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MEAN LABELING PATTERN OF SOME GRAPHS

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Abstract

A graph consists of a pair of $V(G)$, $E(G)$ where $V(G)$ is the vertex set and $E(G)$ is the edge set of G. Graph labeling applied in science and few of them are communication. A survey on recent conjecture and open problem in labeling graph is presented in the paper by J. A. Gallian. The Concept of mean labeling was introduced by Somasundaram and Ponraj in 2003. Many research papers had published in this topic. In this paper we have established a general format of mean labeling pattern of some Graphs. The graphs are $J(m, n)$, $J(m, n) - e$, $Gl(n)$, $F(P_n)$, Y_n.

1. Introduction

Throughout this paper graph G refers to simple graph. For basic definitions and notations in graph theory follow Bondy Murthy [1]. The concept of mean labeling was introduced by S. Somasundaram and Ponraj [7]. In this paper we investigate the existence of mean labeling of some graph.

Definition 1.1. The Jelly fish graph $J(m, n)$ is obtained from a 4 cycle u, v'_0, v, v_0 by joining uv with an edge and appending m pendent edges $\{v_1, v_2, v_3, \ldots, v_m\}$ to v_0 an n pendent edges $\{v'_1, v'_2, \ldots, v'_n\}$ to v'_0. If

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$G(V, E) = J(m, n)$. Then G has $p = m + n + 4$ vertices and $q = m + n + 5$ edges.

Definition 1.2. A walk is an alternating sequence of vertices and connecting edges. A walk can end on the same vertex on which it began or on a vertex.

A walk is called a path if all its vertices are distinct.

Definition 1.3. A closed walk in which no vertices, except the end vertices, are repeated is called the cycle and the number of edges in a cycle is called its length.

Definition1.4. A F-tree $F(P_n)$ is a graph obtained from a path on $n \geq 3$ vertices by appending two pendent edges one to an end vertex and the other vertex adjacent to an end vertex.

Definition 1.5. A Y-tree is a graph obtained from a path by appending an edge to a vertex of a path adjacent to an end point and it is denoted by Y_n where n is the number of vertices in the tree.

2. Main Results

Theorem 2.1. *For $m, n \geq 1$ Jelly fish $J(m, n)$ is a mean graph.*

Proof. Let $G = J(m, n)$ be a graph with $|V(G)| = m + n + 4$, and $|E(G)| = m + n + 5$, $N(u) = \{v'_0, u', v_0\}$

$$N(v_0) = \{u, u', v_i / j = 1, 2, 3, \ldots, m\}; N(v'_0) = \{u, u', v'_j / j = 1, 2, 3, \ldots, n\}$$

The vertex set of the graph is given below,

$$V(G) = \{u, u', v_i, v'_j / j = 1, 2, 3, \ldots, n\}$$

Vertex set labeling of graph G is given below,

$$ML(v_0) = 0; ML(v'_0) = m + n + 3; ML(v_i) = 2i - 1; ML(v'_1) = ML(v'_0) + 1;$$

$$ML(v'_j) = \{2i / i = 2, 3, \ldots, n\}$$

$$ML(u) = ML(v_m) + 1; ML(u') = ML(u) + 2$$

Above graph satisfies the mean labeling condition.

Therefore, the graph G is mean graph.

Theorem 2.2. *For* $m, n \geq 1$ $J(m, n) - e(e = uv)$ *admits mean graph where* $m = n$.

Proof. Let $G = J(m, n) - e$ be a graph with $|V(G)| = m + n + 4$ and $|E(G)| = m + n + 4$

$$N(u) = \{v_0, v_0'\}; N(v_0) = \{v_i, u, u'; 1 \leq i \leq m\}$$

The vertex set of the graph G is defined by,

$$V(G) = \{u, u', v_i, v_j'; 0 \leq i \leq m, 0 \leq j \leq n\}$$

The edge set of the graph G is defined by,

$$E(G) = \{e_i; 1 \leq i \leq m + n + 4\}$$

$$f(e_i) = \{v_0 v_i \cup v_0 u \cup v_0 u' \cup v_0' v_j' \cup v_0' u \cup v_0' u'\}$$

The vertex labeling pattern of the graph G is given below,

$$ML(v_i) = \{2i - 1, 1 \leq i \leq m\}; ML(v_0) = 0$$

$$ML(v_0') = m + n + 4; ML(v_j') = \{2j; 1 \leq j \leq n\}$$

$$ML(e_i) = \{i; 1 \leq i \leq m + n + 4\}.$$

This concludes that G is satisfying a mean labeling condition.

Therefore $G = J(m, n) - e$ is a mean graph.

Theorem 2.3. *The Globe graph* $Gl(n)$ *is a mean graph.*

Proof. Let $G = Gl(n)$ be a graph with $|V(G)| = n + 2; |E(G)| = 2n$.

The vertex set of the graph is given below, $V(G) = \{v_i; 0 \leq i \leq 2n\}$.

The edge set of the graph is given below, $E(G) = \{e_{i1}; 1 \leq i \leq 2n\}$

Labeling pattern of the graph is given below,

$$ML(v_0) = 0; ML(v_i) = \{2i - 1; 1 \leq i \leq n\}; ML(v_n) = 2n$$

Edge labeling is given below, $ML(e_i) = \{i; 1 \le i \le n\}$

Graph G satisfies the condition of mean labeling,

Therefore, graph G is mean graph.

Theorem 2.4. *Let G be a graph obtained by joining a pendent vertex with a vertex of degree two of a comb graph. Then G is a mean graph.*

Proof. Let G be a graph with $|V(G)| = 2n + 2; |E(G)| = 2n + 1$.

The vertex set of the graph G is given below, $V(G) = \{v_i, u_j; 0 \le i \le n+1, 0 < j \le m\}$.

The graph G obtained from joining the vertex v_0 to v_1 and v_n to v_{n+1}.

The edge set of the graph G is given below,

$$E(G) = \{e_i, e_1'; 1 \le i \le i \le n\}$$

$$f(e_1) = \{v_i, v_{i+1}; 0 \le i \le n+1\}; f(e_i') = \{v_i u_j; 1 \le i, j \le n\}$$

$$N(v_i) = \{v_{i-1} v_{i+1}, u_j; 0 \le i \le n+1 \text{ and } 1 \le j \le n\}$$

$$N(v_0) = \{v_1\}; N(v_{2+1}) = \{v_n\}$$

The mean labeling pattern of graph G is given below,

$$ML(v_i) = \{2i; 0 \le i \le n\}; ML(v_{n+1}) = \{v_{n+1}\}$$

$$ML(u_j) = \{2j - 1; 1 \le j \le n\}$$

$$ML(e_i) = \{2i - 1; 1 \le i \le n+1\}; ML(e_i') = \{2i; 1 \le i \le n\}$$

The vertex and edge labeling are distinct.

Hence the graph G is mean graph.

Theorem 2.5. *Let G be a graph obtained by attaching pendent edges to both sides of each vertex of a path P_n then G is a mean graph.*

Proof. Consider a graph G which is obtained by attaching pendent edges to both sides of each vertex of a path P_n.

The graph G with $|V(G)| = 3n$ and $|E(G)| = 3n - 1$.

The vertex set of the graph G is $V(G) = \{v_i, v'_i, v''_i; 1 \le i \le n\}$.

Let v_i and v''_i be the pendent vertices adjacent to $v_i; 1 \le i \le n$.

$$N(v_i) = \{v'_i, v''_i, v_{i+1}, v_{i-1}; 1 \le i \le n\}$$

$$N(v_1) = \{v_2, v'_1, v''_1\}; N(v_n) = \{v_{n-1}, v'_n, v''_n\}$$

The labeling pattern of the graph G is given below,

$$ML(v_i) = \{3i - 2; 1 \le i \le n\}; ML(v'_i) = \{3i - 1; 1 \le i \le n\}$$

$$ML(v''_i) = \{3i - 3; 1 \le i \le n\}; ML(e_i) = \{3i - 1; 1 \le i \le n\}$$

$$ML(e'_i) = \{3i; 1 \le i \le n\}; ML(e''_i) = \{3i - 2; 1 \le i \le n\}$$

The graph G is mean graph.

Theorem 2.6. *The graph F-Tree admits mean graph.*

Proof. Let $G = F(P_n)$ be a graph with $|V(G)| = n + 2$ and $|E(G)| = n + 1$.

The vertex set of the graph is $|V(G)| = \{v_i, v'_n, v'_{n-1}; 1 \le i \le n\}$

The pendent vertex are v'_n, v'_{n-1} is join to v_n and v_{n-1}.

The edge set of the graph is

$$E(G) = \{e_i, e'_1, e'_2; 1 \le i \le i \le n - 1\}$$

$$N(v_1) = \{v_2\}; N(v_i) = \{v_{i-1}, v_{i+1}; 1 < i < n\}; N(v_n) = \{v'_n, v_{n-1}\}$$

$$f(e_i) = \{v_i v_{i+1}; 1 < i < n - 2\}; f(e_{n-1}) = \{v_n v_{n-1}\};$$

$$f(e'_1) = \{v_n v'_n\}; f(e'_2) = \{v_{n-1} v'_{n-1}\}$$

The labeling pattern of the graph G is given below,

$$ML(v_i) = \{i - 1; 1 \le i \le n - 1\}$$

$$ML(v_i) = \{i + 1; i = n\}$$

$$ML(v'_{n-1}) = \{v_{n-1} - 1\}; ML(v'_n) = v_n - 1.$$

$$ML(e_i) = \{i; 1 \le i \le n-2\}; ML(e_1') = n; ML(e_2') = n-2.$$

Clearly all the vertex and edges are using to labeled the above labeling pattern.

Thus $F(P_n)$ is mean graph.

Theorem 2.7. *The graph Y-tree is a mean graph.*

Proof. Let $G = Y_n$ be a graph with $|V(G)| = n$ and $|E(G)| = n-1$.

The vertex v_1 is adjacent to v_3

The vertex set of the graph G is $V(G) = \{v_i; 1 \le i \le n\}$.

The edge set of the graph G is

$$E(G) = \{e_i; 1 \le i \le n-1\}$$

$$N(v_1) = \{v_3\}; N(v_i) = \{v_i v_{i+1}; 1 \le i \le n\}$$

$$F(e_1) = \{v_1 v_3\}; F(e_i) = \{v_i v_{i+1}; 1 \le i \le n\}$$

The labeling pattern of the graph G is given below,

$$ML(v_i) = \{i-1; 1 \le i \le n\}$$

$$ML(e_i) = \{i; 1 \le i \le n\}$$

Thus the graph G is Mean graph.

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