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NEUTROSOPHIC ALMOST RESOLVABLE AND IRRESOLVABLE SPACES

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Abstract

The aim of this paper is to develop many characterizations of Neutrosophic (N) almost resolvable and irresolvable spaces and also the condition under that a Neutrosophic almost resolvable space becomes a Neutrosophic baire space. The interrelations between Neutrosophic almost resolvable spaces and other spaces also are mentioned.

1. Introduction

In order to cope with uncertainties, the thought of fuzzy sets and fuzzy set operations was introduced by Zadeh [7]. The speculation of fuzzy topological space was studied and developed by C. L. Chang [2]. The paper of Chang sealed the approach for the following tremendous growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed.

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Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentine Smarandache [4] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. Thus neutrosophic set was framed and it includes the parts of truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) severally. Neutrosophic sets deals with non normal interval of]–0 1 +[. The concept of intuitionistic fuzzy almost resolvable spaces and irresolvable spaces was introduced by S. Sharmila [5].

2. Preliminaries

Definition 2.1 [4]**.** Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$A = \{\langle\, x, T_A(x), I_A(x), F_A(x)\,\rangle : x \in X\}$, where $T_A, I_A, F_A, : U \to [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership.

2.4. Definition 2.2 [4]**.** The complement of a Neutrosophic set A on R Denoted by A^C or A^* and is defined as $A^C = \{\langle\, x, F_A(x), 1 - I_A(x), T_A(x)\,\rangle : x \in X\}$.

2.5 Definition 2.3 [4]**.** A Neutrosophic topology on a nonempty set R is a family of N sets in R satisfying the following axioms

(1) $0, 1 \in \tau$

(2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$

(3) $\cup R_i \in \tau$ for any $R_i : i \in I$

Hence (R, τ) is a Neutrosophic Topological Space (NTS).

3. Neutrosophic Almost Resolvable and Irresolvable Spaces

Definition 3.1. A NTS is called a Neutrosophic almost resolvable space if

$\bigcup_{i=1}^{\infty} K_i = 1$, where K_i's are NS s in (R, τ) are such that $\text{NInt}(K_i) = 0$. Otherwise (R, τ) is called Neutrosophic almost irresolvable space.

Example 3.2. Let $R = \{p, q\}$. Let $A3, A4, A5$ and $A6$ be the Neutrosophic sets defined on R as follows. $A1 = \{[p, 0.2, 0.5, 0.6], [q, 0.4, 0.7, 0.5]\}$, $A2 = \{[p, 0.5, 0.1, 0.4], [q, 0.5, 0.5, 0.1]\}$. Then, clearly $\tau = \{0, A1, A2, 1\}$ is a Neutrosophic topology on R. Now consider the Neutrosophic sets defined on R as follows $A3 = \{[p, 0.3, 0.6, 0.5], [q, 1, 0, 0.5]\}$, $A4 = \{[p, 1, 0, 7, 0], [q, 0.2, 0.5, 0]\}$, $A5 = \{[p, 0.4, 0.4, 0.2], [q, 0.5, 0.2, 0.3]\}$, $A6 = \{[p, 0.2, 0, 0.3], [q, 0.2, 0.7, 0.5]\}$. Then, $\text{NInt}(A3) = 0$, NInt(A4), NInt(A5) = 0 and $\text{NInt}(A6) = 0$ and $\{(A3) \cup (A4) \cup (A5) \cup (A6)\} = 1$. Hence (R, τ) is a Neutrosophic almost resolvable space.

Example 3.3. Let $R = \{p, q\}$. Let $A3, A4, A5$ and $A6$ be the Neutrosophic sets defined on R as follows. $A1 = \{[p, 0.2, 0.5, 0.6], [q, 0.4, 0.7, 0.5]\}$, $A2 = \{[p, 0.5, 0.1, 0.4], [q, 0.5, 0.5, 0.1]\}$. Then, clearly $\tau = \{0, A1, A2, 1\}$ is a Neutrosophic topology on R. Now consider the Neutrosophic sets defined on R as follows $A3 = \{[p, 0.2, 0.5, 0.4], [q, 0.4, 0.5, 0.5]\}$, $A4 = \{[p, 0.4, 0.6, 0], [q, 0.2, 0.5, 0.2]\}$, $A5 = \{[p, 0.2, 0.5, 0.1], [q, 0.1, 0.2, 0.3]\}$, $A6 = \{[p, 0.5, 0.2, 0.5], [q, 0.2, 0.7, 0.5]\}$. Then, $\text{NInt}(A3) = 0$, NInt(A4), = 0, NInt(A5) = 0 and $\text{NInt}(A6) = 0$ and $\{(A3) \cup (A4) \cup (A5) \cup (A6)\} \neq 1$. Hence (R, τ) is a Neutrosophic almost irresolvable space.

Theorem 3.4. *If* $\bigcap_{i=1}^{\infty} K_i = 0$, *where* K_i*'s are dense sets in* (R, τ), *then* (R, τ) *is a Neutrosophic almost resolvable space.*

Proof. Suppose that $\bigcap_{i=1}^{\infty} K_i = 0$, where $\text{NCl}(K_i) = 1$ in (R, τ). Then we have $1 - \bigcap_{i=1}^{\infty} K_i = 1 - 0 = 1$, where $1 - \text{NCl}(K_i) = 0$. This implies that $\bigcup_{i=1}^{\infty} (1 - K_i) = 1$, where $\text{NInt}(1 - K_i) = 0$. Let $1 - K_i = L_i$, then we have $\bigcup_{i=1}^{\infty} L_i = 1$, where $\text{NInt}(L_i) = 0$ in (R, τ). Hence (R, τ) is Neutrosophic almost resolvable space.

Definition 3.5. A NTS (R, τ) is called a Neutrosophic hyper-connected space if every Neutrosophic open set is Neutrosophic dense in (R, τ). That is $\text{NCI}(K_i) = 1$ for all $K_i \in \tau$.

Theorem 3.6. *If* $\bigcap_{i=1}^{\infty} K_i = 0$, *where* K_i*'s are Neutrosophic open set in a Neutrosophic hyper-connected space* (R, τ), *then* (R, τ) *is a Neutrosophic almost resolvable space.*

Proof. Suppose that $\bigcap_{i=1}^{\infty} K_i = 0$, where $K_i \in \tau$. Since (R, τ) is a Neutrosophic hyper-connected space, the Neutrosophic open set is a Neutrosophic dense set in (R, τ) for each i. Hence we have $\bigcap_{i=1}^{\infty} K_i = 0$, where $\text{NCl}(K_i) = 1$ in (R, τ). Then by theorem 3.2, (R, τ) is a Neutrosophic almost resolvable space.

Definition 3.7. A Neutrosophic set K in a NTS (R, τ) is called NR_2 space if $K = \bigcap_{i=1}^{\infty} K_i$ where each $K_i \in \tau$.

Definition 3.8. A Neutrosophic set K in a NTS (R, τ) is called NR_2 space if $K = \bigcup_{i=1}^{\infty} K_i$ where each $K_i \in \tau$.

Definition 3.9. A NTS (R, τ) is called Neutrosophic R-space, if countable intersection of NOS s in (R, τ) is Neutrosophic open. That is, every nonzero NR_1 -set in (R, τ) N open in (R, τ).

Theorem 3.10. *If are* $\bigcap_{i=1}^{\infty} K_i = 0$, K_i's NR_1 *-sets in an Neutrosophic hyper-connected space* and *NR-space* (R, τ), *then* (R, τ) *is an Neutrosophic almost resolvable space.*

Proof. Let K_i's be NR_1 -sets in a NR-space (R, τ). Then are Neutrosophic open sets in (R, τ). Hence, we have $\bigcap_{i=1}^{\infty} K_i = 0$, where K_i's are Neutrosophic open sets in an Neutrosophic hyper-connected space (R, τ).

Therefore, by theorem 3.4, (R, τ) is an Neutrosophic almost resolvable space.

Theorem 3.11. If each NS is an NR_2-set in a Neutrosophic almost resolvable space (R, τ), then $\bigcap_{i=1}^{\infty}(1 - K_i) = 0$, where K_i's are Neutrosophic dense sets in (R, τ).

Proof. Let (R, τ) be a Neutrosophic almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i = 1$, where K_i's are such that $\text{NInt}(K_i) = 0$. This implies that $1 - \bigcup_{i=1}^{\infty} K_i = 0$ and $1 - \text{NInt}(K_i) = 1$. Then $\bigcap_{i=1}^{\infty}(1 - K_i) = 0$ and $\text{NCl}(1 - K_i) = 1$. Since K_i's are NR_2-sets, $(1 - K_i)$'s are NR_1-sets in (R, τ). Hence we have $\bigcap_{i=1}^{\infty}(1 - K_i) = 0$, where $(1 - K_i)$'s are Neutrosophic dense and NR_1-sets in (R, τ).

Definition 3.12. A NTS (R, τ) is called Neutrosophic nodec space, if every non-zero Neutrosophic nowhere dense set in (R, τ) is Neutrosophic closed.

Theorem 3.13. *IF the NTS* (R, τ) *is a Neutrosophic first category*, *then* (R, τ) *is a Neutrosophic almost resolvable space*.

Theorem 3.14. *If* (R, τ) *is a Neutrosophic first category space and Neutrosophic nodec space*, *then* (R, τ) *is a Neutrosophic almost resolvable space*.

Theorem 3.15. *If* $\text{NCl}(\text{NInt}(K_i)) = 0.$ *for ach Neutrosophic dense set* K_i *in a Neutrosophic almost resolvable space* (R, τ), *then* (R, τ) is *a Neutrosophic first category space*.

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