Advances and Applications in Mathematical Sciences
Volume 21, Issue 7, May 2022, Pages 3653-3672

ISOMORPHIC PROPERTIES ON INTUITIONISTIC FUZZY *k*-PARTITE HYPERGRAPHS

K. K. MYITHILI and R. KEERTHIKA

Department of Mathematics (CA)
Vellalar College for Women
Erode-638012, Tamilnadu, India
E-mail: mathsmyth@gmail.com

Department of Mathematics
Vellalar College for Women
Erode-638012, Tamilnadu, India
E-mail: keerthibaskar18@gmail.com

Abstract

Hypergraph is generalization of a graph, which can have an edge with more than two vertices. A k-partite hypergraph is a hypergraph whose vertices can be partitioned into k different independent sets. Any two IFk-PHGs (Intuitionistic fuzzy k-partite hypergraphs) has been considered and its isomorphism property is discussed in this paper. It has been proved that the isomorphism between two IFk-PHGs satisfies an equivalence relation and also their size and order are same.

1. Introduction

Euler was the first researcher who found graph theory in 1736. Graph concepts is used to solve many problems in various fields such as optimization techniques, computer science, number theory and algebra. Graph is a combination of a set of vertices V and a collection of subsets of V. In 1976, Berge [4] introduced the concepts of graph and hypergraph theory. The concept of an intuitionistic fuzzy set (IFS) was introduced in 1983 [3] and the concept of an intuitionistic fuzzy graph (IFG) introduced in 1994 [12]. The intuitionistic fuzzy matrix (IFM) notions and index is introduced in [1, 2]. The notions of fuzzy graphs and fuzzy hypergraphs was developed in [5].

2020 Mathematics Subject Classification: 05C72.
Keywords: Intuitionistic fuzzy k-partite hypergraphs (IFk-PHGs), Isomorphism, Weak isomorphism, Co-weak isomorphism, Equivalence relation.
Received January 13, 2022; Accepted February 22, 2022

The concept of intuitionistic fuzzy graphs (IFGs) and intuitionistic fuzzy hypergraphs (IFHGs) have been examined in [8, 10]. In [7], Nagoorgani and Malarvizhi conferred the isomorphism between fuzzy graphs. The authors defined the ideas of isomorphism on fuzzy hypergraphs in [11]. R. Parvathi, S. Thilagavathi, K. T. Atanassov [9] refined the ideas of Isomorphism on Intuitionistic fuzzy directed hypergraphs.

Finally in [6], the authors initiated the concepts of k-partite graphs in intuitionistic fuzzy hypergraphs. In this paper, the isomorphism between two intuitionistic fuzzy k-partite hypergraphs was examined with some of their properties.

2. Preliminaries

Basic definitions relating to intuitionistic fuzzy set (IFS), intuitionistic fuzzy hypergraph (IFHG) and IFk-PHGs are discussed in this section.

Definition 2.1 [12]. Let a set E be fixed. An *intuitionistic fuzzy set* (IFS) V in E is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle / v_i \in E\}$, where the function $\mu_i : E \to [0, 1]$ and $\nu_i : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E$, $0 \le \mu_i(v_i) + \nu_i(v_i) \le 1$.

Definition 2.2 [8]. Consider a fixed set E and $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle \mid v_i \in V\}$ be an IFS. Six types of Cartesian products of n subsets (crisp sets) $V_1, V_2, \ldots, V_n$ of V over E are defined as follows

$$V_{i_1} \times_1 V_{i_2} \times_1 V_{i_3} \ldots \times_1 V_{i_n} =$$

$$\left\{\left\langle (v_1, v_2, \ldots, v_n), \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \right\rangle \middle| v_1 \in V_1, v_2 \in V_2, \ldots, v_n \in V_n\right\},$$

$$V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} \ldots \times_2 V_{i_n} = \left\{\left\langle (v_1, v_2, \ldots, v_n), \sum_{i=1}^{n} \mu_i - \sum_{i \ne j} \mu_i \mu_j\right.\right.$$

$$+ \sum_{i \ne j \ne k} \mu_i \mu_j \mu_k - \ldots + (-1)^{n-2}$$

$$\sum_{i \ne j \ne k \ldots \ne n} \mu_i \mu_j \mu_k \ldots \mu_n + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \rangle \mid v_1 \in V_1,$$

$$v_2 \in V_2, \ldots, v_n \in V_n\},$$

$$V_{i_1} \times_3 V_{i_2} \times_3 V_{i3} \ldots \times_3 V_{i_n} = \left\{\left\langle (v_1, v_2, \ldots, v_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \nu_i - \sum_{i \neq j} \nu_i \nu_j\right.\right.$$

$$+ \sum_{i \neq j \neq k} \nu_i \nu_j \nu_k - \ldots + (-1)^{n-2}$$

$$(-1)^{n-2} \sum_{i \neq j \neq k \ldots \neq n} \nu_i \nu_i \nu_k \ldots \nu_n + (-1)^{n-1} \prod_{i=1}^{n} \nu_i \rangle \mid v_1 \in V_1, v_2 \in V_2, \ldots, v_n \in V_n\},$$

$$V_{i_1} \times_4 V_{i_2} \times_4 V_{i_3} \ldots \times_4 V_{i_n} = \{\langle (v_1, v_2, \ldots, v_n), \min(\mu_1, \mu_2, \ldots, \mu_n),$$

$$\max(\nu_1, \nu_2, \ldots, \nu_n) \rangle \mid v_1 \in V_1, v_2 \in V_2, \ldots, v_n \in V_n\},$$

$$V_{i_1} \times_5 V_{i_2} \times_5 V_{i_3} \ldots \times_5 V_{i_n} = \{\langle (v_1, v_2, \ldots, v_n), \max(\mu_1, \mu_2, \ldots, \mu_n),$$

$$\min(\nu_1, \nu_2, \ldots, \nu_n) \rangle \mid v_1 \in V_1, v_2 \in V_2, \ldots, v_n \in V_n\},$$

$$V_{i_1} \times_6 V_{i_2} \times_6 V_{i_3} \ldots \times_6 V_{i_n} = \{\langle (v_1, v_2, \ldots, v_n), \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \nu_i}{n} \rangle \mid v_1 \in V_1,$$

$$v_2 \in V_2, \ldots, v_n \in V_n\}$$

It must be noted that $v_i \times_s v_j$ is an IFS, where $s = 1, 2, 3, 4, 5, 6$.

Definition 2.3 [10]. An intuitionistic fuzzy hypergraph (IFHG) is an ordered pair $H = \langle V, E \rangle$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$, is a finite set of intuitionistic fuzzy vertices,

(ii) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of crisp subsets of V,

(iii) $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_i)) : \mu_j(v_i), \nu_j(v_i) \geq 0$ and $\mu_j(v_i) + \nu_j(v_i) \leq 1\}$, $j = 1, 2, \ldots, m$,

(iv) $E_j \neq \emptyset$, $j = 1, 2, \ldots, m$,

(v) $\bigcup_j \operatorname{supp}(E_j) = V$, $j = 1, 2, \ldots, m$.

Here, the hyperedges E_j are crisp sets of intuitionistic fuzzy vertices, $\mu_j(v_i)$ and $\nu_j(v_i)$ represents the membership and non-membership degrees of vertex v_i to E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(v_{ij}, \mu_j(v_i), \nu_j(v_j))$. The sets (V, E) are crisp sets.

Definition 2.4 [6]. The IFk-PHG $\mathcal{H}$ is an ordered triple $\mathcal{H} = (V, E, \psi)$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set of vertices,

(ii) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of intuitionistic fuzzy subsets of V,

(iii) $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_i)) : \mu_j(v_i), \nu_j(v_i) \geq 0$ and $\mu_j(v_i) + \nu_j(v_i) \leq 1\}$, $j = 1, 2, \ldots, m$,

(iv) $E_j \neq \emptyset$, $j = 1, 2, \ldots, m$,

(v) $\bigcup_j \operatorname{supp}(E_j) = V$, $j = 1, 2, \ldots, m$.

(vi) For all $v_i \in E_k$ there exists k-disjoint hyperedge ψ_i, $i = 1, 2, \ldots, k \ni$ no two vertices in the same hyperedge are adjacent where $E_k = \bigcap_{i=1}^{k} \psi_i = \emptyset$.

3. Notations

Throughout this chapter the following notations were considered.

Let $\mathcal{H}$ denotes intuitionistic fuzzy k-partite hypergraph (IFk-PHG), then

(i) $\langle \mu_{k_i}, \nu_{k_i} \rangle$ represents the membership and non-membership degrees of the vertex $v_i \in V$, $V \subseteq \mathcal{H}$ such that $0 \leq \mu_{k_i} + \nu_{k_i} \leq 1$.

(ii) $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle$ represents the membership and non-membership degrees of the edge $(v_i, v_j) \in V \times V \subseteq \mathcal{H}$ such that $0 \leq \mu_{k_{ij}} + \nu_{k_{ij}} \leq 1$. That is, $\mu_{k_{ij}}$ and $\nu_{k_{ij}}$ are the degrees of membership and non-membership of i^{th} vertex in j^{th} edge of $\mathcal{H}$.

Note: Throughout this paper, it is assumed that the fourth cartesian product

$$V_{i_1} \times_4 V_{i_2} \times_4 V_{i_3} \dots \times_4 V_{i_n} = \{\langle (v_1, v_2, \dots, v_n), \min(\mu_1, \mu_2, \dots, \mu_n),$$
$$\max(\nu_1, \nu_2, \dots, \nu_n)\rangle \mid v_1 \in V_1, v_2 \in V_2, \dots, v_n \in V_n\rangle,$$

is used to find the edge membership and non-membership $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle$.

4. Isomorphism - Basic Properties

Definition 4.1. Let $\mathcal{H} = (V, E, \psi)$ be an intuitionistic fuzzy k-partite hypergraph. The index matrix representation of IFk-PHG is of the form $[V, \psi \subset V \times V]$ where $V = \{v_1, v_2, \dots, v_n\}$ and

$$\psi = \{\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle\} \equiv$$

$$\begin{array}{c|cccc} & v_1 & v_2 & \cdots & v_n \\ \hline v_1 & \psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\ v_2 & \psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ v_n & \psi_{n1} & \psi_{n2} & \cdots & \psi_{nn} \end{array}$$

where $\psi_{ij} = \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle \in [0, 1] \times [0, 1] (1 \le i, j \le n)$, the k-partite edge between two vertices v_i and v_j is indexed by $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle$. The values of $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle$ of an IFk-PHG can be determined by the fourth cartesian product from definition 2.2.

Definition 4.2. The order of an IFk-PHG $\mathcal{H} = (V, E, \psi)$ is defined to be

$$O(\mathcal{H}) = O_{(\mu_k, \nu_k)}(\mathcal{H}) = \sum_{v_i \in V} \langle \mu_{k_i}, \nu_{k_i} \rangle (v_i).$$

Definition 4.3. The size of an IFk-PHG $\mathcal{H} = (V, E, \psi)$ is defined to be

$$S(\mathcal{H}) = S_{(\mu_k, \nu_k)}(\mathcal{H}) = \sum_{v_i, v_j \in V} \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j).$$

Definition 4.4. The degree of a vertex v in an IFk-PHG, $\mathcal{H}$ is denoted by

$d_{\mathcal{H}}(v)$ and defined by $d_{\mathcal{H}}(v) = (d_{\mu}(v), d_{\nu}(v))$ where $d_{\mu}(v) = \mu_k(\psi_j)$, $d_{\nu}(v) = \nu_k(\psi_j)$ if $v \in \psi_j$.

Definition 4.5. Consider two IFk-PHGs $\mathcal{H} = (V, E, \psi)$ and $\mathcal{H}' = (V', E', \psi')$. A mapping $f : \mathcal{H} \to \mathcal{H}'$ is said to be a bijective map

(i) if $\mu_{k_i}(v_i), \mu_{k_i}(v_j) \in \mathcal{H}$ and $f(\mu_{k_i}(v_i)), f(\mu_{k_i}(v_j)) \in \mathcal{H}'$ then $\mu_{k_i}(v_i) = \mu_{k_i}(v_j)$ and if $\nu_{k_i}(v_i), \nu_{k_i}(v_j) \in \mathcal{H}$, $f(\nu_{k_i}(v_i)), f(\nu_{k_i}(v_j)) \in \mathcal{H}'$ then $\nu_{k_i}(v_i) = \nu_{k_i}(v_j)$;

(ii) if $\mu_{k_i}(v_i), \nu_{k_i}(v_j) \in \mathcal{H}'$ then $\mu_{k_i}(v_i) = f(\mu_{k_i}(v_j))$ and $\nu_{k_i}(v_i) = f(\nu_{k_i}(v_j))$ for some $v_j \in \mathcal{H}$.

Definition 4.6. The Homomorphism of two IFk-PHGs $\mathcal{H} = (V, E, \psi)$ and $\mathcal{H}' = (V', E', \psi')$ is a mapping $h : V \to V'$ which satisfies

(i) $\mu_{k_i}(v_i) \leq \mu'_{k_i}(h(v_i); \nu_{k_i}(v_i) \geq \nu'_{k_i}(h(v_i))$ for every $v_i \in V$ and

(ii) $\mu_{k_{ij}}(v_i, v_j) \leq \mu'_{k_{ij}}(h(v_i), h(v_j)); \nu_{k_{ij}}(v_i, v_j) \geq \nu'_{k_{ij}}(h(v_i), h(v_j))$ for every $v_i, v_j \in V$.

Definition 4.7. Consider two IFk-PHGs $\mathcal{H} = (V, E, \psi)$ and $\mathcal{H}' = (V', E', \psi')$. An isomorphism between two IFk-PHGs $\mathcal{H}$ and $\mathcal{H}'$ denoted by $\mathcal{H} \cong \mathcal{H}'$ is a bijective map $I : V \to V'$ which satisfies the following condition

(i) $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$ and

(ii) $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (I(v_i), I(v_j))$ for every $v_i, v_j \in V$.

Example 4.1. Let H and H' be two IFHG as shown below.

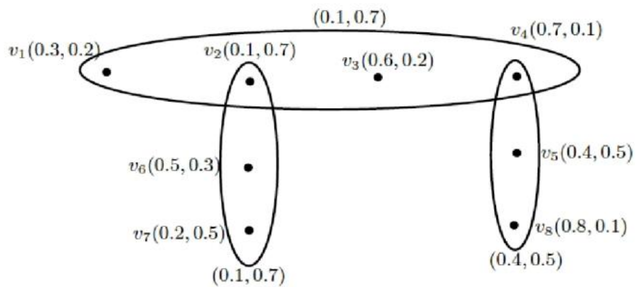

Figure 1. H

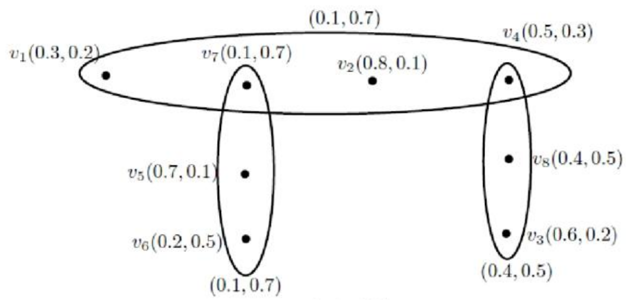

Figure 2. H'

From this we can construct an IFk-PHG, $\mathcal{H}$ and $\mathcal{H}'$. The corresponding graph is shown below.

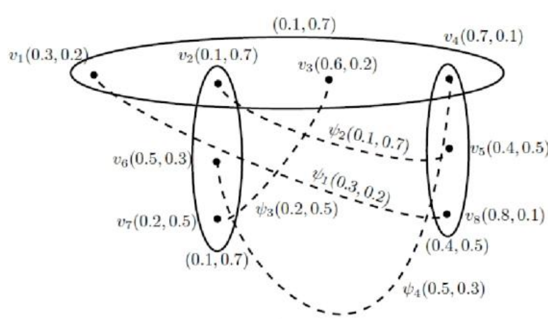

Figure 3. $\mathcal{H}$

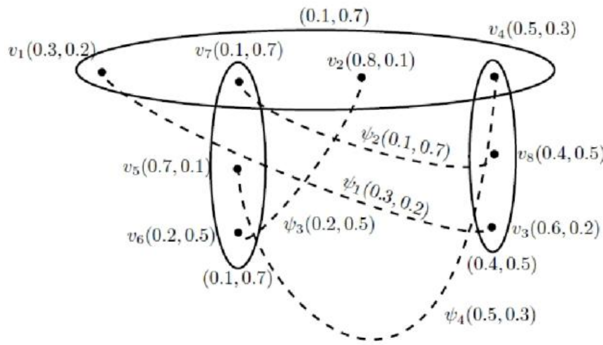

Figure 4. $\mathcal{H}'$

The graphs shown in Figure 3 and Figure 4 are isomorphic IFk-PHGs.

Definition 4.8. The Weak isomorphism of two IFk-PHGs $\mathcal{H} = (V, E, \psi)$ and $\mathcal{H}' = (V', E', \psi')$ is defined as $I : V \to V'$ is a bijective homomorphism that satisfies $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$ for every $v_i \in V$.

Note 1. In weak isomorphism only vertices in IFk-PHGs are equal.

Definition 4.9. The Co-weak isomorphism of two IFk-PHGs $\mathcal{H} = (V, E, \psi)$ and $\mathcal{H}' = (V', E', \psi')$ is defined as $I : V \to V'$ is a bijective homomorphism that satisfies

$$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (I(v_i), I(v_j)) \text{ for every } v_i, v_j \in V.$$

Note 2. In co-weak isomorphism only partite edges in IFk-PHGs are equal.

5. Isomorphism between two Intuitionistic Fuzzy k-partite Hypergraphs

The index matrix representation of two isomorphic intuitionistic fuzzy k-partite hypergraphs is analyzed in this section.

Steps involved to identify isomorphisms between two IFk-PHGs $\mathcal{H}$ and $\mathcal{H}'$.

(i) $\mathcal{H}$ and $\mathcal{H}'$ have same number of vertices (V)

(ii) $\mathcal{H}$ and $\mathcal{H}'$ have same number of hyperedges (E)

(iii) $\mathcal{H}$ and $\mathcal{H}'$ have same number of disjoint sets (ψ) and

(iv) $\mathcal{H}$ and $\mathcal{H}'$ have same number of vertices with the same degrees $(d_{\mathcal{H}})$.

Let the two IFk-PHGs $\mathcal{H}_1 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $\mathcal{H}_2 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ given in Figure 5 and Figure 6 as follows.

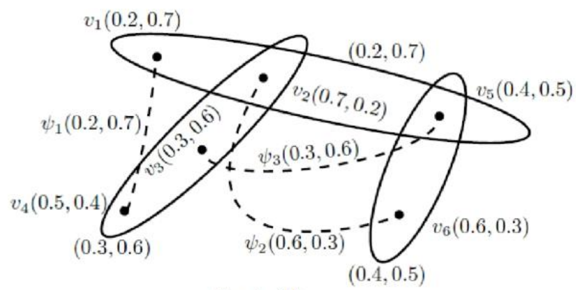

Figure 5. $\mathcal{H}_1$

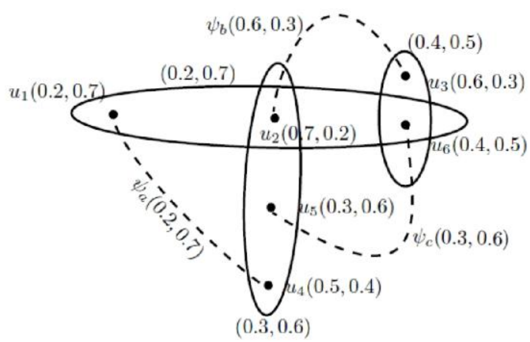

Figure 6. $\mathcal{H}_2$

The index matrix of $\mathcal{H}_1$ is $\mathcal{H}_1 = [V, \mu_{k_{ij}}, \nu_{k_{ij}}]$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle \equiv$

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
v_2	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.6, 0.3\rangle$
v_3	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0, 1\rangle$
v_4	$\langle 0.2, 0.7\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
v_5	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
v_6	$\langle 0, 1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$

The index matrix of $\mathcal{H}_2$ is $\mathcal{H}_2 = [V, \mu_{k_{ij}}, \nu_{k_{ij}}]$ where $V = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ and

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle \equiv$

	u_1	u_2	u_3	u_4	u_5	u_6
u_1	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_2	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_3	$\langle 0, 1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_4	$\langle 0.2, 0.7\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_5	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.6\rangle$
u_6	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0, 1\rangle$

Hence the calculated value of $d_{\mathcal{H}}(v_i)$, $d_{\mathcal{H}}(u_i)$ are displayed below:

$d_{\mathcal{H}}(v_1) = (0.2, 0.7)$ $\quad d_{\mathcal{H}}(u_1) = (0.2, 0.7)$

$d_{\mathcal{H}}(v_2) = (0.6, 0.3)$ $\quad d_{\mathcal{H}}(u_2) = (0.6, 0.3)$

$d_{\mathcal{H}}(v_3) = (0.3, 0.6)$ $\quad d_{\mathcal{H}}(u_3) = (0.6, 0.3)$

$d_{\mathcal{H}}(v_4) = (0.2, 0.7)$ $\quad d_{\mathcal{H}}(u_4) = (0.2, 0.7)$

$d_{\mathcal{H}}(v_5) = (0.3, 0.6)$ $\quad d_{\mathcal{H}}(u_5) = (0.3, 0.6)$

$d_{\mathcal{H}}(v_6) = (0.6, 0.3)$ $\quad d_{\mathcal{H}}(u_6) = (0.3, 0.6)$

$d_{\mathcal{H}}(u_1) = d_{\mathcal{H}}(u_4) = d_{\mathcal{H}}(v_1) = d_{\mathcal{H}}(v_4)$, we must have either

(i) $f(u_1) = v_1$ and $f(u_4) = v_4$ or (ii) $f(u_1) = v_4$ and $f(u_4) = v_1$ perhaps either will work.

Also, $d_{\mathcal{H}}(u_2) = d_{\mathcal{H}}(u_3) = d_{\mathcal{H}}(v_2) = d_{\mathcal{H}}(v_6)$, so we must have either

(i) $f(u_2) = v_2$ and $f(u_3) = v_6$ or (ii) $f(u_2) = v_6$ and $f(u_3) = v_2$

Finally, since $d_{\mathcal{H}}(u_5) = d_{\mathcal{H}}(u_6) = d_{\mathcal{H}}(v_3) = d_{\mathcal{H}}(v_5)$, we must have either

(i) $f(u_5) = v_3$ and $f(u_6) = v_5$ or (ii) $f(u_5) = v_5$ and $f(u_6) = v_3$

The mapping are $1 \to 1; 4 \to 4; 2 \to 2; 3 \to 6; 5 \to 3; 6 \to 5$.

The index matrix of $\mathcal{H}_1$ gives the index matrix of $\mathcal{H}_2$ by using this mapping. Using the above permutation, recalculate the index matrix by changing the labels of the hypergraph $\mathcal{H}_2$ to produce another hypergraph $\mathcal{H}_2^*$. Hence the resulting index matrix of $\mathcal{H}_2^*$ (after labeling of $\mathcal{H}_2$) becomes,

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle \equiv$

	u_1	u_2	u_3	u_4	u_5	u_6
u_1	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_2	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.6, 0.3\rangle$
u_3	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0, 1\rangle$
u_4	$\langle 0.2, 0.7\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_5	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.6\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
u_6	$\langle 0, 1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$

which is same as $\mathcal{H}_1$ Hence $\mathcal{H}_1 \cong \mathcal{H}_2$.

6. Some properties of isomorphism on IF*k*-PHGs

In this section, the properties of isomorphism on IFk-PHGs are discussed by using order, size and degree of $\mathcal{H}$. It has also been proved that isomorphism between IFk-PHGs preserves an equivalence relation.

Theorem 6.1. *In any two isomorphic IFk-PHGs, their size and order are same.*

Proof. If $I : \mathcal{H} \to \mathcal{H}'$ is an isomorphism between the IFk-PHGs $\mathcal{H}$ and $\mathcal{H}'$ with the underlying sets V and V' respectively, then $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$ for every $v_i \in V$.

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (I(v_i), I(v_j))$ for every $v_i, v_j \in V$.

We know that

$$
\begin{aligned}
O(\mathcal{H}) &= O_{(\mu_k, \nu_k)}(\mathcal{H}) \\
&= \sum_{v_i \in V} \langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) \\
&= \sum_{v_i \in V} \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i)) \\
&= O_{(\mu_k, \nu_k)}(\mathcal{H}') \\
&= O(\mathcal{H}')
\end{aligned}
$$

Similarly, $S(\mathcal{H}) = S_{(\mu_k, \nu_k)}(\mathcal{H})$

$$
\begin{aligned}
&= \sum_{v_i, v_j \in V} \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) \\
&= \sum_{v_i, v_j \in V} \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (I(v_i), I(v_j)) \\
&= S_{(\mu_k, \nu_k)}(\mathcal{H}') \\
&= S(\mathcal{H}')
\end{aligned}
$$

Hence, their order and size are same.

Corollary 6.1. *If order and size of the two IFk-PHG are same then it is not necessary that they are isomorphic.*

Theorem 6.2. *In any two weak isomorphic IFk-PHGs, their order are same, but the converse part need not be true.*

Proof. Consider two IFk-PHGs $\mathcal{H}$ and $\mathcal{H}'$. Given that $\mathcal{H}$ and $\mathcal{H}'$ are weak isomorphic.

(i.e.) $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$ for every $v_i \in V$, where I is a mapping from $V \to V'$.

We know that

$$O(\mathcal{H}) = O_{(\mu_k, \nu_k)}(\mathcal{H})$$

$$= \sum_{v_i \in V} \langle \mu_{k_i}, \nu_{k_i} \rangle (v_i)$$

$$= \sum_{v_i \in V} \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$$

$$= O_{(\mu_k, \nu_k)}(\mathcal{H}')$$

$$= O(\mathcal{H}')$$

This shows that order is same.

On the other hand, assume that $O(\mathcal{H}) = O(\mathcal{H}')$

(i.e.) $\sum_{v_i \in V} \langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \sum_{v_i \in V} \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$

But it is not necessary that $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (I(v_i))$

Hence, the IFk-PHGs of same order need not be weak isomorphic.

Example for the above theorem: Let $\mathcal{H} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $\mathcal{H}' = \{v'_1, v'_2, v'_3, v'_4, v'_5, v'_6, v'_7, v'_8\}$ be two IFk-PHGs as shown in Figure 7 and Figure 8 respectively.

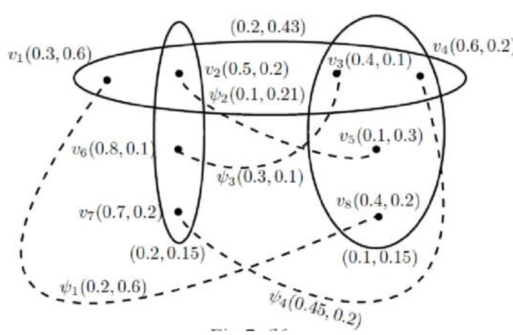

Figure 7. $\mathcal{H}$

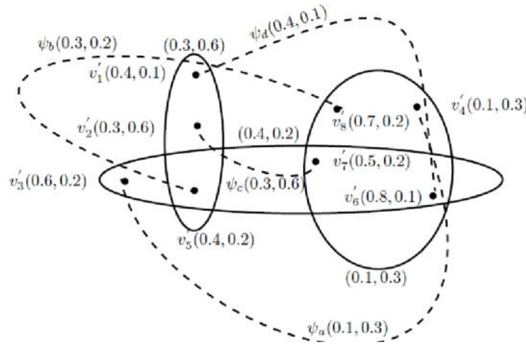

Figure 8. $\mathcal{H}'$

Remark 6.1. In any two co-weak isomorphic IFk-PHGs, their sizes are same. But the converse part need not be true.

Theorem 6.3. *If $\mathcal{H}$ and $\mathcal{H}'$ are isomorphic IFk-PHGs then the value of $d_{\mathcal{H}}(v_i)$ are same.*

Proof. Let $I : V \to V'$ be isomorphic of $\mathcal{H}$ onto $\mathcal{H}'$. Then by isomorphism definition, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (I(v_i), I(v_j))$ for every $v_i, v_j \in V$.

$$d_\mu(v_i) = \mu_k(\psi_j) \text{ if } v_i \in \psi_j$$

$$= \mu'_k(\psi_j)$$

$$= d_{\mu'}(I(v_i))$$

Similarly, $d_\nu(v_i) = \nu_k(\psi_j)$ if $v_i \in \psi_j$

$$= \nu'_k(\psi_j)$$

$$= d_{\nu'}(I(v_i))$$

Hence, the degrees of their vertices are same.

Corollary 6.2. *Converse of the above theorem need not be true.*

(*i.e.*) *if the degree of vertices of two IFk-PHGs are same then the two IFk-PHGs need not be isomorphic.*

Example:

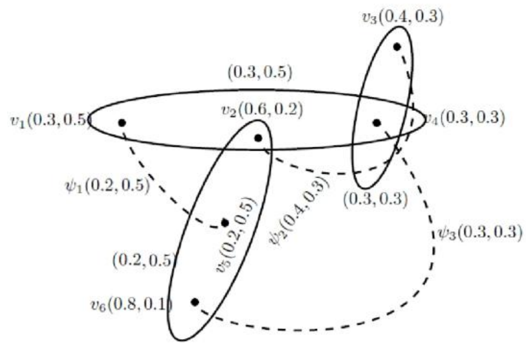

Figure 9. $\mathcal{H}$

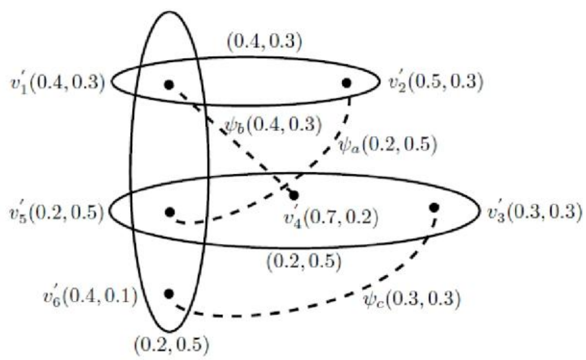

Figure 10. $\mathcal{H}'$

Theorem 6.4. *Isomorphism between IFk-PHGs satisfies an equivalence relation.*

Proof. Let $\mathcal{H} = (V, E, \psi)$, $\mathcal{H}' = (V', E', \psi')$ and $\mathcal{H}'' = (V'', E'', \psi'')$ be an IFk-PHGs.

(i) **Reflexive:**

Consider the identity map $h : V \to V$, such that $h(v_i) = v_i$, for every $v_i \in V$. This h is an bijective map which satisfies $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu_{k_i}, \nu_{k_i} \rangle (h(v_i))$, for every $v_i \in V$.

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (h(v_i), h(v_j))$ for every $v_i, v_j \in V$.

Hence, h is an isomorphism of an IFk-PHG onto itself.

(ii) **Symmetric:**

Let $h : V \to V'$ and $h(v_i) = v_i', v_i \in V$ be an isomorphism of $\mathcal{H}$ onto $\mathcal{H}'$ satisfying,

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (h(v_i))$ for every $v_i \in V$.

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (h(v_i), h(v_j))$ for every $v_i, v_j \in V$.

From $h(v_i) = v_i'$, we have $h^{-1}(v_i') = v_i$ for every $v_i' \in V'$.

Therefore, $\langle \mu_{k_i}, \nu_{k_i} \rangle (h^{-1}(v_i')) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (v_i')$ for every $v_i' \in V'$.

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (h^{-1}(v_i'), h^{-1}(v_j')) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v_i', v_j')$ for every $v_i', v_j' \in V'$.

This implies that $h^{-1} : V' \to V$ is a bijective map and $\mathcal{H}' \cong \mathcal{H}$.

(iii) **Transitive:**

Let the two isomorphisms be $h : V \to V'$ and $g : V' \to V''$ of the IFk-PHGs, $\mathcal{H}$ onto $\mathcal{H}'$ and $\mathcal{H}'$ onto $\mathcal{H}''$ respectively. Then we need to prove that $(g \circ h)$ is a 1-1, onto map from V to V'' where $(g \circ h)(v_i) = g(h(v_i))$ for every $v_i \in V$.

As h is an isomorphism we have $h(v_i) = v_i'$ for every $v_i \in V$.

Also, $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (h(v_i))$ for every $v_i \in V$.

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (h(v_i), h(v_j))$ for every $v_i, v_j \in V$.

Using $h(v_i) = v_i'$ we have,

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (v_i')$

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v_i', v_j')$.

As g is an isomorphism from V' to V'' we have $g(v_i') = v_i'', v_i' \in V'$ and

$\langle \mu'_{k_i}, \nu'_{k_i} \rangle (v'_i) = \langle \mu''_{k_i}, \nu''_{k_i} \rangle (g(v'_i))$ for all $v'_i \in V'$.

$\langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v'_i, v'_j) = \langle \mu''_{k_{ij}}, \nu''_{k_{ij}} \rangle (g(v'_i), g(v'_j)$ for every $v'_i, v'_j \in V'$.

Using the above equations and also by using $h(v_i) = v'_i, v_i \in V$ we get,

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (v'_i) = \langle \mu''_{k_i}, \nu''_{k_i} \rangle (v''_i)$, for all $v_i \in V$.

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu''_{k_i}, \nu''_{k_i} \rangle (g(h(v_i)))$, for every $v_i \in V$.

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v'_i, v'_j)$

$= \langle \mu''_{k_{ij}}, \nu''_{k_{ij}} \rangle (g(v'_i), g(v'_j))$, for every $v'_i, v'_j \in V'$

$= \langle \mu''_{k_{ij}}, \nu''_{k_{ij}} \rangle (g(h(v_i)), g(h(v_j)))$, for every $v_i, v_j \in V$.

Therefore $g \circ h$ is an isomorphism between $\mathcal{H}$ and $\mathcal{H}''$. Hence isomorphism between IFk-PHGs preserves equivalence relation.

Theorem 6.5. *Any two IFk-PHGs with weak isomorphism satisfies the partial order relation.*

Proof. Let $\mathcal{H} = (V, E, \psi)$, $\mathcal{H}' = (V', E', \psi')$, and $\mathcal{H}'' = (V'', E'', \psi'')$, be an IF$k$-PHGs.

(i) **Reflexive:**

Let the identity map be $h : V \to V$ such that $h(v_i) = v_i$, for every $v_i \in V$. This h is an bijective map which satisfies $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu_{k_i}, \nu_{k_i} \rangle (h(v_i))$, for every $v_i \in V$.

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) = \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (h(v_i), h(v_j))$ for all $v_i, v_j \in V$.

Hence, h is a weak isomorphism of an IFk-PHG to itself.

Hence $\mathcal{H}$ is weak isomorphic to itself.

(ii) **Anti Symmetric:**

Let h be weak isomorphic between $\mathcal{H}$ and $\mathcal{H}'$ and let g be weak isomorphic between $\mathcal{H}'$ and $\mathcal{H}$. (i.e.) $h : V \to V'$ is a bijective map $h(v_i) = v'_i$, $v_i \in V$ satisfying

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (h(v_i))$ for every $v_i \in V$.

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) \leq \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (h(v_i), h(v_j))$ for every $v_i, v_j \in V$ and $g : V' \to V$ is a bijective map satisfying

$\langle \mu'_{k_i}, \nu'_{k_i} \rangle (v'_i) = \langle \mu_{k_i}, \nu_{k_i} \rangle (g(v'_i))$ for every $v_i \in V$.

Also, $\langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v'_i, v'_j) \leq \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (g(v'_i), g(v'_j))$ for every $v_i, v_j \in V$.

The above inequalities implies that $\mathcal{H}$ and $\mathcal{H}'$ are identical.

(iii) **Transitive:**

Let $h : V \to V'$ and $g : V' \to V''$ be two weak isomorphism of the IFk-PHGs, $\mathcal{H}$ onto $\mathcal{H}'$ and $\mathcal{H}'$ onto $\mathcal{H}''$ respectively. Then we need to prove that $g \circ h$ is a 1-1, onto map from V to V'' where $(g \circ h)(v_i) = g(h(v_i))$ for every $v_i \in V$.

As h is a weak isomorphism we have $h(v_i) = v'_i$ for every $v_i \in V$.

Also, $\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (h(v_i))$ for every $v_i \in V$.

$\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) \leq \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (h(v_i), h(v_j))$ for every $v_i, v_j \in V$.

As g is a weak isomorphism from V' to V'' we have $g(v'_i) = v''_i$, $v'_i \in V'$ and

$\langle \mu'_{k_i}, \nu'_{k_i} \rangle (v'_i) = \langle \mu''_{k_i}, \nu''_{k_i} \rangle (g(v'_i))$ for every $v'_i \in V'$.

$\langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v'_i, v'_j) \leq \langle \mu''_{k_{ij}}, \nu''_{k_{ij}} \rangle (g(v'_i), g(v'_j))$ for every $v'_i, v'_j \in V'$

Using the above equations we get,

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu'_{k_i}, \nu'_{k_i} \rangle (v'_i) = \langle \mu''_{k_i}, \nu''_{k_i} \rangle (v''_i)$ for every $v_i \in V$.

$\langle \mu_{k_i}, \nu_{k_i} \rangle (v_i) = \langle \mu''_{k_i}, \nu''_{k_i} \rangle (g(h(v_i)))$, for every $v_i \in V$

Also, $\langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle (v_i, v_j) \leq \langle \mu'_{k_{ij}}, \nu'_{k_{ij}} \rangle (v'_i, v'_j)$

$$\leq \langle \mu''_{k_{ij}}, \nu''_{k_{ij}} \rangle (g(v'_i), g(v'_j)) \text{ for every } v'_i, v'_j \in V'$$

$$= \langle \mu''_{k_{ij}}, \nu''_{k_{ij}} \rangle (g(h(v_i)), g(h(v_j))) \text{ for every } v_i, v_j \in V$$

Therefore $g \circ h$ is a weak isomorphism between $\mathcal{H}$ and $\mathcal{H}''$. Thus transitivity is satisfied and it is weak isomorphic. Hence it satisfies partial order relation.

7. Conclusion

The concept of isomorphism plays a vital role in graph theory. A bijective correspondence which preserves adjacent relationship between vertex sets of two IFk-PHGs. The size, degree and order of the vertices of the isomorphic IFk-PHGs are discussed and some of its properties are also analyzed and extended in the index matrix representation. It is verified that there exists an equivalence relation between all isomorphic IFk-PHGs and partial order relation for all weak isomorphic IFk-PHGs.

References

[1] K. Atanassov, Index Matrices: Towards an Augmented Matrix Calculus, Springer, Cham, 2014.

[2] K. Atanassov Generalized index matrices, Comptes rendus de l'Academie Bulgare des Sciences 40(11) (1987), 15-18.

[3] K. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, June 1983 (Deposed in Central Sci. -Techn. Library of Bulg. Acad. of Sci., 1697/84) (in Bulg.), Reprinted: Int. Journal Bioautomation 20(S1) (2016), S1-S6.

[4] C. Berge, Graphs and Hypergraphs, North-Holland, New York, 1976.

[5] Mordeson N. John, S. Nair, Premchand, Fuzzy Graphs and Fuzzy Hypergraphs, New York, Physica-Verlag, 2000.

[6] K. K. Myithili and R. Keerthika, Types of Intuitionistic Fuzzy k-partite Hypergraphs, AIP Conference Proceedings, Volume 2261, 2020, 030012-1 - 030012-13; https://doi.org/10.1063/5.0017108.

[7] A. Nagoorgani and J. Malarvizhi, Isomorphism on fuzzy graphs, International Journal of Computational and Mathematical Sciences 2 (2008), 200-206.

[8] R. Parvathi and M. G. Karunambigai, Intuitionistic fuzzy graphs, Proceedings of 9th Fuzzy days International Conference on Computational Intelligence, Advances in Soft

Computing: Computational Intelligence, Theory and Applications, Springer-Verlag, New York 20 (2006), 139-150.

[9] R. Parvathi, S. Thilagavathi and K. T. Atanassov, Isomorphism on intuitionistic fuzzy directed hypergraphs, International Journal of Scientific and Research Publications 3 (2013), 1-8.

[10] R. Parvathi, S. Thilagavathi and M. G. Karunambigai, Intuitionistic fuzzy hypergraphs, Bulgarian Academy of Sciences, Cybernetics and Information Technologies 9(2) (2009) 46-53.

[11] C. Radhamani and C. Radhika, Isomorphism on Fuzzy Hypergraphs, IOSR Journal of Mathematics 2 (2012), 24-31.

[12] A. Shannon and K. Atanassov, A first step to a theory of the intuitionistic fuzzy graphs, Proc. of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.), Sofia, Sept. 28-30 (1994) 59-61.