, y_2)$ be the coordinate of the two points then the

(i) Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(ii) Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

6. Illustration

Let us consider OCFN activities duration time in the following project network with five nodes.

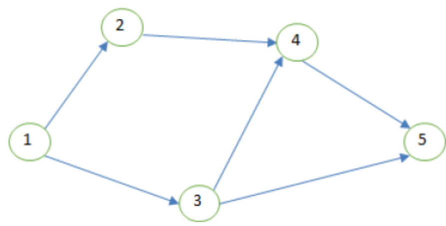

Figure 4. Project Network.

Calculation

We need to modify a fuzzy number to a standard expected time using the ranking method. To discover the fuzzy critical path, the conventional method will be utilized. The project network is made up of five nodes, each with its own set of hourly activities.

(a) Left Skewed Quadrilateral Fuzzy number (LSQDFN)

Case (i)**.** LSQDFN-1

Table1. OCFN, LSQDFN 1.

Node	OCFN	Slope $\begin{pmatrix} m_1, m_2, m_3, m_4, \\ m_5, m_6, m_7 \end{pmatrix}$	LSQDFN 1
1	(3.5,0.1),(4,0.2),(5,0.3), (6,0.4),(8,0.5),(10,0.6), (12,0.7),(14,0.8)	(0.2,0.1,0.1,0.05, 0.05, 0.05, 0.05)	(3.5,0.1),(4,0.2), (6,0.4), (14,0.8)
2	(8,0.1),(10,0.2),(13,0.3), (16,0.4),(20,0.5),(24,0.6), (28,0.7),(32,0.8)	(0.05,0.033,0.033, 0.025,0.025, 0.025,0.025)	(8,0.1),(10,0.2), (16,0.4), 32,0.8)
3	(8,0.1),(9,0.2),(11,0.3),	(0.1,0.05,0.05,0.03,	(8,0.1),(9,0.2),

	(13,0.4),(16,0.5),(19,0.6),(22,0.7),(25,0.8)	0.03,0.03,0.03)	(13,0.4),(25,0.8)
4	(4.5,0.1),(5,0.2),(6,0.3),(7,0.4),(10,0.5),(13,0.6),(16,0.7),(19,0.8)	(0.2,0.1,0.1,0.03,0.03, 0.03,0.03)	(4.5,0.1),(5,0.2),(7,0.4), (19,0.8)
5	(1,0.1),(2,0.2),(4,0.3),(6,0.4),(9,0.5),(12,0.6),(15,0.7),(18,0.8)	(0.1,0.05,0.05,0.03,0.03,0.03,0.03)	(1,0.1),(2,0.2),(6,0.4), (18,0.8)

Table 2. Calculation-Duration time.

$A_{FN}(ij)$	Activity duration			
	For OCFN	Expected time	For LSQDFN 1	Expected time
1-2	(0, 1, 8, 9, 9, 9, 9, 9)	7.36	(0, 9, 9, 9)	9
1-3	(3, 4, 1, 0, 1, 2, 3, 4)	1.86	(3, 0, 3, 4)	4.25
2-4	(12, 12, 12, 12, 11, 10, 9, 8)	10.93	(12, 12, 9, 8)	15.25
3-4	(9, 7, 5, 3, 1, 1, 3, 5)	3.64	(13, 9, 4.5, 3.5)	8.5
3-5	(3.5, 0, 3.5, 7, 10.5, 14, 17.5, 19.5)	9.14	(7, 7, 7, 7)	17.63
4-5	(3.5, 3, 2, 1, 1, 1, 1, 1)	10.75	(3.5, 3, 1, 1)	16.88

Table 3. Calculation-Earliest, Latest times.

$A_{FN}(ij)$	For OCFN			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	7.36	0	7.36
1-3	0	1.86	12.79	14.65
2-4	7.36	18.29	7.36	18.29
3-4	1.86	5.5	14.65	18.29
3-5	1.86	11	19.9	29.04
4-5	18.29	29.04	18.29	29.04

Table 4. Calculation-Earliest, Latest time.

$A_{FN}(ij)$	For LSQDFN-2			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	9	0	9
1-3	0	4.25	8.37	12.62
2-4	9	24.25	9	24.25
3-4	4.25	12.75	15.75	24.25
3-5	4.25	21.88	23.5	41.13
4-5	24.25	41.13	24.25	41.13

Table 5. Completion Time.

Path	Total Completion Time	
	Under OCFN	Under LSQDFN-I
1-2-4- 5	54.69	74.38
1-3-4- 5	36.4	58.13
1-3-5	7.36	26.13

The LSQDFN -1 conditions $(m_1 \neq m_2, m_2 = m_3, m_4 = m_5 = m_6 = m_7)$ are satisfied t in this case, and table 7 shows that the 1-2-4-5 is a critical path.

Case (ii)**.** LSQDFN-2

Table 6. OCFN, LSQDFN 2.

Node	OCFN	Slope $\begin{pmatrix} m_1, m_2, m_3, m_4, \\ m_5, m_6, m_7 \end{pmatrix}$	LSQDFN 2
1	(0,0.1),(1,0.2),(3.5,0.3), (6,0.4),(8.5,0.5), (12,0.6), (15.5,0.7),(19,0.8)	(0.01,0.04,0.04, 0.04,0.029,0.029, 0.029)	(0,0.1),(1,0.2), (8.5,0.5),(19,0.8)

2	(2,0.1),(3,0.2),(5,0.3),(7,0.4),(9,0.5),(14,0.6),(19,0.7), (24,0.8)	(0.1,0.05,0.05,0.05,0.02,0.02,0.02)	(2,0.1),(3,0.2),(9,0.5), (24,0.8)
3	(3,0.1),(4,0.2),(7.5,0.3),(11,0.4),(15,0.5),(19,0.6),(23,0.7),(27,0.8)	(0.1,0.029,0.029,0.029,0.025,0.025,0.025)	(3,0.1),(4,0.2),(15,0.5),(27,0.8)
4	(2,0.1),(5,0.2),(7,0.3),(9,0.4),(11,0.5),(13.5,0.6),(16,0.7),(18.5,0.8)	(0.03,0.05,0.05,0.05,0.04,0.04,0.04)	(2,0.1),(5,0.2),(11,0.5), (18.5,0.8)
5	(1,0.1),(2,0.2),(4,0.3),(6,0.4),(9,0.5),(12,0.6),(15,0.7),(18,0.8)	(0.07,0.05,0.05,0.05,0.04,0.04,0.04)	(3.5,0.1),(5,0.2),(11,0.5), (18.5,0.8)

Table 7. Calculation-Duration time.

$A_{FN}(ij)$	Activity duration			
	For OCFN	Expected time	For LSQDFN 1	Expected time
1-2	(2, 2, 1.5, 1, 0.5, 2, 3.5, 5)	1.86	(2, 2, 0.5, 5)	4.13
1-3	(3, 3, 4, 5, 6.5, 7, 7.5, 8)	5.54	(4, 2, 0, 6)	5.5
2-4	(0, 2, 2, 2, 2, 0.5, 3, 5.5)	2	(0, 2, 2, 5.5)	3.75
3-4	(1, 1, 0.5, 2, 4, 5.5, 7, 8.5)	3.46	(1, 1, 4, 8.5)	6
3-5	(2.5,1, 0.5, 2, 4, 5.5, 7, 8.5)	3.57	(2.5, 1, 4, 8.5)	6.75
4-5	(9, 8, 3.5, 1, 1.5, 2, 2.5, 3)	3.21	(9, 8, 1.5, 3)	8.38

Table 8. Calculation-Earliest, Latest times.

$A_{FN}(ij)$	For OCFN			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	1.86	5.14	7
1-3	0	5.54	0	5.54
2-4	1.86	3.86	7	9
3-4	5.54	9	5.54	9
3-5	2	5.57	8.64	12.21

4-5	9	12.21	9	12.21

Table 9. Calculation-Earliest, Latest time.

$A_{FN}(ij)$	For LSQDFN-2			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	4.13	0	4.13
1-3	0	5.5	3.62	1.88
2-4	4.13	7.88	4.13	7.88
3-4	5.5	11.5	1.88	7.88
3-5	5.5	12.25	9.51	16.26
4-5	7.88	16.26	7.88	16.26

Table 10. Completion Time.

Path	Total Completion Time	
	Under OCFN	Under LSQDFN-I
1-2-4- 5	52.58	72.5
1-3-4- 5	42.08	59
1-3-5	20.54	32

The LSQDFN -2 conditions $(m_1 \neq m_2, m_2 = m_3, m_4 = m_5 = m_6 = m_7)$ are satisfied t in this case, and table 7 shows that the 1-2-4-5 is a critical path.

(b) Right Skewed Quadrilateral Fuzzy number (RSQDFN)

Case (i)**.** RSQDFN-1

Table 11. OCFN, RSQDFN 1.

Node	OCFN	Slope $\begin{pmatrix} m_1, m_2, m_3, m_4, \\ m_5, m_6, m_7 \end{pmatrix}$	RSQDFN 1
1	(2,0.1),(5,0.2),(14,0.3),	(0.03,0.03,0.03,0.02	(2,0.1),(17,0.4),

	(17,0.4),(20.5,0.5),(24,0.6), (27,0.7),(31.5,0.8)	8,0.028,0.028,0.025)	(27,6,0.4),(14,0.8)
2	(2,0.1),(4,0.2),(6,0.3), (8,0.4),(11.5,0.5),(15,0.6), (18.5,0.7),(22.5,0.8)	(0.05,0.05,0.05,0.02 8,0.028,0.028,0.025)	(2,0.1),(8,0.2), (18.5,0.7),(22.5,0.8)
3	(5,0.1),(9,0.2),(13,0.3), (17,0.4),(21.5,0.5),(26,0.6), (30.5, 0.7),(35.5,0.8)	(0.025,0.0250.025, 0.02,0.020.02,0.028)	(5,0.1),(17,0.4), (30,0.7),(35.5,0.8)
4	(14,0.1),(16,0.2),(18,0.3), (20,0.4),(22.5,0.5),(25,0.6), (27.5,0.7),(30.5,0.8)	(0.05,0.05,0.05,0.04, 0.04,0.04,0.03)	(14,0.1),(20,0.4), (27.5,0.7, (30.5,0.8)
5	(8.5,0.1),(9,0.2),(9.5,0.3), (10,0.4),(11,0.5),(12,0.6), (13,0.7),(14.5,0.8)	(0.2,0.2,0.2,0.1, 0.1,0.1,0.07)	(8.5,0.1),(10,0.4), (13,0.7), (14.5,0.8)

Table 12. Calculation-Duration time.

$A_{FN}(ij)$	Activity duration			
	For OCFN	Expected time	For LSQDFN 1	Expected time
1-2	(4.5, 6, 8, 10, 12, 14, 16, 18)	11.04	(4.5, 6, 10, 18)	15.25
1-3	(4.5, 5, 6, 7, 8, 9, 10, 11)	7.54	(4, 2, 0, 6)	10.75
2-4	(3.5, 5, 7, 9, 10, 11, 12, 13)	8.96	(0, 2, 2, 5.5)	11.75
3-4	(3.5, 4, 5, 6, 6, 6, 6, 6)	5.46	(1, 1, 4, 8.5)	7.25
3-5	(7, 7, 7, 7, 7, 7, 7, 7)	7	(2.5, 1, 4, 8.5)	10.5
4-5	(3.5, 3, 2, 1, 1, 1, 1, 1)	1.54	(9, 8, 1.5, 3)	3.25

Table 13. Calculation-Earliest, Latest times.

$A_{FN}(ij)$	For OCFN			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	11.04	0	11.04
1-3	0	7.54	7	14.54
2-4	11.04	20	11.04	20
3-4	7.54	13	14.54	20

3-5	7.54	14.54	14.54	21.54
4-5	20	21.54	20	21.54

Table 14. Calculation-Earliest, Latest time.

$A_{FN}(ij)$	For LSQDFN-2			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	15.25	0	15.25
1-3	0	10.75	1.75	12.5
2-4	15.25	27	15.25	27
3-4	10.75	18	19.75	27
3-5	10.75	21.25	19.75	30.25
4-5	27	30.25	27	30.25

Table 15. Completion Time.

Path	Total Completion Time	
	Under OCFN	Under LSQDFN-I
1-2-4-5	52.58	72.5
1-3-4-5	42.08	59
1-3-5	20.54	32

The RSQDFN-1 conditions $(m_1 = m_2 = m_3, m_4 = m_5 = m_6, m_6 \neq m_7)$ are satisfied t in this case, and table 11 shows that the 1-2-4-5 is a critical path.

Case (ii)**.** RSQDFN-2

Table 16. OCFN, RSQDFN 2.

Node	OCFN	Slope $\begin{pmatrix} m_1, m_2, m_3, m_4, \\ m_5, m_6, m_7 \end{pmatrix}$	RSQDFN 1
1	(2,0.1),(5,0.2),(8,0.3),	(0.03,0.03,0.03,0.02	(2,0.1),(14,0.5),

	(11,0.4),(14,0.5),(17.5,0.6), (21,0.7),(25.5,0.8)	8,0.028,0.028, 0.025)	(21,0.74), (25,0.8)
2	(2,0.1),(2.5,0.2),(3,0.3),(3.5,0.4),(4,0.5),(5,0.6), (6,0.7),(7.5,0.8)	(0.2,0.2,0.2,0.2, 0.1,0.1,0.07)	(2,0.1),(4,0.5), (6,0.7), (7.5,0.8)
3	(5,0.1),(9,0.2),(13,0.3), (17,0.4),(21.5,0.5),(26,0.6), (30.5,0.7),(35.5,0.8)	(0.025,0.025,0.025, 0.02,0.02,0.01)	(5,0.1),(17,0.4), (30,0.7),(35.5,0.8)
4	(4,0.1),(6,0.2),(8,0.3), (10,0.4),(12.5,0.5),(15,0.6), (18,0.7),(21.5,0.8)	(0.05,0.05,0.05,0.05, 0.03, 0.03,0.02)	(4,0.1),(12.5,0.5), (18,0.7), (21.5,0.8)
5	(1,0.1),(5,0.2),(9,0.3), (13,0.4),(17,0.5),(21.5,0.6), (26,0.7),(33,0.8)	(0.03,0.03,0.03,0.03, 0.02,0.02,0.07)	(1,0.1),(17,0.5), (26,0.7), 33,0.8)

Table 17. Calculation-Duration time.

$A_{FN}(ij)$	Activity duration			
	For OCFN	Expected time	For LSQDFN 1	Expected time
1-2	(0,2.5,5,7.5,10,12.5,15,17.5)	8.75	(0,10,15,18)	15
1-3	(3,4,.5,6,7.5,8.5,9.5,10.51)	6.75	(3,7.5,9.5,11)	11
2-4	(2,3.5,5,6.5,8,10,12,14)	7.54	(2.8,12,14)	13
3-4	(1,3,5,7,9.5,11,12.5,14)	7.96	(1,9.5,12.5,14)	13
3-5	(4,4,4,4,4.5,4.5,5.5,5.5)	4.43	(4,4.5,5.5,5.5)	7.25
4-5	(1,1,3.5,7,9,10,12)	5.96	(1,7,10,12)	10.75

Table 18. Calculation-Earliest, Latest times.

$A_{FN}(ij)$	For OCFN			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	8.75	0	8.75
1-3	0	6.75	1.58	8.33
2-4	8.75	16.29	8.75	16.29
3-4	6.75	14.71	8.33	16.29

3-5	6.75	11.18	17.82	22.25
4-5	0	8.75	0	8.75

Table 19. Calculation-Earliest, Latest time.

$A_{FN}(ij)$	For LSQDFN-2			
	$ES_{FN}(ij)$	$EF_{FN}(ij)$	$LS_{FN}(ij)$	$LF_{FN}(ij)$
1-2	0	15	0	15
1-3	0	11	4	15
2-4	15	28	15	28
3-4	11	24	15	28
3-5	11	18.25	31.5	38.75
4-5	28	38.75	28	38.75

Table 20. Completion Time.

Path	Total Completion Time	
	Under OCFN	Under LSQDFN-I
1-2-4- 5	52.58	72.5
1-3-4- 5	42.08	59
1-3-5	20.54	32

The RSQDFN-2 conditions $(m_1 = m_2 = m_3, m_4 = m_5 = m_6, m_6 \neq m_7)$ are satisfied t in this case, and table 11 shows that the 1-2-4-5 is a critical path.

7. Conclusion

For project decision makers to make the best planning and scheduling decisions, fuzzy critical path length is critical. To find a fuzzy critical path in a project network, we used both right and left skewed QDFN, which is a special case of OCFN, in this study. It should be observed that the propound perspective was supported by an adequate illustration. This concept could be further improved in the future to generalize QDFN of both left and right skewed approaches for use in any sort of optimization issue.

References

[1] D. Stephen Dinagar and N. Rameshan, Solving critical path in project scheduling by using TOPSIS ranking of generalized interval valued octagonal fuzzy numbers, Journal of Physical Sciences 24 (2019), 53-61.

[2] J. Jhonson Savarimuthu and P. Abirami, Octagonal fuzzy number and its corresponding matrices, International Journal of Mathematics and its Applications 6(1) (2018), 75-78.

[3] T. Pathinathan and S. Sathoshkumar, Quadrilateral fuzzy number, International Journal of Engineering and Technology 7(4.10) (2018), 1018-1021.

[4] D. Stephen Dinagar and N. Rameshan, A New approach for fuzzy critical path method using octagonal fuzzy numbers, International Journal of Pure and Applied Mathematics 119 (2018), 13.

[5] D. Stephen Dinagar and D. Abirami, A note on fuzzy critical path analysis in project networks, International Journal of Current Research 6(07) (2014), 7458-7465.