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SOME REMARKS ON SOMEWHERE FUZZY CONTINUOUS FUNCTIONS

G. THANGARAJ[1] and S. SENTHIL[2]

[1]Department of Mathematics
Thiruvalluvar University
Vellore-632 115, Tamilnadu, India

[2]Department of Mathematics
Indo-American College
Cheyyar-604 407, Tamilnadu, India

Abstract

In this paper, several characterizations of fuzzy somewhere dense sets and fuzzy cs dense sets in fuzzy topological spaces are established. The condition for the fuzzy hyperconnected and fuzzy almost P-space to become an fuzzy σ-Baire space is established by means of fuzzy cs dense sets. The conditions for fuzzy topological spaces to become fuzzy Baire spaces are obtained. The conditions for somewhere fuzzy continuous functions to become somewhat fuzzy continuous functions and for somewhat fuzzy nearly continuous functions to become somewhere fuzzy continuous functions, are also obtained.

1. Introduction

In 1965, L. A. Zadeh [26] introduced the concept of fuzzy sets as a new approach for modelling uncertainties. The potential of fuzzy notion was realized by the mathematicians and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by C. L. Chang [6] in 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

Continuity is one of the most important and fundamental properties that

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have been widely used in Mathematics. In the recent years, a considerable amount of research has been done on many types of continuity in general topology. T. M. Al-Shami [1] introduced a new class of sets, namely somewhere dense sets, in topological spaces. The concept of fuzzy somewhere dense sets in fuzzy topological spaces was introduced by G. Thangaraj in [10]. By means of fuzzy somewhere dense sets, a new type of fuzzy continuous functions, called somewhere fuzzy continuous function, is introduced by G. Thangaraj and S. Senthil in [23]. In this paper, several characterizations of fuzzy somewhere dense sets and fuzzy cs dense sets in fuzzy topological spaces are established. It is established that in fuzzy hyperconnected spaces, fuzzy β-open sets coincide with fuzzy somewhere dense sets and fuzzy F_σ-sets in fuzzy almost P-spaces are fuzzy cs dense sets and fuzzy G_δ-sets are fuzzy somewhere dense sets. The condition for the fuzzy hyperconnected and fuzzy almost P-space to become a fuzzy σ-Baire space is established by means of fuzzy cs dense sets. The conditions for fuzzy topological spaces to become fuzzy Baire spaces are obtained. The conditions for somewhere fuzzy continuous functions to become somewhat fuzzy continuous functions and somewhat fuzzy nearly continuous functions to become somewhere fuzzy continuous functions, are also obtained.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$ for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [6]**.** Let λ be any fuzzy set in the fuzzy topological space (X, T). The fuzzy interior, the fuzzy closure and the fuzzy complement of λ are defined respectively as follows:

(i) $Int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$;

(ii) $Cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

(iii) $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

The notions union $\psi = \vee_i(\lambda_i)$ and intersection $\delta = \wedge_i(\lambda_i)$ are defined respectively, for the family $\{\lambda_i / i \in I\}$ of fuzzy sets in $(X\ T)$ as follows:

(iii) $\psi(x) = \sup_i\{\lambda_i(x) / \in I\}$

(iv) $\delta(x) = \inf_i\{\lambda_i(x) / x \in X\}$.

Lemma 2.1[2]. *For a fuzzy set* λ *of a fuzzy topological space X,*

(i) $1 - Int(\lambda) = Cl(1 - \lambda)$ *and* (ii) $1 - Cl(\lambda) = Int(1 - \lambda)$.

Definition 2.2. A fuzzy set λ in a fuzzy topological space (X, T) is called an

(1) fuzzy regular-open if $\lambda = \operatorname{int} cl(\lambda)$ and fuzzy regular-closed if $\lambda = cl \operatorname{int}(\lambda)$ [2].

(2) fuzzy pre-open if $\lambda \leq \operatorname{int} cl(\lambda)$ and fuzzy pre-closed if $\lambda \leq cl \operatorname{int}(\lambda)$ [5].

(4) fuzzy semi-open if $\lambda \leq cl \operatorname{int}(\lambda)$ and fuzzy semi-closed if $\lambda = \operatorname{int} cl(\lambda)$ [2].

(3) fuzzy α-open if $\lambda \leq \operatorname{int} cl \operatorname{int}(\lambda)$ and fuzzy α-closed if $cl \operatorname{int} cl(\lambda) \leq \lambda$ [5].

(3) fuzzy β-open if $\lambda \leq cl \operatorname{int} cl(\lambda)$ and fuzzy β-closed if $\operatorname{int} cl \operatorname{int}(\lambda) \leq \lambda$ [4].

Definition 2.3[12]. A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X, T).

Definition 2.4[12]. A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, $\operatorname{int} cl(\lambda) = 0$, in (X, T).

Definition 2.5[12]. A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.6[12]**.** Let λ be a fuzzy first category set in the fuzzy topological space (X, T). Then, $1-\lambda$ is called a fuzzy residual set in (X, T).

Definition 2.7. Let (X, T) be the fuzzy topological space and (X, T) is called an

(i) Fuzzy Baire if $\operatorname{int}(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T) [14].

(ii) Fuzzy hyper-connected space if every non-null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [8].

(iii) Fuzzy P-space if each fuzzy G_δ-set in (X, T) is fuzzy open in (X, T) [11].

(iv) Fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, $\lambda \in T$ [3].

(v) Fuzzy almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$ where the fuzzy sets (λ_i)'s in (X, T) are such that $\operatorname{int}(\lambda_i) = 0$. Otherwise (X, T) is called an fuzzy almost irresolvable space [25].

(vi) Fuzzy first category space if $1_X = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy nowhere dense sets in (X, T). A fuzzy topological space which is not of fuzzy first category is said to be of fuzzy second category [12].

(vii) Fuzzy almost P-space if for every non-zero fuzzy G_δ-set λ in (X, T), $\operatorname{int}(\lambda) \neq 0$ in (X, T) [17].

(viii) Fuzzy globally disconnected space if each fuzzy semi-open set in (X, T) is fuzzy open [22].

(ix) Fuzzy resolvable space if there exists an fuzzy dense set λ in (X, T) such that $1-\lambda$ is also an fuzzy dense set in (X, T) [13].

(x) Fuzzy open hereditarily irresolvable space if $\operatorname{int} cl(\lambda) \neq 0$, for any non-zero fuzzy set λ defined on X, then $\operatorname{int}(\lambda) \neq 0$, in (X, T) [10].

(xi) Fuzzy strongly irresolvable space if for every fuzzy dense set λ in

(X, T), $cl\,\text{int}(\lambda) = 1$, in (X, T) [24].

(xii) Fuzzy σ-Baire space if $\text{int}(\vee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T) [16].

Definition 2.8[21]**.** Let (X, T) be a fuzzy topological space. A fuzzy set λ defined on X is called an fuzzy Baire set, if $\lambda = (\mu \wedge \delta)$, where μ is an fuzzy open set and δ is an fuzzy residual set in (X, T).

Definition 2.9[19]**.** A fuzzy set λ in a fuzzy topological space (X, T) is called an fuzzy simply* open set if $\lambda = (\mu \wedge \delta)$, where μ is an fuzzy open set and δ is an fuzzy nowhere dense set in (X, T) and $1 - \lambda$ is called an fuzzy simply* closed set in (X, T).

Definition 2.10[12]**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces.

A function $f : (X, T) \to (Y, S)$ is called an somewhat fuzzy continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists an non-zero fuzzy open set μ in (X, T) such that $\mu \leq f^{-1}(\lambda)$. That is, $\text{int}[f^{-1}(\lambda)] \neq 0$, in (X, T).

Definition 2.11[15]**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces.

A function $f : (X, T) \to (Y, S)$ is called an somewhat fuzzy nearly continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists an non-zero fuzzy open set μ in (X, T) such that $\mu \leq cl[f^{-1}(\lambda)]$. That is, $\text{int}\,cl[f^{-1}(\lambda)] \neq 0$, in (X, T).

Definition 2.12[4]**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces.

A function $f : (X, T) \to (Y, S)$ is called an fuzzy β-continuous function if for each fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is an fuzzy β-open set in (X, T).

Definition 2.13[19]**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces.

A function $f : (X, T) \to (Y, S)$ is called an fuzzy simply*-continuous function if for each fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is an fuzzy simply* -open in (X, T).

Definition 2.14[20]**.** Let (X, T) be an fuzzy topological space. A fuzzy set λ defined on X is called an fuzzy pseudo-open set in (X, T) if $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in (X, T) and δ is an fuzzy first category set in (X, T).

Definition 2.15[20]**.** Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called an fuzzy pseudo-continuous function if for each fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is an fuzzy pseudo-open in (X, T).

Definition 2.16[9]**.** Let (X, T) be an fuzzy topological space. A fuzzy set λ defined on X is said to have the property of Baire, if $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is an fuzzy open set, δ is an fuzzy residual set and η is an fuzzy first category set in (X, T).

Definition 2.17[18]**.** Let (X, T) be a fuzzy topological space. A fuzzy set λ defined on X is called an fuzzy B^* set, if λ is an fuzzy set with fuzzy Baire property in (X, T) such that $\operatorname{int} cl(\lambda) \neq 0$, in (X, T).

Theorem 2.1[23]**.** *If λ is a fuzzy simply* open set in the fuzzy topological space (X, T), then λ is an fuzzy somewhere dense set in (X, T).*

Theorem 2.2[9]**.** *If λ is a fuzzy Baire set in the fuzzy globally disconnected and fuzzy P-space (X, T), then λ is an fuzzy open set in (X, T).*

Theorem 2.3[23]**.** *If λ is an non-zero fuzzy β-open set in the fuzzy topological space (X, T), then λ is an fuzzy somewhere dense set in (X, T).*

Theorem 2.4[23]**.** *If λ is an fuzzy simply*-open set in the fuzzy topological space (X, T), then $\operatorname{int}(\lambda) \neq 0$ in (X, T).*

Theorem 2.5[21]. *If λ is a fuzzy Baire set in the fuzzy submaximal and fuzzy P-space (X, T), then λ is an fuzzy open set in (X, T).*

Theorem 2.6[18]. *If* λ *is an fuzzy* B^{*}*-set in the fuzzy topological space* (X, T), *then there exists an fuzzy Baire set* σ *in* (X, T) *such that* $\sigma \leq \lambda$, *in* (X, T).

Theorem 2.7[13]. *If the fuzzy topological space* (X, T) *is an fuzzy open hereditarily irresolvable space, then* $\text{int}(\lambda) = 0$ *for any non-zero fuzzy set* λ *in* (X, T) *implies that* $\text{int}\,cl(\lambda) = 0$.

3. Fuzzy Somewhere Dense SETS

Definition 3.1[10]. Let (X, T) be the fuzzy topological space. A fuzzy set λ defined on X is called an fuzzy somewhere dense set, if $\text{int}\,cl(\lambda) \neq 0$ in (X, T). That is, λ is an fuzzy somewhere dense set in (X, T), if there exists an non-zero fuzzy open set μ in (X, T) such that $\mu \leq cl(\lambda)$ If λ is an fuzzy somewhere dense set in the fuzzy topological space (X, T), then $1 - \lambda$ is called an fuzzy complement of fuzzy somewhere dense set in (X, T). It is to be denoted as fuzzy cs dense set in (X, T) [23].

Proposition 3.1. *If* $\text{int}(\lambda) \neq 0$, *for an fuzzy set* λ *defined on* X *in the fuzzy topological space* (X, T), *then* λ *is an fuzzy somewhere dense set in* (X, T).

Proof. Let λ be an fuzzy set defined on X. Then, $\text{int}(\lambda) \leq \text{int}\,cl(\lambda)$ in (X, T) and $\text{int}(\lambda) \neq 0$ implies that $\text{int}\,cl(\lambda) \neq 0$, in (X, T) and hence (X, T) is an fuzzy somewhere dense set in (X, T).

The following proposition shows that the fuzzy open sets are fuzzy somewhere dense sets in fuzzy topological spaces.

Proposition 3.2. *If* λ *is an non-zero fuzzy open set in the fuzzy topological space* (X, T), *then* λ *is an fuzzy somewhere dense set in* (X, T).

Proof. Let λ be an non-zero fuzzy open set in (X, T) and then, $\text{int}(\lambda) = \lambda \neq 0$, in (X, T). By the proposition 3.1, λ is an fuzzy somewhere dense set in (X, T).

Remark 3.1. It is to be noted that the fuzzy somewhere dense sets in fuzzy topological spaces need not be fuzzy open sets [23].

Proposition 3.3. *If λ is an fuzzy closed set in the fuzzy topological space (X, T), then λ is an fuzzy cs dense set in (X, T).*

Proof. Let λ be an fuzzy closed set in (X, T). Then, $1-\lambda$ is an fuzzy open set in (X, T). By the proposition 3.2, $1-\lambda$ is an fuzzy somewhere dense set in (X, T). Then, $1-(1-\lambda)$ is an fuzzy cs dense set in (X, T). Hence λ is an fuzzy cs dense set in (X, T).

Remark 3.2. The class of fuzzy somewhere dense sets contains all non-zero fuzzy regular-open sets, fuzzy open sets, fuzzy α-open sets, fuzzy semi-open sets, fuzzy pre-open sets and fuzzy β-open sets in fuzzy topological spaces. For,

(i) If λ is a non-zero fuzzy regular-open set in the fuzzy topological space (X, T), then $\lambda = \operatorname{int} cl(\lambda)$ implies that λ is an fuzzy somewhere dense set in (X, T).

(ii) If λ is a non-zero fuzzy pre-open set in (X, T), then $\lambda \leq \operatorname{int} cl(\lambda)$ and $\lambda \neq 0$ implies that $\operatorname{int} cl(\lambda) \neq 0$ and hence λ is an fuzzy somewhere dense set in (X, T).

(iii) If $\lambda \neq 0$ is a non-zero fuzzy α-open set in (X, T), then $\lambda \leq \operatorname{int} cl \operatorname{int}(\lambda) \leq \operatorname{int} cl(\lambda)$ and this implies that $\operatorname{int} cl(\lambda) \neq 0$ and hence λ is an fuzzy somewhere dense set in (X, T).

(iv) If $\lambda \neq 0$ is a non-zero fuzzy β-open set in (X, T), then $\lambda \leq cl \operatorname{int} cl(\lambda)$ and this implies that $\operatorname{int} cl(\lambda) \neq 0$ (for otherwise, $\operatorname{int} cl(\lambda) = 0$ implies that $cl[\operatorname{int} cl(\lambda)] = cl[0] = 0,$ and then $\lambda = 0,$ a contradiction) and hence λ is an fuzzy somewhere dense set in (X, T).

(v). If λ is a non-zero fuzzy semi-open set in (X, T) then $\operatorname{int}(\lambda) \neq 0$ and by the proposition 3.1, λ is an fuzzy somewhere dense set in (X, T).

Proposition 3.4. *If λ is an fuzzy pseudo-open set in the fuzzy topological space (X, T), then λ is an fuzzy somewhere dense set in (X, T).*

Proof. Let λ be an fuzzy pseudo-open set in (X, T). Then, $\lambda = \mu \vee \delta,$

where $\mu \in T$ and δ is an fuzzy first category set in (X, T). Now $\text{int}(\lambda) = \text{int}(\mu \vee \delta) \geq \text{int}(\mu) \vee \text{int}(\delta) \geq \mu \vee \text{int}(\delta) \geq \mu \neq 0$. Hence, $\text{int}(\lambda) \neq 0$ in (X, T) and then, by the proposition 3.1, λ is an fuzzy somewhere dense set in (X, T).

Remark 3.3. In view of the above proposition, one will have the following result: "Fuzzy pseudo-open sets are fuzzy somewhere dense sets in fuzzy topological spaces".

The following proposition shows that the fuzzy somewhere dense sets are fuzzy dense sets in fuzzy hyperconnected spaces.

Proposition 3.5. *If λ is an fuzzy somewhere dense set in the fuzzy hyperconnected space (X, T), then λ is an fuzzy dense set in (X, T).*

Proof. Let λ be an fuzzy somewhere dense set in (X, T). Then, $\text{int}\, cl(\lambda) \neq 0$, in (X, T). This implies that there exists an fuzzy open set μ in (X, T) such that $\mu \leq cl(\lambda)$ and then $cl(\mu) \leq cl[cl(\lambda)] = cl(\lambda)$. Since μ is an fuzzy open set in the fuzzy hyperconnected space (X, T), $cl(\mu) = 1$ in (X, T) and then $1 \leq cl(\lambda)$. That is, $cl(\lambda) = 1$ in (X, T). Hence the fuzzy somewhere dense set λ is an fuzzy dense set in (X, T).

Proposition 3.6. *If λ is a non-zero fuzzy open set in the fuzzy hyperconnected space (X, T), then $1 - \lambda$ is not an fuzzy somewhere dense set in (X, T).*

Proof. Let λ be an fuzzy open set in (X, T). Then, $\text{int}(\lambda) = \lambda$ in (X, T). Since (X, T) is the fuzzy hyperconnected space, the fuzzy open set λ is an fuzzy dense set in (X, T) and thus $cl(\lambda) = 1$, in (X, T). Then $cl(1 - \lambda) = 0$. and this implies that $1 - cl\,\text{int}(\lambda) = 0$ in (X, T). Then $cl\,\text{int}(1 - \lambda) = 0$. Hence $1 - \lambda$ is not an fuzzy somewhere dense set in (X, T).

Proposition 3.7. *If λ is an fuzzy somewhere dense set in the fuzzy hyperconnected space (X, T), then λ is an fuzzy β-open set in (X, T).*

Proof. Let λ be an fuzzy somewhere dense set in (X, T). Then

$\text{int}\, cl(\lambda) \neq 0$, in (X, T). Since (X, T) is the fuzzy hyperconnected space, the fuzzy open set $\text{int}\, cl(\lambda)$ is an fuzzy dense set in (X, T) and hence $cl[\text{int}\, cl(\lambda)] = 1$. This implies that $\lambda \leq cl[\text{int}\, cl(\lambda)]$. Hence λ is an fuzzy β-open set in (X, T).

Remark 3.4. In view of the theorem 2.3 and the proposition 3.7, one will have the following result:

"In fuzzy hyperconnected spaces, fuzzy β-open sets coincide with fuzzy somewhere dense sets."

Proposition 3.8. *If λ is an fuzzy G_δ-set (an fuzzy F_σ-set) in the fuzzy P-space (X, T) then λ is an fuzzy somewhere dense set (an fuzzy cs dense set) in (X, T).*

Proof. Let λ be an fuzzy G_δ-set (an fuzzy F_σ-set) in (X, T). Since (X, T) is the fuzzy P-space, the fuzzy G_δ-set (F_σ-set) λ is an fuzzy open set (an fuzzy closed set) in (X, T). Then, by the proposition 3.2, λ is an fuzzy somewhere dense set in (X, T). (By the proposition 3.3, λ is an fuzzy cs dense set in (X, T).

Proposition 3.9. *If the fuzzy semi-closed set λ is an fuzzy somewhere dense set in the fuzzy topological space (X, T), then $\text{int}(\lambda) \neq 0$, in (X, T).*

Proof. Let λ be an fuzzy semi-closed set in (X, T). Then, $\text{int}\, cl(\lambda) \leq \lambda$ Since λ is an fuzzy somewhere dense set, $\text{int}\, cl(\lambda) \neq 0$, in (X, T). Let $\text{int}\, cl(\lambda) = \mu$ and then μ is an fuzzy open set in (X, T) and hence $\mu \leq \lambda$, implies that $\text{int}(\lambda) \neq 0$ in (X, T).

Proposition 3.10. *If the fuzzy set λ is an fuzzy cs dense set in the fuzzy hyperconnected space (X, T), then* $\text{int}(\lambda) = 0$, in (X, T).

Proof. Let λ be an fuzzy cs dense set in (X, T). Then, $1 - \lambda$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy hyperconnected space, by the proposition 3.5, $1 - \lambda$ is an fuzzy dense set in (X, T) and then $cl(1 - \lambda) = 1$. This implies, by the lemma 2.1, that $1 - \text{int}\, \lambda = 1$ and then $\text{int}(\lambda) = 0$, in (X, T).

Proposition 3.11. *If* λ *is an fuzzy somewhere dense set in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, then the fuzzy cs dense set,* $1-\lambda$ *is an fuzzy nowhere dense set in* (X, T).

Proof. Let λ be an fuzzy somewhere dense set in (X, T). Then, $1-\lambda$ is an fuzzy cs dense set in (X, T). Since (X, T) is an fuzzy hyperconnected space, By the proposition 3.10, $\operatorname{int}(1-\lambda)=0$, in (X, T). Since (X, T) is an fuzzy open hereditarily irresolvable space, $\operatorname{int}(1-\lambda)=0$, implies that $\operatorname{int} cl(1-\lambda)=0$ and hence $1-\lambda$ is an fuzzy nowhere dense set in (X, T). Thus, the fuzzy cs dense set $1-\lambda$ is an fuzzy nowhere dense set in (X, T).

The following proposition shows that the fuzzy pseudo-open sets are fuzzy dense sets in fuzzy hyperconnected spaces.

Proposition 3.12. *If* λ *is an fuzzy pseudo-open set in the fuzzy hyperconnected space* (X, T), *then* λ *is an fuzzy dense set in* (X, T).

Proof. Let λ be an fuzzy pseudo-open set in (X, T). Then, by the proposition 3.4, λ is an fuzzy somewhere dense set in (X, T). Since (X, T) is an fuzzy hyperconnected space, by the proposition 3.5, λ is an fuzzy dense set in (X, T).

Proposition 3.13. *If* λ *is an fuzzy* F_σ*-set in the fuzzy almost P-space* (X, T), *the* λ *is an fuzzy cs dense set in* (X, T).

Proof. Let λ be an fuzzy F_σ-set in (X, T). Then, $1-\lambda$ is an fuzzy G_δ-set in (X, T). Since (X, T) is an fuzzy almost P-space, for the fuzzy G_δ-set $1-\lambda$, $\operatorname{int}(1-\lambda)\neq 0$, in (X, T). Then, by the proposition 3.1, $1-\lambda$ is an fuzzy somewhere dense set in (X, T) and hence λ is an fuzzy cs dense set in (X, T).

Remark 3.5. In view of the above proposition, one will have the following result: "Fuzzy F_σ-sets in fuzzy almost P-spaces are fuzzy cs dense sets and fuzzy G_δ-sets in fuzzy almost P-spaces are fuzzy somewhere dense sets".

The following proposition shows that the fuzzy F_σ-sets in the fuzzy hyperconnected and fuzzy almost P-spaces are fuzzy σ-nowhere dense sets.

Proposition 3.14. *If* λ *is an fuzzy* F_σ*-set in the fuzzy hyperconnected and fuzzy almost P-space* (X, T), *then* λ *is an fuzzy* σ*-nowhere dense set in* (X, T).

Proof. Let λ be an fuzzy F_σ-set in (X, T). Since (X, T) is the fuzzy almost P-space, by the proposition 3.13, λ is an fuzzy cs dense set in (X, T). Also since (X, T) is the fuzzy hyperconnected space, by the proposition 3.10, for the fuzzy cs dense set λ in (X, T), $\text{int}(\lambda) = 0$, in (X, T). Thus λ is an fuzzy F_σ-set such that $\text{int}(\lambda = 0,)$ in (X, T). Hence λ is an fuzzy σ-nowhere dense set in (X, T).

The following proposition gives a condition for the fuzzy hyperconnected and fuzzy almost P-spaces to become fuzzy σ-Baire spaces.

Proposition 3.15. *If* $\text{int}[\vee_{i=1}^{\infty} (\lambda_i) = 0$, *where* (λ_i)*'s are fuzzy* F_σ*-sets in the fuzzy hyperconnected and fuzzy almost P-space* (X, T) *then* (X, T) *is an fuzzy* σ*-Baire space*.

Proof. Let (λ_i)'s be the fuzzy F_σ-sets in (X, T). Since (X, T) is the fuzzy hyperconnected and fuzzy almost P-space, by the proposition 3.14, (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Then, from the hypothesis, $\text{int}[\vee_{i=1}^{\infty} (\lambda_i) = 0$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). This implies that (X, T) is an fuzzy σ-Baire space.

Theorem 3.1[23]. *A fuzzy set* λ *in the fuzzy topological space* (X, T) *is an fuzzy cs dense set if and only if there exists an fuzzy closed set* μ *in* (X, T) *such that* $\text{int}(\lambda) \leq \mu$.

Proposition 3.16. *If* $(\lambda_i)' s(i = 1 \text{ to } \infty)$ *are fuzzy cs dense sets in the fuzzy topological space* (X, T), *then there exist an fuzzy open set* λ *and fuzzy* F_σ*-set* μ *in* (X, T) *such that* $\lambda \leq \mu$ *where* $\lambda = \vee_{i=1}^{\infty} \text{int}(\lambda_i)$ *and* $\mu = \vee_{i=1}^{\infty}(\mu_i)$, $1 - \mu_i \in T$.

Proof. Let $(\lambda_i)' s(i = 1 \text{ to } \infty)$ be fuzzy cs dense sets in (X, T). Then, by the theorem 3.1, there exist fuzzy closed sets μ_i in (X, T) such that $\text{int}(\lambda_i) \leq \mu_i$.

This implies that $\vee_{i=1}^{\infty}\operatorname{int}(\lambda_i) \leq \vee_{i=1}^{\infty}(\mu_i)$. Let $\lambda = \vee_{i=1}^{\infty}\operatorname{int}(\lambda_i)$ and $\vee_{i=1}^{\infty}\operatorname{int}(\lambda_i)$. Then, λ is an fuzzy open set and μ is an fuzzy F_σ- set in (X, T). Thus, there exist an fuzzy open set λ and an fuzzy F_σ-set μ in (X, T) such that $\lambda \leq \mu$.

Proposition 3.17. *If* λ *is an fuzzy somewhere dense set in the fuzzy hyperconnected space* (X, T), *then* $\operatorname{int}(1-\lambda) = 0$ *in* (X, T).

Proof. Let λ be an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy hyperconnected space, by the proposition 3.5 , λ is an fuzzy dense set in (X, T) and hence $cl(\lambda) = 1$ in (X, T) and then $1 - cl(\lambda) = 0$ and this implies, by the lemma 2.1, that $\operatorname{int}(1-\lambda) = 0$ in (X, T).

Proposition 3.18. *If* λ *is an fuzzy somewhere dense set in the fuzzy strongly irresolvable space* (X, T), *if and only if* $\operatorname{int}(\lambda) \neq 0$ *in* (X, T).

Proof. Let λ be an fuzzy somewhere dense set in (X, T). Then, $\operatorname{int} cl(\lambda) \neq 0$, in (X, T). It is claimed that $\operatorname{int}(\lambda) \neq 0$ in (X, T). Suppose that $\operatorname{int}(\lambda) = 0$ in (X, T). Then $cl(1-\lambda) = 1 - \operatorname{int}(\lambda) = 1 - 0 = 1$. Since (X, T) is the fuzzy strongly irresolvable space, $cl(1-\lambda) = 1$ will imply that $cl \operatorname{int}(1-\lambda) = 1$ and hence, by the lemma 2.1, that $1 - \operatorname{int} cl(\lambda) = 1$ in (X, T). Then $\operatorname{int} cl(\lambda) = 0$, a contradiction and therefore $\operatorname{int}(\lambda) \neq 0$ in (X, T).

Conversely, suppose that $\operatorname{int}(\lambda) \neq 0$ in (X, T). Then, by the proposition 3.1, λ is an fuzzy somewhere dense set in (X, T).

Proposition 3.19. *If* λ *is an fuzzy simply*-open set in the fuzzy hyperconnected space* (X, T), *then* λ *is an fuzzy dense set in* (X, T).

Proof. Let λ be an fuzzy simply*-open set in (X, T). Then, by the theorem 2.1, λ is an fuzzy somewhere dense set in (X, T). Then, by the proposition 3.5, λ is an fuzzy dense set in (X, T).

Proposition 3.20. *If* λ *is an fuzzy simply*-open set in the fuzzy topological space* (X, T), *such that* $cl(\lambda) \neq 1$, *then* $Bd(\lambda)$ *is an fuzzy somewhere dense set*, *in* (X, T).

Proof. Let λ be an fuzzy simply*-open set in the fuzzy topological space (X, T). Then, by the theorem 2.4, $\text{int}(\lambda) \neq 0$ in (X, T). From the hypothesis, $cl(\lambda) \neq 1$ in (X, T). Then, $1 - cl(\lambda) \neq 0$ in (X, T). This implies, by the lemma 2.1, that $\text{int}(1 - \lambda) \neq 0$ in (X, T).

Now $\text{int}\, cl[Bd(\lambda)] = \text{int}\, cl[cl(\lambda) \wedge cl(1 - \lambda)]$

$\geq \text{int}\, cl\, cl[\lambda \wedge (1 - \lambda)]$

$= \text{int}\, cl[\lambda \wedge (1 - \lambda)]$

$\geq \text{int}[\lambda \wedge (1 - \lambda)]$

$= \text{int}(\lambda) \wedge \text{int}(1 - \lambda) \geq 0$

This implies that $\text{int}\, cl[Bd(\lambda)] \neq 0,$ in (X, T) and hence $Bd(\lambda)$ is an fuzzy somewhere dense set in (X, T).

Proposition 3.21. *If λ is an fuzzy simply*-open set in the fuzzy topological space (X, T) such that $cl\,\text{int}(\lambda) = 1,$ then $Bd(\lambda)$ is not an fuzzy somewhere dense set, in (X, T).*

Proof. Let λ be an fuzzy simply*-open set in (X, T). Now $cl\,\text{int}(\lambda) \leq cl(\lambda)$ and $cl\,\text{int}(\lambda) = 1$ implies that $cl(\lambda) = 1$ in (X, T) and then $\text{int}\, cl[Bd(\lambda)] = \text{int}\, cl[cl(\lambda) \wedge cl(1 - \lambda)] = \text{int}\, cl[1 \wedge cl(1 - \lambda)] = \text{int}\, cl[cl(1 - \lambda)]$ $incl(1 - \lambda) = 1 - cl\,\text{int}(\lambda) = 1 - 1 = 0.$ Hence $Bd(\lambda)$ is not an fuzzy somewhere dense set, in (X, T).

Proposition 3.22. *If λ is an fuzzy simply*-closed set in the fuzzy topological space (X, T), then λ is an fuzzy cs dense set in (X, T).*

Proof. Let λ be an fuzzy simply*-closed set in (X, T). Then, $1 - \lambda$ is an fuzzy simply*-open set in (X, T). By the theorem 2.1, $1 - \lambda$ is an fuzzy somewhere dense set in (X, T) and hence λ is an fuzzy cs dense set, in (X, T).

The following proposition gives a condition for the fuzzy hyperconnected and fuzzy open hereditarily irresolvable spaces to become fuzzy Baire spaces.

Proposition 3.23. *If* $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, *where* (λ_i)*'s are fuzzy somewhere dense sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space* (X, T), *then* (X, T) *is an fuzzy Baire space*.

Proof. Suppose that (λ_i)'s are fuzzy somewhere dense sets in (X, T). Since (X, T) is an fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, by the proposition 3.11, $[1-(\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T). From the hypothesis $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, $1 - cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 0$, in (X, T). This implies , by the lemma 2.1, that $\text{int}[\vee_{i=1}^{\infty}(1-\lambda_i)] = 0$, and then $\text{int}[\vee_{i=1}^{\infty}(1-\lambda_i)] = 0$, where $[1-(\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T). Hence (X, T) is an fuzzy Baire space.

Proposition 3.24. *If* λ *is an fuzzy open set and* μ *is an fuzzy somewhere dense set in the fuzzy hyperconnected space* (X, T), *then* $\lambda \wedge \mu$ *is an fuzzy somewhere dense set in* (X, T).

Proof. Let $\lambda (\neq 0)$ be an fuzzy open set and μ be an fuzzy somewhere dense set in (X, T). It is claimed that $\text{int}\, cl(\lambda \wedge \mu) \neq 0$ in (X, T). Suppose that $\text{int}\, cl(\lambda \wedge \mu) = 0$. Since $\text{int}(\lambda \wedge \mu) \leq \text{int}\, cl(\lambda \wedge \mu)$, $\text{int}(\lambda \wedge \mu) = 0$ and then $\text{int}(\lambda) \wedge \text{int}(\mu) = 0$ and this will imply that $\text{int}(\mu) \leq 1 - \text{int}(\lambda)$ and then $\text{int}(\mu) \leq 1 - \lambda$ (since $\lambda \in T$). Now $cl\,\text{int}(\mu) \leq cl(1-\lambda)$ will imply that $cl\,\text{int}(\mu) \leq 1 - \text{int}(\lambda)$. Since $\text{int}(\mu)$ is an fuzzy open set in the fuzzy hyperconnected space, $cl\,\text{int}(\mu) = 1$ and then $1 \leq 1 - \text{int}(\lambda)$ and this will imply that $\text{int}(\lambda) = 0$, a contradiction to λ being an non-zero fuzzy open set for which $\text{int}(\lambda) = \lambda \neq 0$. Hence $\text{int}\, cl(\lambda \wedge \mu) \neq 0$. in (X, T). Thus $\lambda \wedge \mu$ is an fuzzy somewhere dense set in (X, T).

Proposition 3.25. *If* $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, *where* (λ_i)*'s are fuzzy cs dense sets in the fuzzy hyperconnected space* (X, T), *then* (X, T) *is the fuzzy almost irresolvable space*.

Proof. Suppose that $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where (λ_i)'s are fuzzy cs dense sets in

(X, T). Since (X, T) is the fuzzy hyperconnected space, by the proposition 3.10, for the fuzzy cs dense sets $(\lambda_i)' s$, $\text{int}(\lambda_i) = 0$, in (X, T). Thus, $\vee_{i=1}^{\infty} (\lambda_i) \neq 1$, where $\text{int}(\lambda_i) = 0$, in (X, T), implies that (X, T) is the fuzzy almost irresolvable space.

The following proposition shows that the fuzzy Baire sets in the fuzzy globally disconnected and fuzzy P-spaces, are fuzzy somewhere dense sets.

Proposition 3.26. *If λ is a fuzzy Baire set in the fuzzy globally disconnected and fuzzy P*-space (X, T), *then λ is an fuzzy somewhere dense set in* (X, T).

Proof. Let $\lambda(\neq 0)$ be an fuzzy Baire set in (X, T). Since (X, T) is the fuzzy globally disconnected and fuzzy P-space, by the theorem 2.2, λ is an fuzzy open set in (X, T). Then, by the proposition 3.2, λ is an fuzzy somewhere dense set in (X, T).

Theorem 3.2[9]. *If λ is an fuzzy residual set in the fuzzy second category but not an fuzzy Baire space* (X, T), *then* $\text{int}(1-\lambda)$ *is an non-zero fuzzy first category set in* (X, T).

Proposition 3.27. *If λ is an fuzzy residual set in the fuzzy second category but not an fuzzy Baire space* (X, T), *then λ is an fuzzy cs dense set in* (X, T).

Proof. Let λ be an fuzzy residual set in (X, T). Since (X,T) is an fuzzy second category but not an fuzzy Baire space, by the theorem 3.2, then $\text{int}(1-\lambda)$ is an non-zero fuzzy first category set in (X, T). That is, $\text{int}(1-\lambda) \neq 0$, in (X, T). Then, by the proposition 3.1, $(1-\lambda)$ is an fuzzy somewhere dense set in (X, T) and then λ is an fuzzy cs dense set in (X, T).

Proposition 3.28. *If λ is an fuzzy first category set in the fuzzy second category but not an fuzzy Baire space* (X, T), *then λ is an fuzzy somewhere dense set in* (X, T).

Proof. Let λ be an fuzzy first category set in (X, T). Then, $1-\lambda$ is an fuzzy residual set in (X, T). Since (X,T) is an fuzzy second category but not an fuzzy Baire space, by the proposition 3.27, $1-\lambda$ is an fuzzy cs dense set and hence λ is an fuzzy somewhere dense set in (X, T).

Proposition 3.29. *If* λ *is an fuzzy first category set in the fuzzy second category but not an fuzzy Baire space* (X, T), *with fuzzy hyper connectedness, then* λ *is an fuzzy dense set in* (X, T).

Proof. Let λ be an fuzzy first category set in (X, T). Since (X, T) is an fuzzy second category but not an fuzzy Baire space, by the proposition 3.28, λ is an fuzzy somewhere dense set in (X, T). Since (X, T) is an fuzzy hyperconnected space, by the proposition 3.5, λ is an fuzzy dense set in (X, T).

Proposition 3.30. *If* λ *is an fuzzy residual set in the fuzzy second category but not an fuzzy Baire space* (X, T) *with fuzzy hyper connectedness, then* $\operatorname{int}(\lambda) = 0$ *in* (X, T).

Proof. Let λ be an fuzzy residual set in (X, T). Since (X, T) is an fuzzy second category but not an fuzzy Baire space, by the proposition 3.27, λ is an fuzzy cs dense set in (X, T). Since (X, T) is an fuzzy hyperconnected space, by the proposition 3.10, $\operatorname{int}(\lambda) = 0$ in (X, T).

Proposition 3.31. *If* λ *is a fuzzy Baire set in the fuzzy submaximal and fuzzy P-space* (X, T), *then* λ *is an fuzzy somewhere dense set in* (X, T).

Proof. Let $\lambda(\neq 0)$ be an fuzzy Baire set in (X, T). Since (X, T) is the fuzzy submaximal and fuzzy P-space, by the theorem 2.5, λ is an fuzzy open set in (X, T). Then, by the proposition 3.2, λ is an fuzzy somewhere dense set in (X, T).

Proposition 3.32. *If* λ *is an fuzzy* B^**-set in the fuzzy globally disconnected and fuzzy P-space* (X, T), *then there exists an fuzzy somewhere dense set* σ *in* (X, T) *such that* $\sigma \leq \lambda$, in (X, T).

Proof. Let λ be an fuzzy B^*-set in (X, T). Then, by the theorem 2.6, there exists an fuzzy Baire set σ in (X, T) such that $\sigma \leq \lambda$, in (X, T). Since (X, T) is the fuzzy globally disconnected and fuzzy P-space , by the proposition 3.26, the fuzzy Baire set σ is an fuzzy somewhere dense set in (X, T). Hence there exists an fuzzy somewhere dense set σ in (X, T) such that $\sigma \leq \lambda$, in (X, T).

Proposition 3.33. *If λ is an fuzzy dense set in the fuzzy submaximal space (X, T), then λ is an fuzzy somewhere dense set in (X, T).*

Proof. Let λ be an fuzzy dense set in (X, T). Since (X, T) is the fuzzy submaximal space, λ is an fuzzy open set in (X, T). By the proposition 3.2, λ is an fuzzy somewhere dense set in (X, T).

Proposition 3.34. *If λ is an fuzzy dense set in the fuzzy nodec and fuzzy open hereditarily irresolvable space (X, T), then λ is an fuzzy somewhere dense set in (X, T).*

Proof. Let λ be an fuzzy dense set in (X, T). Then, $cl(\lambda) = 1$ and this implies that $1 - cl(\lambda) = 0$ and thus $\operatorname{int}(1 - \lambda) = 0$, in (X, T). Since (X, T) is fuzzy open hereditarily irresolvable space, $\operatorname{int}(1 - \lambda) = 0$, implies that $\operatorname{int} cl(1 - \lambda) = 0$. Then, $1 - \lambda$ is an fuzzy nowhere dense set in (X, T). Since (X, T) is an fuzzy nodec space, the fuzzy nowhere dense set $1 - \lambda$ is an fuzzy closed set and then λ is an fuzzy open set in (X, T). By the proposition 3.2, λ is an fuzzy somewhere dense set in (X, T).

Proposition 3.35. *If λ is an non-zero fuzzy regular open set in the fuzzy topological space (X, T), then λ is an fuzzy somewhere dense set and fuzzy cs dense set in (X, T).*

Proof. Let λ be an non-zero fuzzy regular open set in (X, T). Then, $\lambda = \operatorname{int} cl(\lambda)$ and $\lambda \neq 0$, implies that $\operatorname{int} cl(\lambda) \neq 0$ and hence λ is an fuzzy somewhere dense set in (X, T). Also, $\operatorname{int}(\lambda) \leq \lambda = \operatorname{int} cl(\lambda) \leq cl(\lambda)$, in (X, T). Let $cl(\lambda) = \mu$. Thus, for the fuzzy set λ, there exists an fuzzy closed set μ in (X, T) such that $\operatorname{int}(\lambda) \leq \mu$ and then, by the theorem 3.1, λ is an fuzzy cs dense set, in (X, T).

4. Somewhere Fuzzy Continuous Functions

Definition 4.1[23]. A function $f : (X, T) \to (Y, S)$ from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) is called an somewhere fuzzy continuous function if whenever $\text{int}(\lambda) \neq 0,$ for an fuzzy set λ defined on Y, then $f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). That is, $f : (X, T) \to (Y, S)$ is an somewhere fuzzy continuous function if $\text{int}\, cl[f^{-1}(\lambda)] \neq 0$ in (X, T) whenever $\text{int}(\lambda) \neq 0,$ for an fuzzy set λ defined on Y in (Y, S).

Proposition 4.1. *If the function* $f : (X, T) \to (Y, S)$ *is the somewhere fuzzy continuous function from the fuzzy hyperconnected space* (X, T) *into the fuzzy topological space* (Y, S), *then* λ *is an fuzzy* β*-continuous function from* (X, T) *into* (Y, S).

Proof. Let λ be an non-zero fuzzy open set in (Y, S). Then, $\text{int}(\lambda) = \lambda \neq 0,$ in (Y, S). Since $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function from (X, T) into (X, Y), $f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy hyperconnected space, by the proposition 3.7, $f^{-1}(\lambda)$ is an fuzzy β-open set in (X, T). Hence, for the fuzzy open set β in (X, T), $f^{-1}(\lambda)$ is an fuzzy β-open set in (X, T) implies that $f : (X, T) \to (Y, S)$ is an fuzzy β-continuous function from (X, T) into (Y, S).

Proposition 4.2. If $f : (X, T) \to (Y, S)$ *is the fuzzy pseudo-continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S), *then* λ *is the somewhere fuzzy continuous function from* (X, T) *into* (Y, S).

Proof. Let λ be an fuzzy set defined on Y such that $\text{int}(\lambda) \neq 0$ in (Y, S). Then, $\text{int}(\lambda)$ is an non-zero fuzzy open set in (Y, S). Since the function $f : (X, T) \to (Y, S)$ is the fuzzy pseudo-continuous function, $f^{-1}(\text{int}[\lambda])$ is an fuzzy pseudo-open in (X, T). Then, by the proposition 3.4, $f^{-1}(\text{int}[\lambda])$ is an fuzzy somewhere dense set in (X, T). Then, $\text{int}\, cl[f^{-1}(\text{int}[\lambda]] \neq 0$ in (X, T). Now, $\text{int}\, cl[f^{-1}(\text{int}[\lambda])] \leq \text{int}\, cl f^{-1}(\lambda)$ implies that $\text{int}\, cl[f^{-1}(\lambda)] \neq 0$ in (X, T) and hence $f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Hence the function $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function from (X, T) into (Y, S).

Theorem 4.1[23]. *If $f : (X, T) \to (Y, S)$ is an somewhat fuzzy continuous function from the fuzzy topological space (X, T) into another fuzzy topological space (Y, S), then f is the somewhere fuzzy continuous function.*

Theorem 4.2[15]. *If the function $f : (X, T) \to (Y, S)$ from the fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is an somewhat fuzzy continuous function, then f is an somewhat fuzzy nearly continuous function.*

Theorem 4.3[20]. *If $f : (X, T) \to (Y, S)$ is an fuzzy pseudo-continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S), n then f is an somewhat fuzzy continuous function.*

Theorem 4.4[20]. *If $f : (X, T) \to (Y, S)$ is a fuzzy pseudo-continuous function from a fuzzy topological space (X, T) into a fuzzy topological space (Y, S), then f is a fuzzy somewhat fuzzy nearly continuous function from (X, T) into (Y, S).*

Remark 4.1[23]. The somewhere fuzzy continuous function from the fuzzy topological space (X, T) into another fuzzy topological space (Y, S) need not be an somewhat fuzzy continuous function.

Remark 4.2[15]. The somewhat fuzzy nearly continuous function from the fuzzy topological space (X, T) into another fuzzy topological space (Y, S), need not be an somewhat fuzzy continuous function.

Remark 4.3[20]. The somewhat fuzzy nearly continuous function from the fuzzy topological space (X, T) into a fuzzy topological space (Y, S) need not be the fuzzy pseudo-continuous function.

Remark 4.4[23]**.** The somewhere fuzzy continuous function from the fuzzy topological space (X, T) into another fuzzy topological space (Y, S), need not be an fuzzy continuous function.

Proposition 4.3. *If* $f:(X, T) \to (Y, S)$ *is the somewhere fuzzy continuous function, then f is the somewhat fuzzy nearly continuous function from* (X, T) *into* (Y, S).

Proof. Let λ be an non-zero fuzzy open set in (Y, S). Then, $\operatorname{int}(\lambda) = \lambda \neq 0,$ in (Y, S). Since $f:(X, T) \to (Y, S)$ is the somewhere fuzzy continuous function, $f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T) and hence $\operatorname{int} cl[f^{-1}(\lambda)] \neq 0,$ in (X, T). Hence, for the fuzzy open set λ in (Y, S), $\operatorname{int} cl[f^{-1}(\lambda)] \neq 0,$ in (X, T) implies that $f:(X, T) \to (Y, S)$ is the somewhat fuzzy nearly continuous function from (X, T) into (Y, S).

Proposition 4.4. *If* $f:(X, T) \to (Y, S)$ *is the somewhere fuzzy continuous function from the fuzzy hyperconnected space* (X, T) *into the fuzzy topological space* (Y, S) *and* $\operatorname{int}(\lambda) \neq 0,$ *for an fuzzy set* λ *defined on* Y, *then*

(i) $cl[f^{-1}(\lambda)] = 1$ *in* (X, T).

(ii) $\operatorname{int}[f^{-1}(1 - \lambda)] = 0$ *in* (X, T).

Proof (i) Let λ be an fuzzy set defined on Y such that $\operatorname{int}(\lambda) \neq 0,$ in (Y, S). Since the function $f:(X, T) \to (Y, S)$ is the somewhere fuzzy continuous function from (X, T) into $(Y, S) f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy hyperconnected space, by the proposition 3.5, $f^{-1}(\lambda)$ is an fuzzy dense set in (X, T) and hence $cl[f^{-1}(\lambda)] = 1,$ in (X, T).

(ii) From (i), $cl[f^{-1}(\lambda)] = 1,$ in (X, T) and this implies that $1 - cl[f^{-1}(\lambda)] = 0.$ Then, $\operatorname{int}[1 - [f^{-1}(\lambda)] = 0$ and this implies that $\operatorname{int}[f^{-1}(1 - \lambda)] = 0,$ in (X, T).

Proposition 4.5. *If* $f:(X, T)\to(Y, S)$ *is the somewhere fuzzy continuous function from the fuzzy hyperconnected and fuzzy submaximal space* (X, T) *into the fuzzy topological space* (Y, S) *and* $\operatorname{int}(\lambda)\neq 0$ *for an fuzzy set* λ *defined on* Y, *then*

(i) $f^{-1}(\lambda)$ *is an fuzzy open set in* (X, T).

(ii) $f^{-1}(1-\lambda)$ *is an fuzzy nowhere dense set in* (X, T).

Proof (i) Let λ be an fuzzy set defined on Y such that $\operatorname{int}(\lambda)\neq 0$, in (Y, S). Since $f:(X, T)\to(Y, S)$ is the somewhere fuzzy continuous function from the fuzzy hyperconnected space (X, T) into (Y, S), by the proposition 4.4, $cl[f^{-1}(\lambda)]=1, \ldots$ (1), in (X, T). Since (X, T) is the fuzzy submaximal space, for the fuzzy dense set $f^{-1}(\lambda)$, $f^{-1}(\lambda)$ is an fuzzy open set in (X, T).

(ii) From (i), $f^{-1}(\lambda)$ is an fuzzy open set in (X, T) and hence $\operatorname{int}[f^{-1}(\lambda)]=f^{-1}(\lambda)$ and then from the equation (1), $cl\operatorname{int}[f^{-1}(\lambda)]=1$, in (X, T). Now $1-cl\operatorname{int}[f^{-1}(\lambda)]=0$ and then $\operatorname{int}cl[1-[f^{-1}(\lambda)]]=0$, in (X, T). This implies that $\operatorname{int}cl[f^{-1}(1-\lambda)]=0$ and hence $f^{-1}(1-\lambda)$ is an fuzzy nowhere dense set in (X, T).

Example 4.1. Consider the set $X=\{a, b, c\}$. Let $I=[0, 1]$. The fuzzy sets α, β, λ, μ and η are defined on X as follows:

α, $X\to[0, 1]$ is defined as

β, $X\to[0, 1]$ is defined as $\beta(a)=0.4$; $\beta(b)=0.9$; $\beta(c)=0.7$,

λ, $X\to[0, 1]$ is defined as $\lambda(a)=0.4$; $\lambda(b)=0.6$; $\lambda(c)=0.4$,

μ, $X\to[0, 1]$ is defined as $\mu(a)=0.6$; $\mu(b)=0.5$; $\mu(c)=0.4$,

δ, $X\to[0, 1]$ is defined as $\delta(a)=0.6$; $\delta(b)=0.9$; $\delta(c)=0.7$,

η, $X\to[0, 1]$ is defined as $\eta(a)=0.6$; $\eta(b)=0.5$; $\eta(c)=0.7$.

Then, $T = \{0, \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1\}$ and $S = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ are fuzzy topologies on X. On computation one can see that $\operatorname{int}(\alpha) = \mu \neq 0$, $\operatorname{int}(\beta) = \lambda \neq 0$, $\operatorname{int}(\alpha \wedge \beta) = \mu \neq 0$, $\operatorname{int}(\alpha \wedge \beta) = \lambda \wedge \mu \neq 0$, $\operatorname{int}(1 - [\lambda]) = \mu \neq 0$ in (X, S). Now define a function $f : (X, T) \to (Y, S)$ by $f(a) = b; f(b) = c;$ $f(c) = a$. By computation, one can see that $\operatorname{int} cl[f^{-1}(\theta)] \neq 0$, where $\theta = \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1 - \mu, 1 - [\lambda \wedge \mu], \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu,$ in (X, T). This implies that f is an somewhere fuzzy continuous function from (X, T) into (X, S).

Now $1 - \alpha, 1 - \beta, 1 - [\alpha \vee \beta]$ and $1 - [\alpha \wedge \beta]$ are fuzzy nowhere dense sets in (X, T) and $1 - [\alpha \wedge \beta] = (1 - \alpha) \vee (1 - \beta) \vee (1 - [\alpha \vee \beta])$, implies that $1 - (\alpha \wedge \beta)$ is an fuzzy first category set in (X, T). On computation $\alpha \vee (1 - [\alpha \wedge \beta]) = \alpha$, $\beta \vee (1 - [\alpha \wedge \beta]) = \eta$, $(\alpha \vee \beta) \vee (1 - [\alpha \wedge \beta]) = \alpha \vee \beta$, $(\alpha \wedge \beta) \vee (1 - [\alpha \wedge \beta]) = \theta$ and then $\alpha, \eta, \alpha \vee \beta, \theta$ are fuzzy pseudo-open sets in (X, T). For the fuzzy open set $\lambda \vee \mu$ in (X, S), $f^{-1}([\lambda \vee \mu]) = 1 - \lambda$, which is not an fuzzy pseudo-open set in (X, T). implies that f is not an fuzzy pseudo-continuous function from (X, T) into (X, S).

Example 4.2. Let $X = \{a, b, c\}$. Consider the fuzzy sets $\lambda, \mu, \gamma, \alpha$ and β defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 1; \lambda(b) = 0.2; \lambda(c) = 0.7,$

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.3; \mu(b) = 1; \mu(c) = 0.2,$

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.7; \gamma(b) = 0.4; \gamma(c) = 1,$

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.6; \alpha(b) = 0.5; \alpha(c) = 0.7,$

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.7; \beta(b) = 0.3; \beta(c) = 0.5.$

Then, $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \wedge \gamma), \gamma \wedge (\lambda \vee \mu), 1\}$

$S = \{0, \alpha, \beta, \alpha \vee \beta, \alpha \wedge \beta, 1\}$ are fuzzy topologies on X. Define a function $f : (X, T) \to (X, S)$ by $f(a) = a; f(b) = b; f(c) = c$. Then, on computation,

$f^{-1}(\alpha) = \alpha; f^{-1}(\beta) = \beta; f^{-1}(\alpha \vee \beta) = \alpha \vee \beta; f^{-1}(\alpha \wedge \beta) = \alpha \wedge \beta$ in (X, T) and $\operatorname{int}[f^{-1}(\alpha)] = \mu \wedge \gamma \neq 0; \operatorname{int}[f^{-1}(\beta)] = \mu \wedge \gamma \neq 0; \operatorname{int}[f^{-1}(\alpha \vee \beta)] = \gamma \wedge (\lambda \vee \mu) \neq 0.$ Then f is an somewhat fuzzy continuous function from (X, T) into (X, S).

On computation the fuzzy nowhere dense sets in (X, T) are $1 - \lambda, 1 - \mu, 1 - \gamma, (1 - [\lambda \vee \mu]), (1 - [\lambda \vee \gamma]), (1 - [\mu \vee \gamma]), 1 - (\lambda \vee [\mu \wedge \gamma])$ $1 - (\mu \vee [\lambda \wedge \gamma]), 1 - (\gamma \wedge [\lambda \vee \mu])$. Now $1 - (\lambda \wedge \mu) = (1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma)$ $\vee(1 - [\lambda \vee \mu]) \vee (1 - [\lambda \vee \gamma]) \vee (1 - [\mu \vee \gamma]) \vee (1 - (\lambda \vee [\mu \wedge \gamma])), \vee (1 - (\mu \vee [\lambda \wedge \gamma]))$ $\vee(1 - (\gamma \wedge [\lambda \vee \mu]))$ and then $1 - (\lambda \wedge \mu)$ is an fuzzy first category set in (X, T). On computation, the fuzzy pseudo-open sets in (X, T) are

$\eta_1 : X \to [0, 1]$ is defined as $\eta_1(a) = 1; \eta_1(b) = 0.8; \eta_1(c) = 0.8,$

$\eta_2 : X \to [0, 1]$ is defined as $\eta_2(a) = 0.7; \eta_2(b) = 1; \eta_2(c) = 0.8,$

$\eta_3 : X \to [0, 1]$ is defined as $\eta_3(a) = 0.7; \eta_3(b) = 0.8; \eta_3(c) = 1,$

$\eta_4 : X \to [0, 1]$ is defined as $\eta_4(a) = 1; \eta_4(b) = 1; \eta_4(c) = 0.8,$

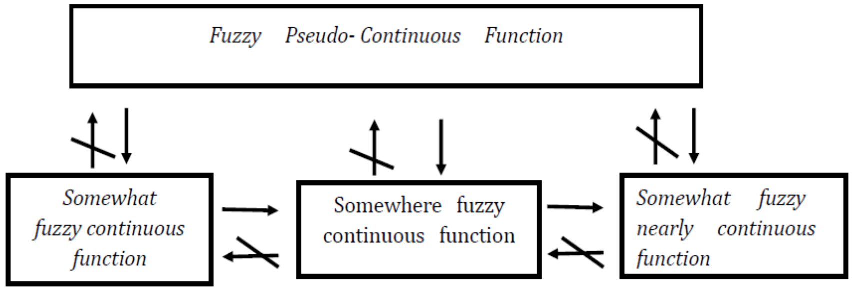
$\eta_5 : X \to [0, 1]$ is defined as $\eta_5(a) = 1; \eta_5(b) = 0.8; \eta_5(c) = 1,$

$\eta_6 : X \to [0, 1]$ is defined as $\eta_6(a) = 0.7; \eta_6(b) = 1; \eta_6(c) = 1,$

$\eta_7 : X \to [0, 1]$ is defined as $\eta_7(a) = 0.7; \eta_7(b) = 0.8; \eta_7(c) = 0.8,$

For the fuzzy open set α in (X, S), $f^{-1}(\alpha) = \alpha$ and α is not an fuzzy pseudo open set in (X, T). Thus the function f is not an fuzzy pseudo-continuous function from (X, T) into (Y, S).

Remarks. The inter-relations between the classes of fuzzy pseudo continuous functions, somewhat fuzzy continuous functions, somewhere fuzzy continuous functions and somewhat fuzzy nearly continuous functions can be diagrammatically represented as follows:



The following proposition gives the condition for the somewhere fuzzy continuous function to become an fuzzy continuous function.

Proposition 4.6. *If $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function from the fuzzy hyperconnected and fuzzy submaximal space (X, T) into the fuzzy topological space (Y, S), then f is the fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let λ be an non-zero fuzzy open set in (Y, S), then $\text{int}(\lambda) = \lambda \neq 0$, in (Y, S). Since $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function from the fuzzy hyperconnected and fuzzy submaximal space (X, T), by the proposition 4.5, $f^{-1}(\lambda)$ is an fuzzy open set in (X, T). Hence, for the fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is an fuzzy open set in (X, T) implies that $f : (X, T) \to (Y, S)$ is the fuzzy continuous function from (X, T) into (Y, S).

Proposition 4.7. *If $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function from the fuzzy hyper connected and fuzzy open hereditarily irresolvable space (X, T) into the fuzzy topological space (Y, S), then $f^{-1}(1 - \lambda)$ is an fuzzy nowhere dense set in (X, T) for the fuzzy set λ defined on Y with $\text{int}(\lambda) \neq 0$, in (Y, S).*

Proof. Let λ be an fuzzy set defined on Y such that $\text{int}(\lambda) \neq 0$, in (Y, S). Since $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function, $f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Then $1 - f^{-1}(\lambda)$ is the fuzzy cs dense set in (X, T). Now $1 - f^{-1}(\lambda) = f^{-1}(1 - \lambda)$ implies that

$f^{-1}(1-\lambda)$ is an fuzzy cs dense set in (X, T). Since (X, T) is the fuzzy hyperconnected and fuzzy hereditarily irresolvable space, by the proposition 3.11, $f^{-1}(1-\lambda)$ is an fuzzy nowhere dense set in (X, T).

Proposition 4.8. *If* $f:(X, T) \to (Y, S)$ *is the somewhere fuzzy continuous function from the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space* (X, T) *into the fuzzy open hereditarily irresolvable space* (Y, S) *and if* $cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 1$, *where* (λ_i)*'s are the fuzzy somewhere dense sets in* (X, T), *then* (X, T) *is the fuzzy Baire space.*

Proof. Let (λ_i)'s $(i = 1 \text{ to } \infty)$ be the fuzzy somewhere dense sets in (Y, S). Then, $\operatorname{int} cl(\lambda_i) \neq 0$, in (Y, S). Since (Y, S) is the fuzzy open hereditarily irresolvable space, $\operatorname{int}(\lambda_i) \neq 0$ in (X, T). Also since $f:(X, T) \to (Y, S)$ is the somewhere fuzzy continuous function, $[f^{-1}(\lambda_i)]$'s are fuzzy somewhere dense sets in (X, T). Then, $\{1-[f^{-1}(\lambda_i)\}$'s are cs dense sets in (X, T). The fuzzy topological space (X, T) being an fuzzy hyperconnected and fuzzy open hereditarily irresolvable, the fuzzy cs dense sets $[1-f^{-1}(\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T). Now the hypothesis, $1-cl\{\wedge_{i=1}^{\infty}[f^{-1}(\lambda_i)]\} = 1$, implies that, $1-cl[\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 0$, and then $\operatorname{int}[1-\wedge_{i=1}^{\infty} f^{-1}(\lambda_i)] = 0$. This implies that $\operatorname{int}[1-\wedge_{i=1}^{\infty}(1-f^{-1}(\lambda_i))] = 0$, in (X, T) and thus $\operatorname{int}[\vee_{i=1}^{\infty}(1-f^{-1}(\lambda_i)] = 0$, where $(1-f^{-1}(\lambda_i))$'s are fuzzy nowhere dense sets in (X, T). Hence (X, T) is an fuzzy Baire space.

The following proposition gives the condition for the somewhere fuzzy continuous function to become an somewhat fuzzy continuous function.

Proposition 4.9. *If* $f:(X, T) \to (Y, S)$ *is the somewhere fuzzy continuous function from the fuzzy strongly irresolvable space* (X, T) *into the fuzzy topological space* (Y, S), *then f is an somewhat fuzzy continuous function from* (X, T) *into* (Y, S).

Proof. Let λ be an non-zero fuzzy open set in (Y, S), then $\text{int}(\lambda) \neq 0$ in (Y, S). Since $f : (X, T) \to (Y, S)$ is the somewhere fuzzy continuous function $f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy strongly irresolvable space, by the proposition 3.18, $\text{int}[f^{-1}(\lambda)] \neq 0$, in (X, T). Hence ,for the fuzzy open set (X, T), $\text{int}[f^{-1}(\lambda)] \neq 0$, implies that f is an somewhat fuzzy continuous function from (X, T) into (Y, S).

The following proposition gives the condition for the somewhat fuzzy nearly continuous function to become an somewhere fuzzy continuous function.

Proposition 4.10. *If* $f : (X, T) \to (Y, S)$ *is the somewhat fuzzy nearly continuous function from the fuzzy strongly irresolvable space* (X, T) *into the fuzzy topological space* (Y, S), *then* f *is an somewhere fuzzy continuous function from* (X, T) *into* (Y, S).

Proof. Let λ be an fuzzy set defined on Y such that $\text{int}(\lambda) \neq 0$ in (Y, S). Then, $\text{int}(\lambda)$ is an non-zero fuzzy open set in (Y, S). Since $f : (X, T) \to (Y, S)$ is the somewhat fuzzy nearly continuous function, $\text{int}\, cl[f^{-1}(\text{int}(\lambda))] \neq 0$. That is, $f^{-1}(\text{int}(\lambda))$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is the fuzzy strongly irresolvable space, by the proposition 3.18, $\text{int}[f^{-1}(\text{int}(\lambda))] \neq 0$. in (X, T). Now $\text{int}[f^{-1}(\text{int}(\lambda))] \leq \text{int}[f^{-1}(\lambda)]$, implies that $[f^{-1}(\lambda_i)] \neq 0$, in (X, T). Hence, for the fuzzy set λ defined on Y such that $\text{int}(\lambda) \neq 0$ in (Y, S), $\text{int}[f^{-1}(\lambda)] \neq 0$, implies that f is an somewhere fuzzy continuous function from (X, T) into (Y, S).

6. Conclusions

In this paper, several characterizations of fuzzy somewhere dense sets and fuzzy cs dense sets in fuzzy topological spaces are established. It is established that non-zero fuzzy open sets are fuzzy somewhere dense sets and fuzzy pseudo-open sets are fuzzy somewhere dense sets in fuzzy topological spaces. It is proved that in fuzzy hyperconnected spaces, fuzzy β-open sets coincide with fuzzy somewhere dense sets and fuzzy F_σ-sets in fuzzy almost P-spaces are fuzzy cs dense sets and fuzzy G_δ-sets in fuzzy almost P-spaces are fuzzy somewhere dense sets. A condition for the fuzzy hyperconnected and fuzzy almost P-space to become an fuzzy σ-Baire space is established by means of fuzzy cs dense sets. The conditions for fuzzy topological spaces to become fuzzy Baire spaces are obtained. It is established that the fuzzy Baire sets in the fuzzy globally disconnected and fuzzy P-spaces, are fuzzy somewhere dense sets. The inter-relations between the classes of fuzzy pseudo-continuous functions, somewhat fuzzy continuous functions, somewhere fuzzy continuous functions and somewhat fuzzy nearly continuous functions are established in this paper. The condition for somewhere fuzzy continuous functions to become somewhat fuzzy continuous functions and the condition for somewhat fuzzy nearly continuous functions to become somewhere fuzzy continuous functions, are also obtained.

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