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A SPREAD OUT OF NEW PARTIAL FEASIBLE AND OPTIMAL PERFECT MATCHING FOR SOLVING INTERVAL-VALUED α-CUT FUZZY LINEAR SUM BOTTLENECK ASSIGNMENT PROBLEM

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Abstract

In this paper, we compare the spreading of new optimal perfect matching to solve interval-value α-cut fuzzy linear sum bottleneck assignment problem. If all the spreading solutions are in minimum cost/time and maximum matching, and we get the matching solution optimal and perfect. Suppose, the solution is in minimum cost/time and minimum matching, the solution is a spread out of new partial feasible matching, if the solution is in maximum cost and minimum matching, we get a spread out of partial feasible matching.

1. Introduction

Let '*J*' jobs and '*P*' machines be given in a balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP) where $\breve{C}_{ij}$ generalized trapezoidal fuzzy numbers. The bottleneck assignment refers to latest completion in the allocation of assignment problem. The interval-valued α-cut of generalized fuzzy linear sum bottleneck assignment problems are minimum cost maximum matching problem. Let $G = (U, V, E)$ be a bipartite graph with edge set E. The edge $[i, j]$ has a cost coefficient $\omega_{\breve{C}_{ij}}$. We obtain perfect matching in G such that the perfect length of an edge in this matching is as small as possible.

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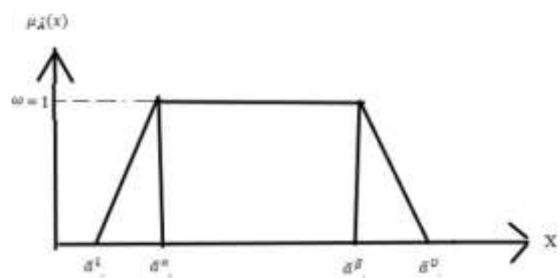
Amit Kumar and Anil Gupta have proposed Assignment and Travelling Salesman problems with Coefficient as LR Fuzzy parameters [1]. In 2019, K. Atanassov proposed extended interval-valued intuitionistic fuzzy index matrices [2]. H. Albrecher approached a note on the asymptotic behaviour of bottleneck problems [3]. Linear bottleneck assignment problems were proposed by Fulkerson, Glicksberg and Gross. In 1999, D. Dubois, and P. Fortemps have proposed computing improved optimal solutions to max-min flexible constraint satisfaction problems [4]. In 1971, R. Garfinkal have proposed improved algorithm for bottleneck assignment problem [5]. In 2004 E. Hansen and G. W. Walster, introduced Global Optimization using interval analysis.

Definition 1.1. Fuzzy number $\breve{A} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U)$ is fuzzy subset of R with membership grade $\mu_{\breve{A}}(x)$ is said to be trapezoidal fuzzy number and then the following membership functions as,

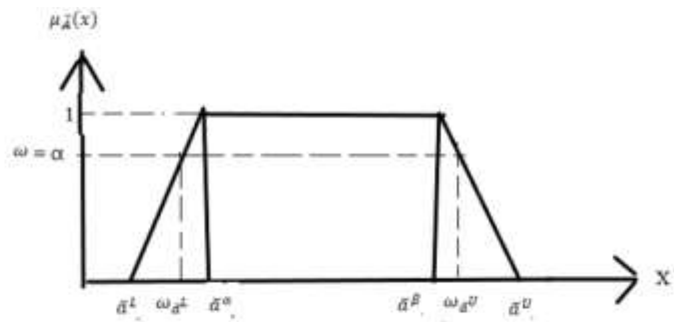
$$\mu_{\breve{A}}(x) = \begin{cases} \dfrac{X - \breve{a}^L}{\breve{a}^\alpha - \breve{a}^L}, \text{ if } \breve{a}^L \leq X \leq \breve{a}^\alpha \\ 1, \, if \; \breve{a}^\alpha \leq X \leq \breve{a}^\beta \qquad \breve{a}^L < \breve{a}^\alpha < \breve{a}^\beta < \breve{a}^U \\ \dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}, \, if \; \breve{a}^\beta \leq X \leq \breve{a}^U \end{cases}$$



Definition 1.2. A fuzzy number $\breve{A} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ is said to be generalized trapezoidal fuzzy number with membership grade $\mu_{\breve{A}}(x)$ then the following membership functions as,

$$\mu_{\breve{A}}(x) = \begin{cases} \omega\left(\dfrac{X - \breve{a}^L}{\breve{a}^\alpha - \breve{a}^L}\right), \text{ if } \breve{a}^L \le X \le \breve{a}^\alpha \\ \omega = 1, \textit{ if } \breve{a}^\alpha \le X \le \breve{a}^\beta \qquad \breve{a}^L < \breve{a}^\alpha < \breve{a}^\beta < \breve{a}^U \\ \omega\left(\dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}\right), \textit{ if } \breve{a}^\beta \le X \le \breve{a}^U \end{cases}$$

Where $\omega \in (0, 1]$.



α-Cut of Generalized Trapezoidal Fuzzy Number

1.3. Arithmetic operations Generalized Trapezoidal Fuzzy Number:

Let $\breve{P} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ and $\breve{Q} = (\breve{b}^L, \breve{b}^\alpha, \breve{b}^\beta, \breve{b}^U, \omega)$ are two generalized trapezoidal fuzzy number then the following operations are,

1. $\breve{P} + \breve{Q} = (\breve{a}^L + \breve{b}^L, \breve{a}^\alpha + \breve{b}^\alpha, \breve{a}^\beta + \breve{b}^\beta, \breve{a}^U + \breve{b}^U, \omega)$ where $\omega = (\min(\omega_1, \omega_2))$

2. $\breve{P} - \breve{Q} = (\breve{a}^L - \breve{b}^U, \breve{a}^\alpha - \breve{b}^\beta, \breve{a}^\beta - \breve{b}^\alpha, \breve{a}^U - \breve{b}^L, \omega)$ where $\omega = (\min(\omega_1, \omega_2))$.

2. α-Cut Generalized Trapezoidal Fuzzy Number

Let $\breve{P} = (\breve{a}^L, \breve{a}^\alpha, \breve{a}^\beta, \breve{a}^U, \omega)$ and $\breve{Q} = (\breve{b}^L, \breve{b}^\alpha, \breve{b}^\beta, \breve{b}^U, \omega)$ are two generalized trapezoidal fuzzy numbers and $*$ be any arithmetic operators, $\breve{P} * \breve{Q}$ define its α-cut, $\alpha_{(\breve{P} * \breve{Q})} = \alpha_{\breve{P}} * \alpha_{\breve{Q}}$. Let $\breve{P} * \breve{Q}$ can be formed as

$\breve{P} * \breve{Q} = \bigcup_{\omega \in [0,\, 1]} \alpha^{(\breve{P} * \breve{Q})}$. $\breve{P}$ and $\breve{Q}$ are fuzzy numbers, $\breve{P} * \breve{Q}$ is also fuzzy numbers.

Consider $\breve{P}$ and $\breve{Q}$ as two generalized fuzzy numbers and then the following membership functions are,

$$\mu_{\breve{P}}(x) = \begin{cases} \omega\left(\dfrac{X - \breve{a}^L}{\breve{a}^\alpha - \breve{a}^L}\right), \ if\ \breve{a}^L \le X \le \breve{a}^\alpha \\ \omega = 1, \ if\ \breve{a}^\alpha \le X \le \breve{a}^\beta \\ \omega\left(\dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}\right), \ if\ \breve{a}^\beta \le X \le \breve{a}^U \end{cases}$$

$$\mu_{\breve{Q}}(x) = \begin{cases} \left(\dfrac{X - \breve{b}^L}{\breve{b}^\alpha - \breve{b}^L}\right), \ if\ \breve{b}^L \le X \le \breve{b}^\alpha \\ \omega = 1, \ if\ \breve{b}^\alpha \le X \le \breve{b}^\beta \\ \omega\left(\dfrac{\breve{a}^U - X}{\breve{a}^U - \breve{a}^\beta}\right), \ if\ \breve{b}^\beta \le X \le \breve{b}^U \end{cases}.$$

Generalized α-Cut trapezoidal fuzzy numbers are defined as,

$$\alpha_{\breve{P}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\alpha, -(\breve{a}^U - \breve{a}^\beta) + \breve{a}^U]$$

$$\alpha_{\breve{Q}} = [\breve{b}^L + (\breve{b}^\alpha - \breve{b}^L)\alpha, -(\breve{b}^U - \breve{b}^\beta) + \breve{b}^U].$$

Let us assume that if $\alpha = \omega$ then, the following generalized α-Cut trapezoidal fuzzy numbers are formed as,

$$\omega_{\breve{P}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega, -(\breve{a}^U - \breve{a}^\beta) + \breve{a}^U]$$

$$\omega_{\breve{Q}} = [\breve{b}^L + (\breve{b}^\alpha - \breve{b}^L)\omega, -(\breve{b}^U - \breve{b}^\beta) + \breve{b}^U].$$

2.1.Generalized α-Cut fuzzy numbers to Fuzzy Interval

$$\omega_{\breve{c}_{11}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{11}^L d_{11}^U],$$

$$\omega_{\breve{c}_{12}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{12}^L\, d_{12}^U]$$

$$\omega_{\breve{c}_{13}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{13}^L d_{13}^U],$$

$$\omega_{\breve{c}_{14}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{14}^L\, d_{14}^U]$$

$$\omega_{\breve{c}_{21}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{21}^L d_{21}^U],$$

$$\omega_{\breve{c}_{22}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{22}^L\, d_{22}^U]$$

$$\omega_{\breve{c}_{23}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{23}^L d_{23}^U],$$

$$\omega_{\breve{c}_{24}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{24}^L\, d_{24}^U]$$

$$\omega_{\breve{c}_{31}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{31}^L d_{31}^U],$$

$$\omega_{\breve{c}_{12}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{32}^L\, d_{32}^U]$$

$$\omega_{\breve{c}_{33}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{33}^L d_{33}^U],$$

$$\omega_{\breve{c}_{34}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{34}^L\, d_{34}^U]$$

$$\omega_{\breve{c}_{41}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{41}^L d_{41}^U],$$

$$\omega_{\breve{c}_{12}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{42}^L\, d_{42}^U]$$

$$\omega_{\breve{c}_{43}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{43}^L d_{43}^U],$$

$$\omega_{\breve{c}_{44}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{44}^L\, d_{44}^U].$$

2.2. Interval-valued α-Cut of Generalized Fuzzy Linear Sum Bottleneck can be modelled as

$$Min \max_{1 \le i,\ j \ge n} \omega_{\breve{c}_{ij}} x_{ij}$$

Such that

$$\sum_{j=1}^{n} x_{ij} = 1 \ (i = 1, 2, \dots, n)$$

$$\sum_{j=1}^{n} x_{ij} = 1 \ (j = 1, 2, \ldots, n)$$

$$x_{ij} \in \{0, 1\} (i, j = 1, 2, \ldots, n).$$

Definition 2.3. An interval value $\omega_{\tilde{c}_{ij}} = [a_{ij}^L d_{ij}^U] \in R$ is said to be interval value fuzzy set with membership grade $\mu_{\tilde{c}_\omega}(x)$ then the following membership functions as,

$$\mu_{\tilde{C}_\omega}(x) = \begin{cases} 0, \ X < a_{ij}^L \\ 1, \ a_{ij}^L < X < d_{ij}^U \\ 0, \ X > d_{ij}^u \end{cases}$$

Where $a_{ij}^L < d_{ij}^U$.

3. Fuzzy Interval Operations

Let $X = [a_{11}^L \, d_{11}^U]$ and $Y = [a_{22}^L \, d_{22}^U]$ are two closed interval values in R then the following operations are as,

(a) $X + Y = [a_{11}^L \, d_{11}^U] + [a_{22}^L \, d_{22}^U] = [a_{11}^L + a_{22}^L \, d_{11}^U + d_{22}^U]$

(b) $X - Y = [a_{11}^L \, d_{11}^U] - [a_{22}^L \, d_{22}^U] = [a_{11}^L - d_{22}^L \, d_{11}^U - a_{22}^U]$

(c) $X \cdot Y = (a_{11}^L \, a_{22}^L, a_{11}^L d_{22}^U, \, d_{11}^U a_{22}^L, \, d_{11}^U a_{22}^L, \, d_{11}^U d_{22}^U) \max (a_{11}^L a_{22}^L, \, a_{11}^L a_{22}^U, d_{11}^U a_{22}^L, \, d_{11}^U d_{22}^U)$

(d) $X/Y = \left[\min\left(\frac{a_{11}^L}{d_{22}^U}, \frac{a_{11}^L}{a_{22}^L}, \frac{d_{11}^U}{d_{22}^U}, \frac{d_{11}^U}{a_{22}^L} \right) \max\left(\frac{a_{11}^L}{d_{22}^U}, \frac{a_{11}^L}{a_{22}^L}, \frac{d_{11}^U}{d_{22}^U}, \frac{d_{11}^U}{a_{22}^L} \right) \right].$

(e) $X \wedge Y = [a_{11}^L d_{11}^U] \wedge [a_{22}^L d_{22}^U] = [a_{11}^L \wedge a_{22}^L d_{11}^U \wedge d_{22}^U]$

(d) $X \vee Y = [a_{11}^L d_{11}^U] \vee [a_{22}^L d_{22}^U] = [a_{11}^L \vee a_{22}^L d_{11}^U \vee d_{22}^U].$

3.1. Interval-valued α-Cut of Fuzzy Linear Sum Bottleneck Assignment Table:

$[a_{11}^L\, d_{11}^U]$	$[a_{12}^L\, d_{12}^U]$	$[a_{13}^L\, d_{13}^U]$	$[a_{14}^L\, d_{14}^U]$
$[a_{21}^L\, d_{21}^U]$	$[a_{22}^L\, d_{22}^U]$	$[a_{23}^L\, d_{23}^U]$	$[a_{24}^L\, d_{24}^U]$
$[a_{31}^L\, d_{31}^U]$	$[a_{32}^L\, d_{32}^U]$	$[a_{33}^L\, d_{33}^U]$	$[a_{34}^L\, d_{34}^U]$
$[a_{41}^L\, d_{41}^U]$	$[a_{42}^L\, d_{42}^U]$	$[a_{43}^L\, d_{43}^U]$	$[a_{44}^L\, d_{44}^U]$

3.2. α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment

In the first case α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment cost element is $(\omega_{\tilde{c}_{ij}{}^*_M})$ and α-Cut of threshold Fuzzy Linear Sum Bottleneck Assignment are defined as,

$$\omega_{\tilde{c}_{ij}} = \begin{cases} 1, \; if \;\; \omega_{\tilde{c}_{ij}} > \omega_{\tilde{c}_{ij}{}^*_M} \\ 0, \text{ otherwise.} \end{cases}$$

Let $C = \omega_{\tilde{c}_{ij}}$ be $n \times n$ matrix and $\omega_{\tilde{c}_{ij}}{}^{U}_{\varphi(i)n}$ be a fuzzy arbitrary permutation of IFLSBAP.

Spreading solution is $sp(\varphi(i)) = \max\{\min\{\omega_{\tilde{c}_{\varphi(i)n}}\}$.

3.3. Property (i): If two elements of IFLSBAP are in increasing order, then prove that the sum of two elements of IFLSBAP is also in Increasing Order,

Proof. Let $X = [a_{11}^L d_{11}^U]$ and $Y = [a_{22}^L d_{22}^U]$ are two closed interval values in R is IFLSBAP.

Here, $a_{ij}^L < d_{ij}^U$ and $a_{22}^L < d_{22}^U$ are in increasing orders.

We prove that, the sum of two elements of IFLSBAP is also in increasing order, adding X and Y. We get,

$$X + Y = [a_{11}^L d_{11}^U] + [a_{22}^L d_{22}^U] = [a_{11}^L + a_{22}^L d_{11}^U + d_{22}^U].$$

We see that, Here, $a_{11}^L + a_{22}^L < d_{11}^U + d_{22}^U$ and $a_{ij}^L < d_{ij}^U$ and $a_{22}^L < d_{22}^U$.

Therefore, $[a_{11}^L d_{11}^U] + [a_{22}^L d_{22}^U] < [a_{11}^L + a_{22}^L d_{11}^U + d_{22}^U]$. Hence, if two elements of IFLSBAP are in increasing order then the sum of two elements of IFLSBAP are also in increasing order.

4. Algorithm

Solving optimal perfect matching and feasible partial matching by using generalized α-cut trapezoidal fuzzy numbers we present in the following step by step procedure.

Step 1. Generalized α-cut trapezoidal fuzzy numbers

Let us take generalized trapezoidal fuzzy number and obtain α-cut of trapezoidal fuzzy numbers, if $\alpha = \omega$, then the following form.

$$\omega_{\breve{A}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U],$$

$$\omega_{\breve{B}} = [\breve{b}^L + (\breve{b}^\alpha - \breve{b}^L)\omega - (\breve{b}^U - \breve{b}^\beta) + \breve{b}^U].$$

Step 2. Compute fuzzy interval values by using generalized α-cut trapezoidal fuzzy numbers:

$$\omega_{\breve{C}_{ij}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{ij}^L d_{ij}^U].$$

Where $a_{ij}^L < d_{ij}^U$ and $a_{ij}^L =$ lower boundary of least value

$d_{ij}^U =$ upper boundary of largest value

Step 3. Forming balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP):

Let us consider balanced interval valued fuzzy linear sum bottleneck assignment problem (IFLSBAP) (number of Machines (M) and number of Jobs (P) are equal i.e., $\sum_{i=1}^n M_i = \sum_{j=1}^n P_j$), if IFLSBAP $\sum_{i=1}^n M_i \neq \sum_{j=1}^n P_j$, We Introduce dummy row i.e., $\sum_{i=1}^n M_i + D_i$ (or), introduce dummy column $\sum_{j=1}^{n} P_j + D_j$ (where D_i = dummy row, D_j = dummy column)

Step 4: Calculate $\omega_{\breve{c}_{ij}}{}_0^L$, $\omega_{\breve{c}_{ij}}{}_n^U$.

Let $\omega_{\breve{c}_{ij}} = [a_{ij}^L d_{ij}^U]$ be $(n \times n)$ interval cost/time matrix;

$$\omega_{\breve{c}_{ij}}{}_0^L = \min_{ij}\{a_{ij}^L d_{ij}^U\},\ \omega_{\breve{c}_{ij}}{}_n^U = \max_{ij}\{a_{ij}^L d_{ij}^U\}.$$

Step 5: Calculate $(\omega_{\breve{c}_{ij}}{}^*)$, $(\omega_{\breve{c}_{ij}}{}_M^*)$

$$\omega_{\breve{c}_{ij}}{}^* = \{\omega_{\breve{c}_{ij}} : \omega_{\breve{c}_{ij}}{}_0^L < \omega_{\breve{c}_{ij}} < \omega_{\breve{c}_{ij}}{}_n^U$$

$$\omega_{\breve{c}_{ij}}{}_M^* = \min\{\omega_{\breve{c}_{ij}} \in \omega_{\breve{c}_{ij}}{}^* : |\{\omega_{\breve{c}_{ij}} \in \omega_{\breve{c}_{ij}}{}^* : \omega_{\breve{c}_{ij}} \leq [a_{ij}^L d_{ij}^U]\} \geq |\ \omega_{\breve{c}_{ij}}{}^*\ |/2.$$

Step 6: Feasibility check: Select the feasible element $\omega_{\breve{c}_{ij}}{}_M^*$, $(\omega_{\breve{c}_{ij}}{}_0^L < \omega_{\breve{c}_{ij}}{}_1^L, \omega_{\breve{c}_{ij}}{}_2^L \ldots \omega_{\breve{c}_{ij}}{}_M^* = 0)$, select the $(\omega_{\breve{c}_{ij}})$ row and assigned at least only one zero, similarly column allocations are $\omega_{\breve{c}_{2j}}, \omega_{\breve{c}_{3j}} \ldots, \omega_{\breve{c}_{nj}}$, next column assigned as $\omega_{\breve{c}_{i1}}, \omega_{\breve{c}_{i2}} \ldots, \omega_{\breve{c}_{in}}$. Each row and each column at least only one zero are assigned. Every row and column has at least one matching, the feasibility is executed. The bipartite matching, if minimum cost and maximum matching is optimal and perfect, otherwise the bipartite graph is feasible and partial matching. Obtain interval valued fuzzy linear sum cost is $\sum_{i=1}^{n} \omega_{\breve{c}}{}_{i_{\varphi(i)}M}^{*} = [a_{i\varphi(i)M^L} d_{i\varphi(i)M^U}]$.

Step 7: Backward calculation: Select the lower feasible elements of $\omega_{\breve{c}_{ij}}{}_M^*$ and determine Feasible IFLSBAP

$\omega_{\breve{c}_{ij}}{}_0^L = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\breve{c}}{}_{i_{\varphi(i)}0}^{L} = [a_{i\varphi(i)0^L} d_{i\varphi(i)0^U}]$.

$\omega_{\breve{c}_{ij}}{}_1^L = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\breve{c}}{}_{i_{\varphi(i)}1}^{L} = [a_{i\varphi(i)1^L} d_{i\varphi(i)1^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{2}^{L} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}_{i_{\varphi(i)}2}}^{L} = [a_{i\varphi(i)2^L} d_{i\varphi(i)2^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{M-1}^{L} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}_{i_{\varphi(i)}m-1}}^{L} = [a_{i\varphi(i)m-1^L} d_{i\varphi(i)m-1^U}]$.

Step 8. Forward calculation: Select the upper feasible elements of $\omega_{\tilde{c}_{ij}}{}_{M}^{*}$ and determine Feasible IFLSBAP

$\omega_{\tilde{c}_{ij}}{}_{m+1}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}_{i_{\varphi(i)}m+1}}^{U} = [a_{i\varphi(i)m+1^L} d_{i\varphi(i)m\mp1^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{m+2}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}_{i_{\varphi(i)}m+2}}^{U} = [a_{i\varphi(i)m+2^L} d_{i\varphi(i)m+2^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{m+3}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}_{i_{\varphi(i)}m+3}}^{U} = [a_{i\varphi(i)m+3^L} d_{i\varphi(i)m+3^U}]$.

$\omega_{\tilde{c}_{ij}}{}_{n}^{U} = 0$, the feasible cost/time is $\sum_{i=1}^{n} \omega_{\tilde{c}_{i_{\varphi(i)}n}}^{U} = [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$.

Step 9. Determine and checking Feasible/Optimal and Partial/Perfect of IFLSBAP.

$$\omega_{\tilde{c}_{ij}}{}_{\delta}^{L} = [a_{i\varphi(i)\delta^L} d_{i\varphi(i)\delta^L}] = \min\{\omega_{\tilde{c}_{ij}}{}_{0}^{L}, \omega_{\tilde{c}_{ij}}{}_{1}^{L}, \ldots, \omega_{\tilde{c}_{ij}}{}_{M}^{*} \ldots \omega_{\tilde{c}_{ij}}{}_{m+2}^{U}, \omega_{\tilde{c}_{ij}}{}_{m+3}^{U} \ldots \omega_{\tilde{c}_{ij}}{}_{n}^{U}\}..$$

If, $\omega_{\tilde{c}_{ij}}{}_{\delta}^{L} = [a_{i\varphi(i)\delta^L} d_{i\varphi(i)\delta^L}] \leq [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is optimal and perfect matching.

If, $\omega_{\tilde{c}_{ij}}{}_{\delta}^{L} = [a_{i\varphi(i)\delta^L} d_{i\varphi(i)\delta^*}] < [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is feasible and partial matching.

If, $\omega_{\breve{c}_{ij}}{}_{\delta}^{U} = [a_{i\varphi(i)\delta^U} d_{i\varphi(i)\delta^U}] \leq [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is feasible and perfect matching.

If, $\omega_{\breve{c}_{ij}}{}_{\delta}^{U} = [a_{i\varphi(i)\delta^U} d_{i\varphi(i)\delta^U}] < [a_{i\varphi(i)n^L} d_{i\varphi(i)n^U}]$ is feasible and partial matching.

Step 10: Stop.

Example: Consider generalized trapezoidal fuzzy numbers $(\breve{C}_{ij})$

(9,13,17,21;0.25)	(15,20,25,30;0.20)	(4, 6, 8, 10; 0.50)	(3, 5, 7, 9; 0.50)
(5,7,9,11;0.50)	(8, 10, 12, 14; 0.50)	(4, 6, 8, 10; 0.50)	(9,13,17,21;0.25)
(2,4,6,8;0.50)	(9, 13, 17, 21; 0.25)	(13,18,23,28;0.20)	(5, 7, 9, 11; 0.50)
(3,5,7,9;0.50)	(6, 8, 10, 12; 0.50)	(9, 13, 17, 21; 0.25)	(4, 6, 8, 10; 0.50)

Generalized α-cut trapezoidal fuzzy numbers is

$$\omega_{\breve{c}_{ij}} = [\breve{a}^L + (\breve{a}^\alpha - \breve{a}^L)\omega - (\breve{a}^U - \breve{a}^\beta) + \breve{a}^U] = [a_{ij}^L d_{ij}^U].$$

If $\alpha = \omega$, compute fuzzy interval values by using generalized α-cut trapezoidal fuzzy numbers:

$\omega_{\breve{c}_{11}} = [10, 20], \omega_{\breve{c}_{12}} = [16, 29], \omega_{\breve{c}_{13}} = [5, 9], \omega_{\breve{c}_{14}} = [4, 8], \omega_{\breve{c}_{21}} = [6, 10], \omega_{\breve{c}_{22}} = [9, 12],$
$\omega_{\breve{c}_{23}} = [5, 9], \omega_{\breve{c}24} = [10, 20], \omega_{\breve{c}_{31}} = [3, 1], \omega_{\breve{c}_{32}} = [10, 20], \omega_{\breve{c}_{33}} = [14, 27], \omega_{\breve{c}_{34}} = [6, 1],$
$\omega_{\breve{c}_{41}} = [4, 8], \omega_{\breve{c}_{42}} = [7, 11], \omega_{\breve{c}_{43}} = [10, 20], \omega_{\breve{c}_{44}} = [5, 9].$

Interval-valued fuzzy linear sum bottleneck assignment problem by using Generalized α-cut trapezoidal fuzzy numbers

Table 2

[10 20]	[16 29]	[5 9]	[4 8]
[6 10]	[9 13]	[5 9]	[10 20]
[3 7]	[10 20]	[14 27]	[6 10]
[4 8]	[7 11]	[10 20]	[5 9]

Case i. $\omega_{\bar{c}_{ij}^*} = \omega_{\bar{c}_{ij}^*} = \{\omega_{\bar{c}_{ij}} : {\omega_{\bar{c}_{ij}}}_0^L < \omega_{\bar{c}_{ij}} < {\omega_{\bar{c}_{ij}}}_n^U = [7, 11]$

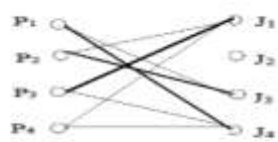
Table 3

[10 20]	[16 29]	0	0
0	[9 13]	0	[10 20]
0	[10 20]	[14 27]	0
0	0	[10 20]	0

$\sum_{i=1}^{n} \omega_{\bar{c}_{i\varphi(i)M}^*} = [a_{i\varphi(i)M^L}\, d_{i\varphi(i)M^U}] = [19, 35]$. The IFLSAP is optimal and perfect matching

Case ii: ${\omega_{\bar{c}_{ij}}}_{M-1}^{*} = \omega_{\bar{c}_{ij}^L} = \{\omega_{\bar{c}_{ij}} : {\omega_{\bar{c}_{ij}}}_0^L < \omega_{\bar{c}_{ij}} < {\omega_{\bar{c}_{ij}}}_n^U = [6, 10]$.

Table 4

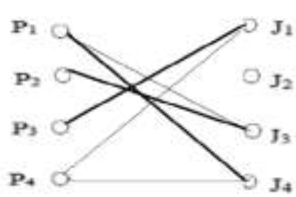


[10 20]	[16 29]	0	0
0	[9 13]	0	[10 20]
0	[[10 20]	[14 27]	0
0	[7 11]	[10 20]	0

Now $[a_{i\varphi(i)M-1^L}\, d_{i\varphi(i)M-1^U}] = [12, 24]$. The IFLSAP is feasible and partial.

Case iii: ${\omega_{\bar{c}_{ij}}}_{M-2}^{*} = \omega_{\bar{c}_{ij}^L} = \{\omega_{\bar{c}_{ij}} : {\omega_{\bar{c}_{ij}}}_0^L < \omega_{\bar{c}_{ij}} < {\omega_{\bar{c}_{ij}}}_n^U = [5, 9]$.

Table 5

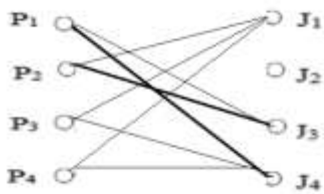


[10 20]	[16 29]	0	0
[6 10]	[9 12]	0	[10 20]
0	[[10 20]	[14 27]	[6 10]
0	[7 11]	[10 20]	0

Now $\sum_{i=1}^{n} \omega_{\bar{c}_{i\varphi(i)M-2}^*} = [a_{i\varphi(i)M-2^L}\, d_{i\varphi(i)M-2^U}] = [12, 24]$. The IFLSAP is feasible and partial.

Case iv: $\omega_{\tilde{c}_{ij}}{}^{L}_{M-3} = \omega_{\tilde{c}_{ij}^{L}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [4, 8]$.

Table 6.

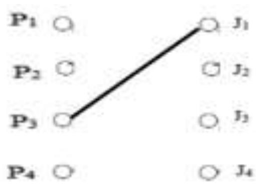


[10 20]	[16 29]	0	0
0	[9 12]	0	[10 20]
0	[[10 20]	[14 27]	0
0	[7 11]	[10 20]	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M-3}} = [a_{i\varphi(i)M-2^{L}}\, d_{i\varphi(i)M-2^{U}}] = [7, 15]$. The IFLSAP is feasible and partial.

Case v: $\omega_{\tilde{c}_{ij}}{}^{L}_{M-4} = \omega_{\tilde{c}_{ij}^{L}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [3, 7]$.

Table 7

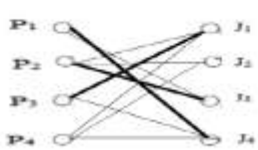


[10 20]	[16 29]	[5 9]	[4 8]
[6 10]	[9 12]	[5 9]	[10 20]
0	[[10 20]	[14 27]	[6 10]
[4 8]	[7 11]	[10 20]	[5 9]

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M-4}} = [a_{i\varphi(i)M-4^{L}}\, d_{i\varphi(i)M-4^{U}}] = [3, 7]$. The IFLSAP is feasible and partial.

Case vi: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+1} = \omega_{\tilde{c}_{ij}^{L}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [9, 13]$.

Table 8

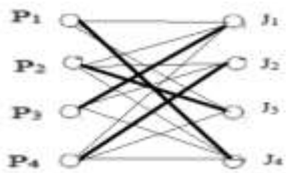


[10 20]	[16 29]	0	0
0	0	0	[10 20]
0	[[10 20]	[14 27]	0
0	0	[10 20]	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^*_{i\varphi(i)M+1}} = [a_{i\varphi(i)M+1^L}\, d_{i\varphi(i)M+1^U}] = [12, 24]$. The IFLSAP is feasible and perfect.

Case vii: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+2} = \omega_{\tilde{c}^L_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [10, 20]$.

Table 9

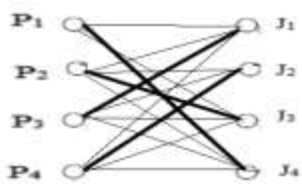


0	[16 29]	0	0
0	0	0	0
0	0	[14 27]	0
0	0	0	0

$\sum_{i=1}^{n} \omega_{\tilde{c}^*_{i\varphi(i)M+2}} = [a_{i\varphi(i)M+2^L}\, d_{i\varphi(i)M+2^U}] = [19, 35]$. The IFLSAP is optimal and perfect.

Case viii: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+3} = \omega_{\tilde{c}^L_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [14, 27]$.

Table 10

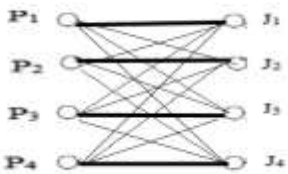


0	[16 29]	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^*_{i\varphi(i)M+3}} = [a_{i\varphi(i)M+3^L}\, d_{i\varphi(i)M+3^U}] = [19, 35]$. The IFLSAP is optimal and perfect.

Case ix: $\omega_{\tilde{c}_{ij}}{}^{L}_{M+4} = \omega_{\tilde{c}^L_{ij}} = \{\omega_{\tilde{c}_{ij}} : \omega_{\tilde{c}_{ij}}{}^{L}_{0} < \omega_{\tilde{c}_{ij}} < \omega_{\tilde{c}_{ij}}{}^{U}_{n} = [16, 29]$.

Table 11



0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Now $\sum_{i=1}^{n} \omega_{\tilde{c}^{*}_{i\varphi(i)M+4}} = [a_{i\varphi(i)M+4^{L}}\, d_{i\varphi(i)M+4^{U}}] = [38, 69]$. the IFLSAP is feasible and perfect.

The optimal perfect schedule is $P_1 \to J_4,\ P_2 \to J_3,\ P_3 \to J_1,\ P_4 \to J_2$.

The spread of new generalized trapezoidal fuzzy optimal perfect assignment cost is $\sum \breve{C}_{i\varphi(i)} = (3, 5, 7, 9, 0.50) + (4, 6, 8, 10; 0.50)$ $+ (2, 4, 6, 8; 0.50) + (6, 8, 10, 12; 0.50) = (15, 23, 33, 39; 0.50)$.

6. Conclusion

We discussed above concepts of optimal perfect matching and partial feasible matching for solving interval valued fuzzy linear sum bottleneck assignment problem by using α-cut of generalized trapezoidal fuzzy number. The machine completed maximum jobs with minimum cost or time then the solution is optimal and perfect, otherwise partial and feasible.

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