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ISOMOPHISM IN SUBDIVISION, TOTAL AND MIDDLE SINGLE VALUED NEUTROSOPHIC GRAPH

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Abstract

In this paper, Subdivision Single Valued Neutrosophic Graph $sd(G)$, Total Single Valued Neutrosophic Graph $tl(G)$ and Middle Single Valued Neutrosophic Graph $M(G)$ for a given Single Valued Neutrosophic Graph $G = (A, B)$ is defined. Some properties and the isomorphism concepts are discussed. Also the relationship of $M(G)$ with $sd(G)$ and $tl(G)$ are discussed.

1. Introduction

Fuzzy set theory and intuitionistic fuzzy sets theory are useful models for dealing with uncertainty and incomplete information. But they may not be sufficient in modeling of indeterminate and inconsistent information encountered in real world. In order to cope with this issue, neutrosophic set theory was proposed by Smarandache as a generalization of fuzzy sets and intuitionistic fuzzy sets. Isomorphism on Single valued neutrosophic graphs are given by Malarvizhi and Divya [4]. Properties and isomorphism of total and middle fuzzy graphs was given by Nagoorgani and Malarvizhi. Properties

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of middle and total intutionistic fuzzy graphs are also discussed [5]. Here, in this paper some properties of subdivision, total and middle Single valued Neutrosophic graphs are discussed and isomorphic relation between them are also have been discussed.

2. Preliminaries

A Single-Valued Neutrosophic graph (SVN graph) is a pair $G = (A, B)$ of the crisp graph $G^* = (V, E)$ (i.e., with underlying set V), where $A : V \to [0, 1]$ is single-valued neutrosophic set in V and $B : V \times V \to [0, 1]$ is single valued neutrosophic relation on V such that

$$T_B(xy) \leq \min \{T_A(x), T_A(y)\},$$

$$I_B(xy) \leq \min \{I_A(x), I_A(y)\},$$

$$F_B(xy) \leq \max \{F_A(x), F_A(y)\},$$

for all $x, y \in V$. A is called single-valued neutrosophic vertex set of G and B is called single-valued neutrosophic edge set of G, respectively.

Given a single-valued neutrosophic graph $G = (A, B)$ of a crisp graph, the order of G is defined as Order $(G) = (O_T(G), O_I(G), O_F(G))$, where $O_T(G) = \sum_{v \in V} T_A(v)$, $O_I(G) = \sum_{v \in V} I_A(v)$, $O_F(G) = \sum_{v \in V} F_A(v)$.

Given a single-valued neutrosophic graph $G = (A, B)$ of a crisp graph $G^* = (V, E)$, the size of G is defined as Size $(G) = (S_T(G), S_I(G), S_F(G))$, where $S_T(G) = \sum_{u \neq v} T_B(u, v), S_I(G) = \sum_{u \neq v} I_B(u, v), S_F(G) = \sum_{u \neq v} F_B(u, v)$.

The degree of a vertex x in an SVNG, $G = (A, B)$ is defined to be sum of the weights of the edges incident at x. It is denoted by $d_G(u)$ and is equal to $(\sum_{u \neq v} T_B(u, v), \sum_{u \neq v} I_B(u, v), \sum_{u \neq v} F_B(u, v))$ for all v adjacent to u in G^*.

Two vertices x and y are said to be neighbors in SVNG if either one of the following conditions hold

i. $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) > 0$

ii. $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) > 0$

iii. $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) > 0$

iv. $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) = 0$

v. $T_B(x, y) = 0, I_B(x, y) = 0, F_B(x, y) > 0$

vi. $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) = 0$

vii. $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) = 0$ for $x, y \in A$.

A single-valued neutrosophic graph $G = (A, B)$ is called strong if the following conditions are satisfied:

$T_B(xy) = \min\{T_A(x), T_A(y)\}$,

$I_B(xy) = \min\{I_A(x), I_A(y)\}$,

$F_B(xy) = \max\{F_A(x), F_A(y)\}$, for all non zero edges $(x, y) \in E$.

A single-valued neutrosophic graph $G = (A, B)$ is called complete if the following conditions are satisfied:

$T_B(xy) = \min\{T_A(x), T_A(y)\}$,

$I_B(xy) = \min\{I_A(x), I_A(y)\}$,

$F_B(xy) = \max\{F_A(x), F_A(y)\}$, for all $x, y \in A$.

Let G and G' be single valued neutrosophic graphs with underlying sets V and V' respectively. A homomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which satisfies

$T_A(u) \leq T_{A'}(h(u)), I_A(u) \leq I_{A'}(h(u)), F_A(u) \leq F_{A'}(h(u))$ for all $u \in V$

$T_B(u, v) \leq T_{A'}(h(u), h(v)), I_B(u, v) \leq I_{B'}(h(u), h(v)), F_B(u, v) \leq F_{B'}(h(u), h(v))$ for all $u, v \in V$.

Let G and G' be single valued neutrosophic graphs with underlying sets V and V' respectively. An isomorphism of single valued neutrosophic graphs,

$h : G \to G'$ is a bijective map $h : V \to V'$ which satisfies

$$T_A(u) = T_{A'}(h(u)),\ I_A(u) = I_{A'}(h(u)),\ F_A(u) = F_{A'}(h(u)) \text{ for all } u \in V$$

$$T_B(u, v) = T_{B'}(h(u), h(v)),\ I_B(u, v) = I_{B'}(h(u), h(v)),\ F_B(u, v) = F_{B'}(h(u), h(v))$$

for all $u, v \in V$. Then G is said to be isomorphic to G'.

A weak isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies

$$T_A(u) = T_{A'}(h(u)),\ I_A(u) = I_{A'}(h(u)),\ F_A(u) = F_{A'}(h(u)) \text{ for all } u \in V.$$

A co-weak isomorphism of single valued neutrosophic graphs, $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies

$$T_B(u, v) = T_{B'}(h(u), h(v)),\ I_B(u, v) = I_{B'}(h(u), h(v)),\ F_B(u, v) = F_{B'}(h(u), h(v))$$

for all $u, v \in V$.

The busy value of the vertex x in G is

$$BV(x) = (BV_{T_A}(x), BV_{I_A}(x), BV_{F_A}(x)) = (\textstyle\sum_i T_A(x) \wedge T_A(x_i),$$

$$\textstyle\sum_i I_A(x) \wedge I_A(x_i), \sum_i F_A(x) \vee F_A(x_i))$$

where x_i are the neighbours of x and the busy value of G is $BV(G) = \sum_i BV(x_i)$ where x_i are the vertices of G.

A vertex in a G is a busy vertex if $(T_A, I_A, F_A)(x) \leq d_G(x)$.

3. Subdivision Single Valued Neutrosophic Graph

Here, we define Subdivision of Single valued Neutrosophic graph and discuss some properties.

Definition 3.1. Let $G : (A, B)$ be a SVN graph with the underlying crisp graph $G^* = (V, E)$. The vertices and edges of G are taken together as vertex set of $sd(G) = (A_{sd}, B_{sd})$, each edge 'e' in G is replaced by a new vertex and that vertex is made as a adjacent of those vertices which lie on 'e' in G. Here

A_{sd} is a SVN subset defined on $V \cup E$ as

$$(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)(x) \text{ if } x \in V$$

$$= (T_B, I_B, F_B)(x) \text{ if } x \in E.$$

The SVN relation B_{sd} on $V \cup E$ is defined as

$$T_{B_{sd}}(x, y) = T_A(x) \wedge T_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

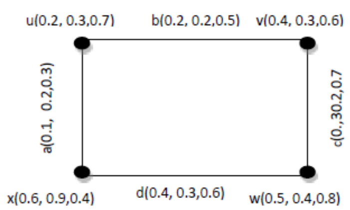
$$I_{B_{sd}}(x, y) = I_A(x) \wedge I_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$F_{B_{sd}}(x, y) = F_A(x) \vee F_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$(T_{B_{sd}}, I_{B_{sd}}, F_{B_{sd}})(x, y)$ is a SVN relation on $(T_{A_{sd}}, I_{A_{sd}}, F_{A_{sd}})$ and hence the pair $sd(G) = (A_{sd}, B_{sd})$, is a SVN graph. This pair is said as subdivision SVN graph of G.



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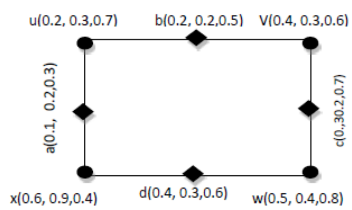


sd(G)

In the above $sd(G)$, $(a, u) = (0.1, 0.2, 0.7)$, $(u, b) = (0.2, 0.2, 0.7)$, $(b, v) = (0.2, 0.2, 0.6)$, $(v, c) = (0.3, 0.2, 0.7)$, $(c, w) = (0.3, 0.2, 0.8)$, $(w, d) = (0.4, 0.3, 0.8)$, $(d, x) = (0.4, 0.3, 0.6)$, $(x, a) = (0.1, 0.2, 0.4)$.

Properties of Subdivision SVN Graph

Theorem 3.2. *Let* $G = (A, B)$ *be SVN graph and* $sd(G)$ *is its subdivision SVN graph, order of* $sd(G) = order\ (G) + size(G)$.

Proof. By definition of $sd(G)$, vertex set of $sd(G)$ is $V \cup E$.

Order of $sd(G) = (O_T(sd(G)), O_I(sd(G)), O_F(sd(G)))$,

$$= \left(\sum_{x \in V \cup E} T_{A_{sd}}(x), \sum_{x \in V \cup E} I_{A_{sd}}(x), \sum_{x \in V \cup E} F_{A_{sd}}(x) \right)$$

$$= \left(\sum_{x \in V} T_{A_{sd}}(x) + \sum_{x \in E} T_{A_{sd}}(x) + \sum_{x \in V} I_{A_{sd}}(x) + \sum_{x \in E} I_{A_{sd}}(x) + \sum_{x \in V} F_{A_{sd}}(x) + \sum_{x \in E} F_{A_{sd}}(x) \right)$$

$$= \left(\sum_{x \in V} T_{A_{sd}}(x), \sum_{x \in V} I_{A_{sd}}(x), \sum_{x \in V} F_{A_{sd}}(x) \right) + \left(\sum_{x \in E} T_{A_{sd}}(x), \sum_{x \in E} I_{A_{sd}}(x), \sum_{x \in E} F_{A_{sd}}(x) \right)$$

$= order\ (G) + size(G).$

Theorem 3.3. *Number of edges in* $sd(G)$ *is equal to twice number of edges in G.*

Proof. As each edge in G is replaced by a new vertex and as exactly two vertices lie on an edge in G, it is adjacent to exactly two vertices in $sd(G)$. Thus number of edges in $sd(G)$ is equal to twice the number of edges in G.

Theorem 3.4. *Let* $G = (A, B)$ *be SVN graph and* $sd(G)$ *is its subdivision SVN graph, size of* $sd(G) =$ *twice the size* (G).

Proof. By definition of $sd(G)$ vertex set of $sd(G)$ is $V \cup E$.

Size of $sd(G) = (S_T(sd(G)), S_I(sd(G)), S_F(sd(G)))$

$= (\sum T_{B_{sd}}(x, y), \sum I_{B_{sd}}(x, y), \sum F_{B_{sd}}(x, y))$ where $x \in V$, $y \in E$ and x lies on y

$= (\sum \min\{T_{A_{sd}}(x), T_{A_{sd}}(y)\}, \sum \min\{I_{A_{sd}}(x), I_{A_{sd}}(y)\}, \sum \max\{F_{A_{sd}}(x), F_{A_{sd}}(y)\})$

$= (\sum \min\{T_A(x), T_B(y)\}, \sum \min\{I_A(x), I_B(y)\}, \sum \max\{F_A(x), F_B(y)\}).$

As each newly added vertex lies on exactly two edges in $sd(G)$, the value of it contributes two times to the sum in the equation

$= (2\sum T_B(y), 2\sum I_B(y), 2\sum F_B(y))$ where $y \in E$

$= 2(\sum T_B(y), \sum I_B(y), \sum F_B(y))$

$= 2(\text{size of } G).$

Theorem 3.5. $sd(G)$ *is a strong Single valued Neutrosophic graph.*

Proof. In $sd(G)$ for each edge (x, y)

$$T_{B_{sd}}(x, y) = T_A(x) \wedge T_B(y) \text{ if } x \in V \text{ and } y \in E \text{ and } x \text{ lies on } y$$

$$= 0 \text{ otherwise}$$

$$I_{B_{sd}}(x, y) = I_A(x) \wedge I_B(y) \text{ if } x \in V \text{ and } y \in E \text{ and } x \text{ lies on } y$$

$$= 0 \text{ otherwise}$$

$$F_{B_{sd}}(x, y) = F_A(x) \vee F_B(y) \text{ if } x \in V \text{ and } y \in E \text{ and } x \text{ lies on } y$$

$$= 0 \text{ otherwise.}$$

Thus each edge is an effective edge, hence $sd(G)$ is a strong SVN graph.

Theorem 3.6. *Let G is a complete SVN graph, but $sd(G)$ need not be complete SVN graph.*

Proof. G is a complete SVN graph, so each vertex is a adjacent to every other vertex, but in $sd(G)$, each newly added vertex is adjacent to exactly two vertices. Hence $sd(G)$ cannot be a complete SVN graph.

Theorem 3.7. *The busy value of $sd(G)$, $BV(sd(G)) = 4(S_T(G), S_I(G), S_F(G))$.*

Proof.

$$BV(sd(G)) = \sum_{x \in V \cup E} BV(x) = \sum_{x \in V} BV(x) + \sum_{x \in E} BV(x).$$

If $x \in V$, then $BV(x) = (\sum_{y_i \in E} T_B(y_i), \sum_{y_i \in E} I_B(y_i), \sum_{y_i \in E} F_B(y_i))$ where y_i is adjacent to x in $sd(G)$,

$$\sum_{x \in V} BV(x) = \left(\sum_{x \in V} \sum_{y_i \in E} T_B(y_i), \sum_{x \in V} \sum_{y_i \in E} I_B(y_i), \sum_{x \in V} \sum_{y_i \in E} F_B(y_i) \right)$$

$$= \left(2 \sum_{y_i \in E} T_B(y_i), 2 \sum_{y_i \in E} I_B(y_i), 2 \sum_{y_i \in E} F_B(y_i) \right).$$

Since each y_i is adjacent to exactly two vertices in V

$$= 2\left(\sum_{y_i \in E} T_B(y_i),\ \sum_{y_i \in E} I_B(y_i),\ \sum_{y_i \in E} F_B(y_i)\right)$$

$$\sum_{x \in V} BV(x) = 2(S_T(G),\ S_I(G),\ S_F(G)).$$

If $x = y_i \in E$ then $BV(y_i) = (T_B(y_i),\ I_B(y_i),\ F(y_i)) + (T_B(y_i),\ I_B(y_i),\ F(y_i))$. Since each $y_i \in E$ is the newly added vertex that is adjacent to exactly two vertices in V and by definition of subdivision SVN graph.

i.e., $BV(y_i) = 2(T_B(y_i),\ I_B(y_i),\ F(y_i))$, thus $\sum_{y_i \in E} BV(x) = 2(\sum_{y_i \in E} T_B(y_i),$ $\sum_{y_i \in E} I_B(y_i), \sum_{y_i \in E} F_B(y_i)) = 2(S_T(G),\ S_I(G),\ S_F(G))$.

Hence $BV(sd(G)) = 4(S_T(G),\ S_I(G),\ S_F(G))$.

Theorem 3.8. *Let G be SVN graph, $d_{sd(G)}(x) = d_G(x)$ if $x \in V$ $= 2(T_B,\ I_B,\ F_B)(x)$ if $x \in E$.*

Proof. By the definition of degree of a vertex given,

Case 1. If $x \in V$ then in $sd(G)$, x is a neighbour of all points y in E on which x lies on the edges of G, in that case. $(T_B,\ I_B,\ F_B)(x,\ y) = (T_B,\ I_B,\ F_B)(y)$. Hence $d_{sd(G)}(x) = (\sum_{y \in E} T_B(y),$ $\sum_{y \in E} I_B(y), \sum_{y \in E} F_B(y)) = d_G(x)$ (since x is on y in G).

Case 2. If $x = y_i \in E$ then in $sd(G)$, it is a newly added vertex that is made as a neighbour of exactly two vertices of V with $(T_B,\ I_B,\ F_B)(x,\ y_i) = (T_B,\ I_B,\ F_B)(y_i)$ for $x \in V$. Hence $d_{sd(G)}(x) = (T_B,\ I_B,\ F_B)(x,\ y_i) + (T_B,\ I_B,\ F_B)(y_i,\ z)$ where x, z are the only two vertices to that y_i is adjacent,

i.e., $d_{sd(G)}(y_i) = (T_B, I_B, F_B)(y_i) + (T_B, I_B, F_B)(y_i) = 2(T_B, I_B, F_B)(y_i)$. Hence $d_{sd(G)}(x) = d_G(x)$ if $x \in V$

$= 2(T_B,\ I_B,\ F_B)(x)$ if $x \in E$.

Theorem 3.9. *In* $sd(G)$ *every* $y \in E$ *is a busy vertex.*

Proof. $d_{sd(G)}(y) = 2(T_B, I_B, F_B)(y) > (T_A, I_A, F_A)_{sd}(y)$, since $(T_A, I_A, F_A)_{sd}(y) = (T_B, I_B, F_B)(y)$ implies $(T_A, I_A, F_A)_{sd}(y) < d_{sd(G)}(y)$ i.e., y is a busy node in $sd(G)$. Thus each y in E is a busy vertex in $sd(G)$.

4. Total Single Valued Neutrosophic Graph

Here, we define Total Single Valued Neutrosophic graph and discuss some properties.

Definition 4.1. Let $G = (A, B)$ be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The pair $tl(G) = (A_{tl}, B_{tl})$ of G is defined as follows. The vertex set of $tl(G)$ is $V \cup E$. The SVN subset A_{tl} is defined on $V \cup E$ as,

$$(T_A, I_A, F_A)_{tl}(x) = (T_A, I_A, F_A)(x) \text{ if } x \in V$$

$$= (T_B, I_B, F_B)(x) \text{ if } x \in E.$$

The SVN relation B_{tl} on $V \cup E$ is defined as

$$T_{B_{tl}}(x, y) = T_B(x, y), I_{B_{tl}}(x, y) = I_B(x, y), F_{B_{tl}}(x, y) = F_B(x, y) \text{ if } (x, y) \in E$$

$$T_{B_{tl}}(x, y) = T_A(x) \wedge T_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ othewise}$$

$$I_{B_{tl}}(x, y) = I_A(x) \wedge I_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$F_{B_{tl}}(x, y) = F_A(x) \vee F_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$T_{B_{tl}}(e, f) = T_B(e) \wedge T_B(f) \text{ if } e, f \in E \text{ and they have a vertex in common}$$

$$= 0 \text{ otherwise}$$

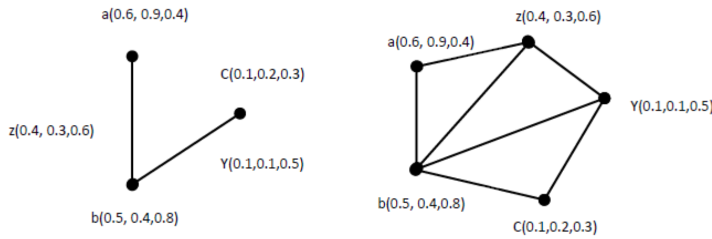
$$I_{B_{tl}}(e, f) = I_B(e) \wedge I_B(f) \text{ if } e, f \in E \text{ and they have a vertex in common}$$

$= 0$ otherwise

$F_{B_{tl}}(e, f) = F_B(e) \vee F_B(f)$ if $e, f \in E$ and they have a vertex in common

$= 0$ otherwise.

Thus by the definition B_{tl} is a single valued neutrosophic relation on A_{tl}. Hence the pair $tl(G) = (A_{tl}, B_{tl})$ is a SVN graph and is termed as Total Single Valued Neutrosophic Graph.



G **tl(G)**

The edge values of $tl(G)$ are, $(a, b) = (0.4, 0.3, .6)$, $(b, c) = (0.1, 0.1, 0.5)$, $(a, z) = (0.4, 0.3, 0.6)$, $(b, z) = (0.4, 0.3, 0.8)$, $(b, y) = (0.1, 0.1, 0.8)$, $(c, y) = (0.1, 0.1, 0.5)$, $(z, y) = (0.1, 0.1, 0.6)$.

Properties of Total SVN Graph.

Theorem 4.2. *Let $G = (A, B)$ be SVN graph and $tl(G)$ is its Total SVN graph, order of $tl(G)$ = order (G) + size (G).*

Proof. By definition of $tl(G)$, vertex set of $tl(G)$ is $V \cup E$.

Order of $tl(G) = (O_T(tl(G)), O_I(tl(G)), O_F(tl(G)))$

$$= \left(\sum_{x \in V \cup E} T_{A_{tl}}(x), \sum_{x \in V \cup E} I_{A_{tl}}(x), \sum_{x \in V \cup E} F_{A_{tl}}(x) \right)$$

$$= \left(\sum_{x \in V} T_{A_{tl}}(x) + \sum_{x \in E} T_{A_{tl}}(x), \sum_{x \in V} I_{A_{tl}}(x) + \sum_{x \in E} I_{A_{tl}}(x), \sum_{x \in V} F_{A_{tl}}(x) + \sum_{x \in E} F_{A_{tl}}(x) \right)$$

$$= \left(\sum_{x \in V} T_{A_{tl}}(x), \sum_{x \in V} I_{A_{tl}}(x), \sum_{x \in V} F_{A_{tl}}(x) \right) + \left(\sum_{x \in E} T_{A_{tl}}(x), \sum_{x \in E} I_{A_{tl}}(x), \sum_{x \in E} F_{A_{tl}}(x) \right)$$

$=$ order (G) + size(G).

Theorem 4.3. *Let* $G = (A, B)$ *be SVN graph and* $tl(G)$ *is its Total SVN graph, size of* $tl(G) = 3size(G) + (\sum_{x, y \in E} T_B(x) \wedge T_B(y),$ $\sum_{x, y \in E} I_B(x) \wedge I_B(y), \sum_{x, y \in E} F_B(x) \vee F_B(y))$

Proof. Size of $tl(G) = (S_T(tl(G)), S_I(tl(G)), S_F(tl(G)))$

$$= \left(\sum_{x, y \in V \cup E} T_{B_{tl}}(x, y), \sum_{x, y \in V \cup E} I_{B_{tl}}(x, y), \sum_{x, y \in V \cup E} F_{B_{tl}}(x, y) \right)$$

$$= \left(\left(\sum_{x, y \in V} T_{B_{tl}}(x, y), \sum_{x, y \in V} I_{B_{tl}}(x, y), \sum_{x, y \in V} F_{B_{tl}}(x, y) \right) \right.$$

$$+ \left(\sum_{x \in V, y \in E} T_{B_{tl}}(x, y), \sum_{x \in V, y \in E} I_{B_{tl}}(x, y), \sum_{x \in V, y \in E} F_{B_{tl}}(x, y) \right)$$

$$\left. + \left(\sum_{x, y \in E} T_{B_{tl}}(x, y), \sum_{x, y \in E} I_{B_{tl}}(x, y), \sum_{x, y \in E} F_{B_{tl}}(x, y) \right) \right) x$$

lies on y in the second parenthesis

$$= \left(\sum_{x, y \in V} T_{B_{tl}}(x, y), \sum_{x, y \in V} I_{B_{tl}}(x, y), \sum_{x, y \in V} F_{B_{tl}}(x, y) \right)$$

$$+ \left(\sum_{x \in V, y \in E} T_A(x) \wedge T_B(y), \sum_{x \in V, y \in E} I_A(x) \wedge I_B(y), \sum_{x \in V, y \in E} F_A(x) \vee F_B(y) \right)$$

$$+ \left(\sum_{x, y \in E} T_B(x) \wedge T_B(y), \sum_{x, y \in E} I_B(x) \wedge I_B(y), \sum_{x, y \in E} F_B(x) \vee F_B(y) \right)$$

$= size(G) + (2\sum T_B(y), 2\sum I_B(y), 2\sum F_B(y)) + (\sum_{x, y \in E} T_B(x) \wedge T_B(y),$ $\sum_{x, y \in E} I_B(x) \wedge I_B(y), \sum_{x, y \in E} F_B(x) \vee F_B(y))$

$$= size(G) + 2size(G) + \left(\sum_{x, y \in E} T_B(x) \wedge T_B(y), \sum_{x, y \in E} I_B(x) \wedge I_B(y), \sum_{x, y \in E} F_B(x) \vee F_B(y) \right)$$

$$= 3size(G) + \left(\sum_{x,\, y \in E} T_B(x) \wedge T_B(y),\ \sum_{x,\, y \in E} I_B(x) \wedge I_B(y),\ \sum_{x,\, y \in E} F_B(x) \vee F_B(y) \right).$$

Theorem 4.4. $d_{tl(G)}(u) = 2d_G(u)$ *if* $u \in V$, $d_{tl(G)}(y_i) =$ *busy value of* y_i *in* $tl(G)$ *if* $y_i \in E$.

Proof. By the definition of degree of a vertex given

Case 1. Let $x \in V$,

$$d_{tl(G)}(x) = \left(\sum_{a \in V} T_{B_{tl}}(x,\, a),\ \sum_{a \in V} I_{B_{tl}}(x,\, a),\ \sum_{a \in V} F_{B_{tl}}(x,\, a) \right)$$

$$+ \left(\sum_{b \in E} T_{B_{tl}}(x,\, b),\ \sum_{b \in E} I_{B_{tl}}(x,\, b),\ \sum_{b \in E} F_{B_{tl}}(x,\, b) \right) \text{ (x lies on edge b)}$$

$$= \left(\sum_{y \in E} T_B(y),\ \sum_{y \in E} I_B(y),\ \sum_{y \in E} F_B(y) \right)$$

$$+ \left(\sum_{b \in E} T_B(x) \wedge T_B(b),\ \sum_{b \in E} I_B(x) \wedge I_B(b),\ \sum_{b \in E} F_B(x) \vee F_B(b) \right)$$

$$= d_G(x) + \left(\sum_{b \in E} T_B(b),\ \sum_{b \in E} I_B(b),\ \sum_{b \in E} F_B(b) \right)$$

$$= d_G(x) + d_G(x) = 2d_G(x).$$

Case 2. If $y_i \in E$,

$$d_{tl(G)}(y_i) = \left(\sum_{a \in V} T_{B_{tl}}(y_i,\, a),\ \sum_{a \in V} I_{B_{tl}}(y_i,\, a),\ \sum_{a \in V} F_{B_{tl}}(y_i,\, a) \right)$$

$$+ \left(\sum_{b \in E} T_{B_{tl}}(y_i,\, b),\ \sum_{b \in E} I_{B_{tl}}(y_i,\, b),\ \sum_{b \in E} F_{B_{tl}}(y_i,\, b) \right)$$

$$= \left(\sum_{a \in V} T_A(a) \wedge T_B(y_i),\ \sum_{a \in V} I_A(a) \wedge I_B(y_i),\ \sum_{a \in V} F_A(a) \vee F_B(y_i) \right)$$

$$+ \left(\sum_{b \in E} T_B(y_i) \wedge T_B(b),\ \sum_{b \in E} I_B(y_i) \wedge I_B(b),\ \sum_{b \in E} F_B(y_i) \vee F_B(b) \right)$$

$$= \text{busy value of } y_i \text{ in } tl(G).$$

5. Middle Single Valued Neutrosophic Graph

Middle Single valued Neutrosophic Graph is defined and some of its properties are discussed.

Definition 5.1. Let $G = (A, B)$ be a SVN graph with its underlying crisp graph $G^* = (V, E)$. The vertices and edges of G are taken together as the vertex set of the pair $M(G) = (A_M, B_M)$ where

$$(T_A, I_A, F_A)_M(x) = (T_A, I_A, F_A)(x) \text{ if } x \in V$$

$$= (T_B, I_B, F_B)(x) \text{ } if \text{ } x \in E$$

$(T_B, I_B, F_B)_M(x, y) = 0$ if both $x, y \in V$

$T_{B_M}(e, f) = T_B(e) \wedge T_B(f)$ if $e, f \in E$ and they have a vertex in common

$= 0$ otherwise

$I_{B_M}(e, f) = I_B(e) \wedge I_B(f)$ if $e, f \in E$ and they have a vertex in common

$= 0$ otherwise

$F_{B_M}(e, f) = F_B(e) \vee F_B(f)$ if $e, f \in E$ and they have a vertex in common

$= 0$ otherwise

$T_{B_M}(x, y) = T_B(y)$ if $x \in V$ and $y \in E$

$= 0$ otherwise

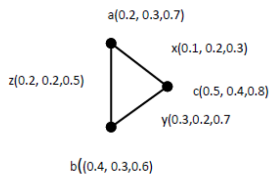
$$I_{B_M}(x, y) = I_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

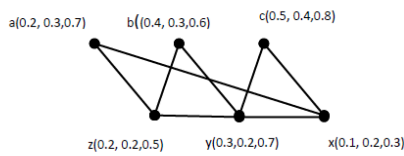
$$F_{B_M}(x, y) = F_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

As A_M is defined only through the values of A and B, $A_M : V \cup E \to [0, 1]$ is well defined SVN subset on $V \cup E$. Also B_M is a SVN relation on A_M is also well defined. Hence the pair $M(G) = (A_M, B_M)$ is a SVN graph and is termed as Middle Single Valued Neutrosophic Graph.



G



M(G)

The edge values of $M(G)$ are, $(a, z) = (0.2, 0.2, 0.5)$, $(a, x) = (0.1, 0.2, 0.3)$, $(b, z) = (0.2, 0.2, 0.5)$, $(b, y) = (0.3, 0.2, 0.7)$, $(c, y) = (0.3, 0.2, 0.7)$, $(c, x) = (0.1, 0.2, 0.3)$, $(z, y) = (0.2, 0.2, 0.7)$, $(y, x) = (0.1, 0.2, 0.7)$.

Properties of a Middle SVN Graph

Theorem 5.2. *Let $G = (A, B)$ be SVN graph and $M(G)$ is its Middle SVN graph, order of $M(G) = order(G) + size(G)$.*

Proof. By definition of $M(G)$ vertex set of $M(G)$ is $V \cup E$.

Order of $tl(G) = (O_T(M(G)), O_I(M(G)), O_F(M(G)))$

$$= \left(\sum_{x \in V \cup E} T_{A_M}(x), \sum_{x \in V \cup E} I_{A_M}(x), \sum_{x \in V \cup E} F_{A_M}(x) \right)$$

$$= \left(\sum_{x \in V} T_{A_M}(x) + \sum_{x \in E} T_{A_M}(x),, \sum_{x \in V} I_{A_M}(x) + \sum_{x \in E} I_{A_M}(x), \right.$$

$$\left.\sum_{x \in V} F_{A_M}(x) + \sum_{x \in E} F_{A_M}(x),\right)$$

$$= order(G) + size(G).$$

Theorem 5.3. *Let* $G = (A, B)$ *be SVN graph and* $M(G)$ *is its Middle SVN graph, size of* $M(G) =$ *twice the size* $(G) + (\sum_{x, y \in E} T_B(x) \wedge T_B(y), \sum_{x, y \in E} I_B(x) \wedge I_B(y), \sum_{x, y \in E} F_B(x) \vee F_B(y))$.

Proof. By definition of $M(G)$, vertex set of $M(G)$ is $V \cup E$.

Size of $M(G) = (S_T(M(G)), S_I(M(G)), S_F(M(G)))$

$$= \left(\sum_{x, y \in V \cup E} T_{B_M}(x, y), \sum_{x, y \in V \cup E} I_{B_M}(x, y), \sum_{x, y \in V \cup E} F_{B_M}(x, y)\right)$$

$$= \left(\left(\sum_{x, y \in V} T_{B_M}(x, y), \sum_{x, y \in V} I_{B_M}(x, y), \sum_{x, y \in V} F_{B_M}(x, y)\right)\right.$$

$$+ \left(\sum_{x \in V,\, y \in E} T_{B_M}(x, y), \sum_{x \in V,\, y \in E} I_{B_M}(x, y), \sum_{x \in V,\, y \in E} F_{B_M}(x, y)\right)$$

$$\left. + \left(\sum_{x, y \in E} T_{B_M}(x, y), \sum_{x, y \in E} I_{B_M}(x, y), \sum_{x, y \in E} F_{B_M}(x, y)\right)\right)$$

$$= \left(\sum_{x, y \in V} T_{B_M}(x, y), \sum_{x, y \in V} I_{B_M}(x, y), \sum_{x, y \in V} F_{B_M}(x, y)\right)$$

$$+ \left(\sum_{x \in V,\, y \in E} T_A(x) \wedge T_B(y), \sum_{x \in V,\, y \in E} I_A(x) \wedge I_B(y), \sum_{x \in V,\, y \in E} F_A(x) \vee F_B(y)\right)$$

$$+ \left(\sum_{x,\, y \in E} T_B(x) \wedge T_B(y), \sum_{x,\, y \in E} I_B(x) \wedge I_B(y), \sum_{x,\, y \in E} F_B(x) \vee F_B(y)\right)$$

$$= 0 + \left(2 \sum T_B(y), 2 \sum I_B(y), 2 \sum F_B(y)\right)$$

$$+\left(\sum_{x,\ y\in E} T_B(x)\wedge T_B(y),\ \sum_{x,\ y\in E} I_B(x)\wedge I_B(y),\ \sum_{x,\ y\in E} F_B(x)\vee F_B(y)\right)$$

$$= 2size(G)+\left(\sum_{x,\ y\in E} T_B(x)\wedge T_B(y),\ \sum_{x,\ y\in E} I_B(x)\wedge I_B(y),\ \sum_{x,\ y\in E} F_B(x)\vee F_B(y)\right).$$

Theorem 5.4. *Number of edges in $M(G)$ is equal to sum of the number of edges of $L(G)$ and twice the number of edges in G.*

Proof. As each edge in G is contributing two edges to $M(G)$ and the pairs of adjacent edges in G, contribute an edge to $M(G)$.

Number of edges in $M(G) =$ two times the number of edges of G + *No* of pair wise adjacent edges in $G^* =$ twice the number of edges of $G +$ Number of edges in $L(G)$.

Theorem 5.5. *$M(G)$ is a strong SVN graph.*

Proof. Consider an edge (x, y) in $M(G)$. From the definition of $M(G)$ we have two cases.

Case 1.

$T_{B_M}(x, y) = T_B(y)$ if $x \in V$ and $y \in E$

$I_{B_M}(x, y) = I_B(y)$ if $x \in V$ and $y \in E$

$F_{B_M}(x, y) = F_B(y)$ if $x \in V$ and $y \in E$.

Also $T_B(y) = T_{A_M}(y)$ as $y \in E$ (by definition of $M(G)$)

$= T_{A_M}(y) \wedge T_{A_M}(x)$ as x lies on y

$I_B(y) = I_{A_M}(y)$ as $y \in E$ (by definition of $M(G)$)

$= I_{A_M}(y) \wedge I_{A_M}(x)$ as x lies on y

$F_B(y) = F_{A_M}(y)$ as $y \in E$ (by definition of $M(G)$)

$= F_{A_M}(y) \vee F_{A_M}(x)$ as x lies on y.

Hence $T_{B_M}(x, y) = T_{A_M}(y) \wedge T_{A_M}(x)$ where $x \in V$ and $y \in E$

$I_{B_M}(x, y) = I_{A_M}(y) \wedge I_{A_M}(x)$ where $x \in V$ and $y \in E$

$F_{B_M}(x, y) = F_{A_M}(y) \vee F_{A_M}(x)$ where $x \in V$ and $y \in E$

Case 2.

$T_{B_M}(x, y) = T_B(x) \wedge T_B(y)$ if $x, y \in E$ and they have a vertex in common

$I_{B_M}(x, y) = I_B(x) \wedge I_B(y)$ if $x, y \in E$ and they have a vertex in common

$F_{B_M}(x, y) = F_B(x) \vee F_B(y)$ if $x, y \in E$ and they have a vertex in common

$T_{B_M}(x, y) = T_{A_M}(x) \wedge T_{A_M}(y)$ by the definition of A_M

$I_{B_M}(x, y) = I_{A_M}(x) \wedge I_{A_M}(y)$ by the definition of A_M

$F_{B_M}(x, y) = F_{A_M}(x) \vee F_{A_M}(y)$ by the definition of A_M.

Hence by the above cases $M(G)$ is a strong SVN graph.

Theorem 5.6. *$M(G)$ is not a complete SVN graph even if G is a complete SVN graph.*

Proof. Given G is a complete SVN graph, then every pair of vertices are adjacent in G^*. But by the definition of $M(G)$ no two nodes in $V(G)$ are neighbours in $M(G)$. So $M(G)$ is not a complete SVN graph.

Theorem 5.7. $d_{M(G)}(x) = d_G(x)$ *if* $x \in V$,

$$d_{M(G)}(y_i) = 2\left(\sum T_B(y_i), \sum I_B(y_i), \sum F_B(y_i)\right)$$

$$+\left(\sum \min\{T_B(y_i), T_B(y_j)\}, \sum \min\{I_B(y_i), I_B(y_j)\}, \sum \max\{F_B(y_i), F_B(y_j)\}\right)$$

if $y_i,\ y_j \in E$ and are adjacent in G^*.

Proof. By the definition of degree of a vertex,

Case 1. Let $x \in V$.

$$d_{M(G)}(x) = \left(\sum_{y_i \in E} T_{B_M}(x,\ y_i),\ \sum_{y_i \in E} I_{B_M}(x,\ y_i),\ \sum_{y_i \in E} F_{B_M}(x,\ y_i)\right)$$ where x lies on y_i

$$= \left(\sum_{y_i \in E} T_B(y_i),\ \sum_{y_i \in E} I_B(y_i),\ \sum_{y_i \in E} F_B(y_i)\right)$$

$$= d_G(x).$$

Case 2. Let $x \in E$. If $x = y_i$, then

$$d_{M(G)}(x) = d_{M(G)}(y_i) = \left(\sum_{a \in V \cup E} T_{B_M}(y_i, a),\ \sum_{a \in V \cup E} I_{B_M}(y_i, a),\ \sum_{a \in V \cup E} F_{B_M}(y_i, a)\right)$$

$$= \left(\sum_{a \in V} T_{B_M}(y_i, a), \sum_{a \in V} I_{B_M}(y_i, a), \sum_{a \in V} F_{B_M}(y_i, a)\right)$$

$$+ \left(\sum_{y_j \in E} T_{B_M}(y_i, y_j),\ \sum_{y_j \in E} I_{B_M}(y_i, y_j),\ \sum_{y_j \in E} F_{B_M}(y_i, y_j)\right)$$

$$= 2(\sum T_B(y_i),\ 2\ \sum I_B(y_i),\ 2\sum F_B(y_i))$$

$$+ (\sum \min\{T_B(y_i), T_B(y_j)\}, \sum \min\{I_B(y_i), I_B(y_j)\}, \sum \max\{F_B(y_i), F_B(y_j)\})$$

$$= 2(\sum T_B(y_i),\ \sum I_B(y_i),\ \sum F_B(y_i))$$

$$+ (\sum \min\{T_B(y_i), T_B(y_i)\}, \sum \min\{I_B(y_i), I_B(y_i)\}, \sum \max\{F_B(y_i), F_B(y_j)\}).$$

Theorem 5.8. *Busy value of* $M(G) = 4$ *size* $(G) + 2$ $(\sum_{x, y \in E} T_B(x) \wedge T_B(y), \sum_{x, y \in E} I_B(x) \wedge I_B(y), \sum_{x, y \in E} F_B(x) \vee F_B(y))$.

Proof. Busy value of $M(G) = \sum_{x \in V \cup E} BV(x)$

$$= \sum_{x \in V \cup E} d_{M(G)}(x) \text{ (as } M(G) \text{ is strong)}$$

$$= 2\,size\,(M(G))$$

$$= 2(2size(G) + (\sum_{x, y \in E} T_B(x) \wedge T_B(y), \sum_{x, y \in E} I_B(x) \wedge I_B(y),$$

$$\sum_{x, y \in E} F_B(x) \vee F_B(y))$$

$$= 4size(G) + 2(\sum_{x, y \in E} T_B(x) \wedge T_B(y), \sum_{x, y \in E} I_B(x) \wedge I_B(y),$$

$$\sum_{x, y \in E} F_B(x) \vee F_B(y)).$$

Note.

Order of $sd(G) =$ Order of $tl(G) =$ Order of $M(G) =$ order of $G +$ size of G.

6. Isomorphic Relationship between $T(G)$, $sd(G)$ and $M(G)$

Theorem 6.1. *If G is a SVN graph then $sd(G)$ is weak isomorphic to $tl(G)$.*

Proof. Let $G = (A, B)$ be a SVN graph with its underlying crisp graph $G^* = (V, E)$. By the definition of $sd(G)$, A_{sd} is a SVN subset defined on $V \cup E$ as

$$(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)(x) \text{ if } x \in V$$

$$= (T_B, I_B, F_B)(x) \text{ if } x \in E. \quad (1)$$

The SVN relation B_{sd} on $V \cup E$ is defined as

$$T_{B_{sd}}(x, y) = T_A(x) \wedge T_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$I_{B_{sd}}(x, y) = I_A(x) \wedge I_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$F_{B_{sd}}(x, y) = F_A(x) \vee F_B(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise.}$$

Using (1) in the above equation,

$$T_{B_{sd}}(x, y) = T_{A_{sd}}(x) \wedge T_{A_{sd}}(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$I_{B_{sd}}(x, y) = I_{A_{sd}}(x) \wedge I_{A_{sd}}(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise}$$

$$F_{B_{sd}}(x, y) = F_{A_{sd}}(x) \vee F_{A_{sd}}(y) \text{ if } x \in V \text{ and } y \in E$$

$$= 0 \text{ otherwise.}$$

Define a map 'g' from $sd(G)$ to $tl(G)$ as identity map $g : V \cup E \to V \cup E$, g be bijection satisfying

$(T_A, I_A, F_A)_{tl}(g(x)) = (T_A, I_A, F_A)_{tl}(x) = (T_A, I_A, F_A)(x) = (T_A, I_A, F_A)_{sd}(x)$ if $x \in V$

$(T_A, I_A, F_A)_{tl}(g(x)) = (T_A, I_A, F_A)_{tl}(x) = (T_B, I_B, F_B)(x) = (T_A, I_A, F_A)_{sd}(x)$ if $x \in E$

That is $(T_A, I_A, F_A)_{tl}(g(x)) = (T_A, I_A, F_A)_{sd}(x)$ if $x \in V \cup E$.

Case 1.

If $x, y \in V, (T_B, I_B, F_B)_{tl}(g(x), g(y)) = (T_B, I_B, F_B)_{tl}(x, y) = (T_B, I_B, F_B)(x, y)$ if $x, y \in V$. By the definition of $sd(G)$, $(T_B, I_B, F_B)_{sd}(x, y) = 0$ if $x, y \in V$.

That implies $(T_B, I_B, F_B)_{sd}(x, y) \leq (T_B, I_B, F_B)_{tl}(g(x), g(y))$ if $x, y \in V$.

Case 2.

If $x \in V$ and $y = e \in E$, then

$T_{B_{tl}}(g(x), g(e)) = T_{B_{tl}}(x, e) = \min\{T_A(x), T_B(e)\}$ if $x \in V$, $e \in E$ and x lies on e

$= 0$ otherwise

$I_{B_{tl}}(g(x), g(e)) = I_{B_{tl}}(x, e) = \min\{I_A(x), I_B(e)\}$ if $x \in V$, $e \in E$ and x lies on e

$= 0$ otherwise

$F_{B_{tl}}(g(x), g(e)) = F_{B_{tl}}(x, e) = \max\{F_A(x), F_B(e)\}$ if $x \in V$, $e \in E$ and x lies on e

$= 0$ otherwise

$(T_B, I_B, F_B)_{sd}(x, y) = (T_B, I_B, F_B)_{tl}(g(x), g(y))$ if $x \in V$, $e \in E$ using the definition of $sd(G)$.

Case 3.

If $x = e_i$, $y = e_j \in E$ then

$T_{B_{tl}}(e_i, e_j) = \min\{T_B(e_i), T_B(e_j)\}$ if e_i, e_j have a vertex in common

$I_{B_{tl}}(e_i, e_j) = \min\{I_B(e_i), I_B(e_j)\}$ if e_i, e_j have a vertex in common

$F_{B_{tl}}(e_i, e_j) = \max\{F_B(e_i), F_B(e_j)\}$ if e_i, e_j have a vertex in common

$= 0$ otherwise

$$T_{B_{sd}}(e_i, e_j) \leq T_{B_{tl}}(e_i, e_j) \text{ if } e_i, e_j \in E$$

$$I_{B_{sd}}(e_i, e_j) \leq I_{B_{tl}}(e_i, e_j) \text{ if } e_i, e_j \in E$$

$$F_{B_{sd}}(e_i, e_j) \leq F_{B_{tl}}(e_i, e_j) \text{ if } e_i, e_j \in E.$$

Thus from the cases we get

$T_{B_{sd}}(x, y) \leq T_{B_{tl}}(x, y)$ if $x, y \in V \cup E$

$I_{B_{sd}}(x, y) \leq I_{B_{tl}}(x, y)$ if $x, y \in V \cup E$

$F_{B_{sd}}(x, y) \leq F_{B_{tl}}(x, y)$ if $x, y \in V \cup E$

Therefore $g : sd(G) \to tl(G)$ is a weak isomorphism.

Theorem 6.2. *$sd(G)$ is weak isomorphic with $M(G)$.*

Proof. Consider the identity map $g : sd(G) \to M(G)$ as $g : V \cup E \to V \cup E$

$(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

$(T_A, I_A, F_A)_M(x) = (T_A, I_A, F_A)(x)$ if $x \in V$

$= (T_B, I_B, F_B)(x)$ if $x \in E$

$(T_A, I_A, F_A)_M(g(x)) = (T_A, I_A, F_A)_M(x)$ for all $x \in V \cup E$.

Hence $(T_A, I_A, F_A)_{sd}(x) = (T_A, I_A, F_A)_M(g(x))$ for all $x \in V \cup E$. (2)

Case 1.

$T_{B_{sd}}(x, e) = \min\{T_{B_{sd}}(x), T_{B_{sd}}(e)\}$ if $x \in V, e \in E$ and $= \min\{T_B(x), T_B(e)\}$ $x \in V, e \in E$ and $T_{B_{sd}}(x, e) = T_B(e)$ by the definition of SVN relation and x lies on e.

Also, $T_{B_M}(x, e) = T_B(e)$ if $x \in V, e \in E$.

Similarly, $I_{B_M}(x, e) = I_B(e)$ if $x \in V, e \in E$.

$F_{B_M}(x, e) = F_B(e)$ if $x \in V, e \in E$.

Hence $(T_B, I_B, F_B)_{sd}(x, e) = (T_B, I_B, F_B)_M(g(x), g(e))$ if $x \in V, e \in E$.

Case 2.

$T_{B_{sd}}(e_i, e_j) = 0$ even if the edges e_i, e_j are neighbours in G

$I_{B_{sd}}(e_i, e_j) = 0$ even if the edges e_i, e_j are neighbours in G

$F_{B_{sd}}(e_i, e_j) = 0$ even if the edges e_i, e_j are neighbours in G

$T_{B_M}(e_i, e_j) = \min\{T_B(e_i), T_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$I_{B_M}(e_i, e_j) = \min\{I_B(e_i), I_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$F_{B_M}(e_i, e_j) = \max\{F_B(e_i), F_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

Also, $T_{B_M}(g(e_i), g(e_j)) = T_{B_M}(e_i, e_j)$, $I_{B_M}(g(e_i), g(e_j)) = I_{B_M}(e_i, e_j)$, $F_{B_M}(g(e_i), g(e_j)) = F_{B_M}(e_i, e_j)$. That is, $T_{B_{sd}}(e_i, e_j) = 0 \le \min\{T_B(e_i), T_B(e_j)\} = T_{B_M}(g(e_i), g(e_j))$ if the edges e_i, e_j are neighbours in G

$I_{B_{sd}}(e_i, e_j) = 0 \le \min\{I_B(e_i), I_B(e_j)\} = I_{B_M}(g(e_i), g(e_j))$ if the edges e_i, e_j are neighbours in G

$F_{B_{sd}}(e_i, e_j) = 0 \le \max\{F_B(e_i), F_B(e_j)\} = F_{B_M}(g(e_i), g(e_j))$ if the edges e_i, e_j are neighbours in G else $(T_B, I_B, F_B)_{sd}(e_i, e_j) = 0 = (T_B, I_B, F_B)_M (g(e_i), g(e_j))$.

Case 3.

$(T_B, I_B, F_B)_{sd}(x, y) = 0$ if $x, y \in V$

$(T_B, I_B, F_B)_M(x, y) = 0$ if $x, y \in V$

$(T_B, I_B, F_B)_{sd}(x, y) = 0 = (T_B, I_B, F_B)_M(x, y)$ if $x, y \in V$.

From these three cases

$$(T_B, I_B, F_B)_{sd}(x, y) \le (T_B, I_B, F_B)_M(g(x), g(y)) \text{ for all } x, y \in V \cup E \quad (3)$$

g being the bijection and from (2) and (3) $sd(G)$ is weak isomorphic with $M(G)$.

Theorem 6.3. *$M(G)$ is weak isomorphic with $tl(G)$.*

Proof. Consider the identity map $g : M(G) \to tl(G)$ as $g : V \cup E \to V \cup E$.

By the definition of A_{tl} in $tl(G)$ and A_M in $M(G)$ we have

$$(T_A, I_A, F_A)_M(x) = (T_A, I_A, F_A)_{tl}(g(x)) \text{ for all } x \in V \cup E. \tag{4}$$

Case 1.

$T_{B_M}(e_i, e_j) = \min\{T_B(e_i), T_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$I_{B_M}(e_i, e_j) = \min\{I_B(e_i), I_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$F_{B_M}(e_i, e_j) = \max\{F_B(e_i), F_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$T_{B_{tl}}(e_i, e_j) = \min\{T_B(e_i), T_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$I_{B_{tl}}(e_i, e_j) = \min\{I_B(e_i), I_B(e_j)\}$ if the edges e_i, e_j are neighbours in G

$F_{B_{tl}}(e_i, e_j) = \max\{F_B(e_i), F_B(e_j)\}$ if the edges e_i, e_j are neighbours in G.

Hence $(T_B, I_B, F_B)_M(x, y) = (T_B, I_B, F_B)_{tl}(x, y) = (T_B, I_B, F_B)_{tl}(g(x), g(y))$ if $x, y \in E$.

Case 2.

$(T_B, I_B, F_B)_M(x, y) = 0$ if $x, y \in V$

$(T_B, I_B, F_B)_{tl}(x, y) = (T_B, I_B, F_B)(x, y)$ if $x, y \in V$

$(T_B, I_B, F_B)_M(x, y) = 0 \leq (T_B, I_B, F_B)(x, y) = (T_B, I_B, F_B)_{tl}(x, y) = (T_B, I_B, F_B)_{tl}(g(x), g(y))$ if $x, y \in V$.

Case 3.

$(T_B, I_B, F_B)_M(x, e) = (T_B, I_B, F_B)(x, e)$ if $x \in V$, $e \in E$ an x lies on e

$T_{B_{tl}}(x, e) = \min\{T_B(x), T_B(e)\}$ if $x \in V$, $e \in E$ an x lies on e

$I_{B_{tl}}(x, e) = \min\{I_B(x), I_B(e)\}$ if $x \in V$, $e \in E$ an x lies on e

$F_{B_{tl}}(x, e) = \max\{F_B(x), F_B(e)\}$ if $x \in V$, $e \in E$ an x lies on e.

So, $(T_B, I_B, F_B)_M(x, e) = (T_B, I_B, F_B)_{tl}(x, e) = (T_B, I_B, F_B)_{tl}(g(x), g(e))$ if $x \in V$, $e \in E$. From the above three cases,

$$(T_B, I_B, F_B)_M(x, y) \leq (T_B, I_B, F_B)_{tl}(g(x), g(y)) \text{ for all } x \in V \cup E \quad (5)$$

'g' being a bijection and from (4) and (5), $M(G)$ is weak isomorphic with $tl(G)$.

References

[1] A. Nagoor Gani and J. Malarvizhi, Isomorphism on fuzzy graphs, International Journal of Computational and Mathematical Sciences, Jan. 2008.

[2] M. Akram, Operations on single-valued nutrosophic graphs, Journal of Uncertain Systems, January 2017.

[3] M. Akram, Bipolar fuzzy graphs, Information Sciences 181 (2011), 5548-5564.

[4] Ali Asghar Talebi, Hossein Rashmanlou, Complement and Isomorphism on Bipolar Fuzzy Graphs, Fuzzy Information and Engineering 6 (2014), 505-522.

[5] S. Broumi, M. Talea, A. Bakali and F. Smarandache, Single-valued neutrosophic graphs, Journal of New Theory 10 (2016), 86-101.

[6] J. Malarvizhi and G. Divya, Isomorphism and Complement on Single Valued Neutrosophic Graphs, American International Journal of Research in Science, Technology, Engineering and Mathematics - 2019.

[7] M. Naga Maruthi Kumari and R. Chandrasekar, Isomorphism on Interval-Valued Fuzzy Graphs, IOSR Journal of Mathematics, (Jan-Feb, 2016), 24-31.

[8] Nagoor gani and J. Anu, Properties on Total and Middle Intuitionstic fuzzy graph, Internatioinal Journal of Fuzzy Mathematical Archive, 2015.

[9] S. Broumi, A. Bakali, M, Talea and F. Smarandache, Isolated Single Valued Neutrosophic Graphs, Neutrosophic Sets and Systems 11 (2016), 74-78.

[10] S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L. H. Son and D. Koley, Uniform Single Valued Neutrosophic Graphs, Neutrosophic Sets and Systems 17 (2017), 42-49.