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A NOTE ON HESITANT TRIANGULAR FUZZY NUMBER IN ASSIGNMENT PROBLEM USING SCORE FUNCTION

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Abstract

Hesitant Triangular Fuzzy Numbers (HTFSs), which is an extension of HFSs to cover fuzzy sets, intuitionistic fuzzy sets, and fuzzy multi-sets as special cases. In this paper, we discuss about the basis concept of Hesitant triangular fuzzy number in assignment problem using score function and numerical examples are verified.

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1. Introduction

Researchers have been increasingly interested in fuzzy multi criteria decision making since Zadeh [4] introduced the basic model of fuzzy decision-making based on fuzzy mathematics theory. Torra [6, 7] suggested the idea of a reluctant Fuzzy set as a generalisation of a Fuzzy set. The more values obtained from decision makers or experts, which belong to [0, 1], the more the Hesitant Fuzzy collection will represent the original knowledge provided by the decision makers as much as possible. As a result, the Hesitant Fuzzy set can be thought of as a more rigorous set that encourages decision makers to be more flexible in their decisions. In Assignment problems, it's helpful to deal with all of the possible values given by evaluations of each alternative with respect to each criterion. As a result, the Hesitant Fuzzy principle has been widely implemented in a variety of fields. The paper is organized as follows: In section II, we summarise some underlying definitions and basic results of fuzzy numbers. Section III, deals with the concept of hesitant triangular fuzzy number and basic theorems are presented. Section IV, we discussing the main results of triangular hesitant fuzzy number and mathematical model.

Section V, we discuss the hesitant fuzzy number in assignment problem using score function and numerical example.

2. Preliminaries

In this section, we studied the basic definition of fuzzy environment, which is relevant to this project.

Definition 2.1 [Fuzzy set]**.** Let X be an universe of discourse, then a fuzzy set is defined as $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x)\rangle x \in X\}$ which is characterized by a membership function $\mu_{\widetilde{A}} : X \to [0, 1]$ where $\mu_{\widetilde{A}}$ denotes the degree of membership of the element x to the set $\widetilde{A}$.

Definition 2.2 [Fuzzy Number]**.** A fuzzy number $\widetilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_{\widetilde{A}}(X_0)$ is piecewise continuous.

(ii) There exist at least one $x_0 \in R$ with $\mu_{\widetilde{A}}(X_0) = 1.$

(iii) $\widetilde{A}$ must be normal and convex.

Definition 2.3 [Convex Fuzzy set]**.** A fuzzy set $\widetilde{A}$ is convex iff, for any $x_1, x_2 \in X$, the membership function of $\widetilde{A}$ satisfies the inequality.

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)), 0 \leq \lambda \leq 1.$$

Definition 2.4 [T-Norm]**.** A t-norm $\widetilde{H}^{ht}$ is a function $\widetilde{H}^{ht} : [0, 1] \times [0, 1] \to [0, 1]$ having the following four properties:

(1) $\widetilde{H}^{ht}(x, 1) = x$ (Neutral element)

(2) $\widetilde{H}^{ht}(x, y) \leq \widetilde{H}^{ht}(x, z)$ if $y \leq z$ (Monotonicity)

(3) $\widetilde{H}^{ht}(x, \ ) \leq \widetilde{H}^{ht}(y, x)$ (Commutativity)

(4) $\widetilde{H}^{ht}(x, \widetilde{H}^{ht}(y, z)) = \widetilde{H}^{ht}(\widetilde{H}^{ht}(x, y), z)$ (Associativity)

(5) For all $x, y, z \in [0, 1].$

Definition 2.5 [T-Conorm]**.** A t-conorm is a function $\widetilde{O}^{ht} : [0,1]^2 \to [0,1]$ such that for $x, y, z \in [0, 1].$

(1) $x\widetilde{O}^{ht}y = y\widetilde{O}^{ht}x$ (Commutativity)

(2) $(x\widetilde{O}^{ht}y)\widetilde{O}^{ht}z = x\widetilde{O}^{ht}(y\widetilde{O}^{ht}z)$ (Associativity)

(3) $x \leq y$ implies $x\widetilde{O}^{ht}z \leq y\widetilde{O}^{ht}z$ (Monotonicity)

(4) $0\widetilde{O}^{ht}x = x$ (Idendity)

Definition 2.6 [Hesitant Fuzzy set]**.** Given a fixed set X, then a hesitant fuzzy set (HFS) on X is a function that when applied to X returns a subset of values in $[0, 1]$. To express the HFS by a mathematical symbol:

$$\widetilde{E} = (\langle x, \widetilde{h}_E^{ht}(x) > / x \in X).$$

Where $h_E(x)$ a set of some is values in $[0, 1]$, and denotes the possible membership degree of the element $x \in X$ to the set E; $\tilde{h}^{ht}(x) = \tilde{h}_E^{ht}(x)$ is called a hesitant fuzzy element (HFE).

Example 2.7. Suppose that $X = \{x_1, x_2, x_3\}$ is the discourse set, and $\tilde{h}_M^{ht}(x_1) = \{0.8, 0.5, 0.3\}$, $\tilde{h}_M^{ht}(x_2) = \{0.6, 0.4\}$ and $\tilde{h}_M^{ht}(x_3) = \{0.6, 0.3, 0.2\}$ are the HFSs for $x_i (i = 1, 2, 3)$ to a set $\tilde{E}$. Then $\tilde{E}$ can be considered as HFS; i.e.

$$\tilde{E} = \{\langle x_1, \{0.8, 0.5, 0.3\}\rangle, \langle x_2, \{0.6, 0.4\}\rangle, \langle x_3, \{0.6, 0.3, 0.2\}\rangle\}.$$

Definition 2.8 [Triangular Fuzzy Number]**.** A Triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a^L, a^M, a^U). The membership function μ is defined as,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \; x < a^L \\ \dfrac{x - a^L}{a^M - a^L}, \; a^L \leq x \leq a^M \\ \dfrac{x - a^U}{a^M - a^U}, \; a^M \leq x \leq a^U \\ 0, \; x \geq a^U \end{cases}$$

Where $0 < a^L \leq a^M \leq a^U$, a^L and a^U stand for the lower and upper values of the support of $\tilde{A}$ respectively and a^M stands for the middle value.

Definition 2.9 [Arithmetic operation on Triangular Fuzzy Number]**.** The four operations that can be performed on triangular fuzzy numbers are as follows. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

Addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

Multiplication:

$$\tilde{A} \times \tilde{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)).$$

Division:

$$\widetilde{A}/\widetilde{B} = (\min(a_1/b_1,\ a_1/b_3,\ a_3/b_1,\ a_3/b_3),\ a_2/b_2,\ \max(a_1/b_1,\ a_1/b_3,$$

$$a_3/b_1,\ a_3/b_3).$$

Definition 2.10 [Hesitant Triangular Fuzzy set]. Let X be a fixed set, a hesitant triangular fuzzy set (HTFs) on X is in terms of a function when applied to each x in X and returns a subset of values in [0, 1].

To express the HTFs by a mathematical symbol:

$$\widetilde{E} = \{\langle x, \widetilde{h}_E^{ht}(x)\rangle / x \in X\}.$$

Where $\widetilde{h}_E^{ht}(x)$ is a set of some possible triangular fuzzy values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set $\widetilde{E}$.

Definition 2.11 [Arithmetic operations on HFE]. Given three HTFSs $\widetilde{A}^{ht}$, $\widetilde{B}^{ht}$, $\widetilde{C}^{ht}$ and $\lambda > 0$ defined their operations as follows:

(1) $\widetilde{A}^{ht^{\lambda}} = \bigcup_{\gamma \in \widetilde{A}^{ht}} \{((\gamma^L)^{\lambda},\ (\gamma^M)^{\lambda},\ (\gamma^R)^{\lambda}\};$

(2) $\lambda\widetilde{A}^{ht} = \bigcup_{\gamma \in \widetilde{A}^{ht}} \{(1-(1-\gamma^L)^{\lambda}, 1-(1-\gamma^M)^{\lambda}, 1-(1-\gamma^R)^{\lambda})\};$

(3) $\widetilde{B}^{ht} \oplus \widetilde{C}^{ht} = \bigcup_{\gamma_1 \in \widetilde{B}^{ht},\ \gamma_2 \in \widetilde{C}^{ht}} \{(| \gamma_1^L + \gamma_2^L - \gamma_1^L\gamma_2^L |, | \gamma_1^M + \gamma_2^M - \gamma_1^M\gamma_2^M |,$

$| \gamma_1^R + \gamma_2^R - \gamma_1^R\gamma_2^R |\};$

(4) $\widetilde{B}^{ht} \ominus \widetilde{C}^{ht} = \bigcup_{\gamma_1 \in \widetilde{B}^{ht},\ \gamma_2 \in \widetilde{C}^{ht}} \{(| \gamma_1^L + \gamma_2^L - \gamma_1^L\gamma_2^L |, | \gamma - \gamma_2^M + \gamma_1^M\gamma_2^M |,$

$| \gamma_1^R + \gamma_2^R - \gamma_1^R\gamma_2^R |\};$

(5) $\widetilde{B}^{ht} \otimes \widetilde{C}^{ht} = \bigcup_{\gamma_1 \in \widetilde{B}^{ht},\ \gamma_2 \in \widetilde{C}^{ht}} \{(\gamma_1^L\gamma_2^L,\ \gamma_1^M\gamma_2^M,\ \gamma_1^R\gamma_2^R)\}$

(6) $\widetilde{A}^{ht^c} = \bigcup_{\gamma \in \widetilde{A}^{ht}} \{1 - \gamma\}$

(7) $\widetilde{B}^{ht} \cup \widetilde{C}^{ht} = \bigcup_{\gamma_1 \in \widetilde{B}^{ht},\ \gamma_2 \in \widetilde{C}^{ht}} Max\{\gamma_1,\ \gamma_2\}$

(8) $\tilde{B}^{ht} \cap \tilde{C}^{ht} = \bigcup_{\gamma_1 \in \tilde{B}^{ht},\, \gamma_2 \in \tilde{C}^{ht}} Min\,\{\gamma_1, \gamma_2\}$.

Definition 2.12 [Zero Triangular Hesitant Fuzzy Number]**.** If $\tilde{h}^{ht} = (0, 0, 0)$ then $\tilde{h}^{ht}$ is said to be Zero Triangular hesitant fuzzy number. It is denoted by $\tilde{0}^{ht}$.

Definition 2.13 [Unit Triangular Hesitant Fuzzy Number]**.** If $\tilde{h}^{ht} = (1, 1, 1)$ then $\tilde{h}^{ht}$ is said to be Unit Triangular hesitant fuzzy number. It is denoted by $\tilde{1}^{ht}$.

Definition 2.14 [Fuzzy Feasible Solution]**.** A set of non-negative allocation $\tilde{X}^{ht}$ that satisfies (in the sense equivalent) the row and the column restrictions is known as Fuzzy Feasible Solution.

Definition 2.15 [Fuzzy Degenerate Basic Feasible Solution]**.** A Fuzzy feasible solution to a Fuzzy Transportation problem with m sources and n destinations is said to be a Fuzzy basic feasible solution if the number of positive allocations are $(m + n - 1)$. If the number of allocations in a Fuzzy basic solution is less than $(m + n - 1)$, it is called Fuzzy degenerate basic feasible solution.

Definition 2.16 [Fuzzy Optimal Solution]**.** A Fuzzy feasible solution is said to be Fuzzy optimal solution if it minimizes the total Fuzzy Transportation cost.

3. Basic Theorems

In this section, we deals some basic theorems of hesitant fuzzy number.

Definition 3.1 [Score Function]**.** Let $\tilde{A}^{ht}$ be an HFE, with a score function of h defined by,

$$S(\tilde{A}^{ht}) = \left| \frac{1}{l(\tilde{A}^{ht})} \sum_{\gamma \in \tilde{A}^{ht}} \gamma \right|.$$

Where $l(\tilde{A}^{ht})$ is the number of elements in $\tilde{A}^{ht}$.

Let $\widetilde{A}_1^{ht}$ and $\widetilde{A}_2^{ht}$ be two HFSs, then

If $S(\widetilde{A}_1^{ht}) > S(\widetilde{A}_2^{ht})$, then $\widetilde{A}_1^{ht} > \widetilde{A}_2^{ht}$.

If $S(\widetilde{A}_1^{ht}) = S(\widetilde{A}_2^{ht})$, then $\widetilde{A}_1^{ht} = \widetilde{A}_2^{ht}$.

3.2. Fundamental Theorems of Hesitant Fuzzy Assignment Problem

The solution to Hesitant Fuzzy Assignment Problem is fundamentally based on the following three theorems.

Theorem 3.2.1 [*Hesitant Fuzzy Reduction Theorem on Assignment Problem*]**.** *The Hesitant Fuzzy Assignment minimizes the total Hesitant Fuzzy cost for the new Hesitant Fuzzy cost matrix; it also minimizes the total Hesitant fuzzy cost for the original Hesitant Fuzzy cost matrix. If the addition (subtraction) of a constant Hesitant Fuzzy number to a every Hesitant Fuzzy element of a row (or column) of the Hesitant Fuzzy cost matrix* $\widetilde{C}_{ij}^{ht}$, *where the Hesitant Fuzzy cost* $(\widetilde{C}_{ij}^{ht})$ *is represented either by normal or non-normal Triangular Hesitant Fuzzy Number.*

Proof. Let $\widetilde{X}_{ij}^{ht}$ minimizes the total Hesitant Fuzzy cost

$$\widetilde{Z}^{ht} = \sum_{i=1}^{n}\sum_{j=1}^{n}\widetilde{C}_{ij}^{ht}\widetilde{X}_{ij}^{ht}.$$

Over all $\widetilde{X}_{ij}^{ht}$ such that $\widetilde{X}_{ij}^{ht} \geq 0$ and

$$\sum_{i=1}^{n}\widetilde{X}_{ij}^{ht} = \sum_{j=1}^{n}\widetilde{X}_{ij}^{ht} = 1.$$

It is to be shown that the Assignment $\widetilde{X}_{ij}^{ht}$ also minimizes new total Hesitant Fuzzy cost

$$\widetilde{Z}'^{ht} = \sum_{i=1}^{n}\sum_{j=1}^{n}(\widetilde{C}_{ij}^{ht}\Theta\widetilde{u}_{i}^{ht}\Theta\widetilde{v}_{j}^{ht})\widetilde{X}_{ij}^{ht}.$$

For all $i, j = 1, 2, \ldots, n,$ where $\tilde{u}_i^{ht}$ and $\tilde{v}_j^{ht}$ are Fuzzy constants subtracted from i^{th} row and j^{th} column of the cost matrix $(\tilde{C}_{ij}^{ht})$.

To prove this, it may be written as

$$\tilde{Z}'^{ht} = \sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{C}_{ij}^{ht}\tilde{X}_{ij}^{ht})\,\Theta\left(\sum_{i=1}^{n}\tilde{u}_i^{ht}\sum_{i=1}^{n}\tilde{X}_{ij}^{ht}\right)\Theta\left(\sum_{i=1}^{n}\tilde{v}_i^{ht}\sum_{i=1}^{n}\tilde{X}_{ij}^{ht}\right).$$

Using the above equations, we get

$$\tilde{Z}^{ht} = \tilde{Z}^{ht}\ \Theta \sum_{i=1}^{n}\tilde{u}_i^{ht}\sum_{i=1}^{n}\tilde{v}_i^{ht}.$$

The terms that are subtracted from $\tilde{Z}^{ht}$ to give $\tilde{Z}'^{ht}$ are independent of $\tilde{X}_{ij}'^{s}$, it follows that $\tilde{Z}'^{ht}$ is minimized whenever $\tilde{Z}^{ht}$ is minimized, and conversely.

Theorem 3.2.2. *If $\tilde{X}_{ij}^{ht}, i = 1, 2, \ldots, n; j = 1, 2, \ldots, n$ is an Optimal solution for an Assignment problem with cost $(\tilde{C}_{ij}^{ht})$, then it is also Optimal for the problem with cost $\tilde{C}_{ij}'^{ht}$ when $\tilde{C}_{ij}'^{ht} = (\tilde{C}_{ij}^{ht})$ for $i, j = 1, 2, \ldots, n; j \neq k$ $\tilde{C}_{ij}'^{ht} = (\tilde{C}_{ij}^{ht}) - \tilde{A}^{ht}$ is a Hesitant Fuzzy constant.*

Proof. We have $\tilde{Z}'^{ht} = \sum_i \sum_j \tilde{C}_{ij}'^{ht}\tilde{X}_{ij}^{ht}$

$$= \sum_i \left(\sum_{j\neq k}\tilde{C}_{ij}'^{ht} + \tilde{C}_{ik}'^{ht}\right)\tilde{X}_{ik}^{ht}$$

$$= \sum_i \left(\sum_{j\neq k}\tilde{C}_{ij}'^{ht} + \tilde{C}_{ik}'^{ht} - \tilde{A}^{ht}\right)\tilde{X}_{ij}^{ht}$$

$$= \sum\sum \tilde{C}_{ij}^{ht}\tilde{X}_{ik}^{ht} - \tilde{A}^{ht}\sum \tilde{X}_{ij}^{ht}$$

$$= \tilde{Z}^{ht} = \tilde{A}^{ht} \text{ since } (\textstyle\sum \tilde{X}_{ik}^{ht} = 1).$$

Thus if $(\tilde{X}_{ij}^{ht})$ minimizes $\tilde{Z}^{ht}$ so will it give $\tilde{Z}'^{ht}$.

Theorem 3.2.3. *In an Assignment Problem with Hesitant Fuzzy cost* $(\widetilde{C}_{ij}^{ht})$, *if all* $(\widetilde{C}_{ij}^{ht}) \geq 0$ *then a feasible solution* $(\widetilde{X}_{ij}^{ht})$, *which satisfies,* $\sum_{i=1}^{n} \sum_{i=1}^{n} \widetilde{C}_{ij}^{ht} \widetilde{X}_{ij}^{ht} = 0$, *is Optimal for the problem.*

Proof. Since all $(\widetilde{C}_{ij}^{ht}) \geq 0$ and all $(\widetilde{X}_{ij}^{ht}) \geq 0$. The Objective function $\widetilde{Z}^{ht} = \sum \sum \widetilde{C}_{ij}^{ht}$ cannot be negative. The minimum possible value that $\widetilde{Z}^{ht}$ can attain is 0.

Thus, any feasible solution $(\widetilde{X}_{ij}^{ht})$ that satisfies $\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{C}_{ij}^{ht} \widetilde{X}_{ij}^{ht} = 0$, will be Optimal.

4. Main Results

In this section, we deals mathematical model of triangular hesitant fuzzy number and algorithm of hesitant fuzzy assignment problem.

4.1. Main results on Triangular Hesitant Fuzzy Number

Suppose there are n activities (jobs, tasks, or sources) to be performed and n services facilities (men, machine, labourers etc.) are available for doing this activities. Assume that each service facility can perform one activity at a time. The objective of the problem is to assign these activities to the service facilities on one to one basis in such a way so that the total time or total cost involved is minimized and total sale or total profit is maximized or the total satisfaction of the group is maximized.

4.2. Mathematical model of Triangular Hesitant Fuzzy Number

Let the i^{th} person is assigned to the j^{th} job and is denoted by $\widetilde{X}_{ij}^{ht}$ and $\widetilde{C}_{ij}^{ht}$ be the corresponding Triangular Hesitant Fuzzy cost of assigning the i^{th} person to the j^{th} job. Since the objective is to minimize the overall Triangular Hesitant Fuzzy cost for performing all jobs, the mathematical model of this Triangular Hesitant Fuzzy Assignment problem is as follows:

Minimize $\widetilde{Z}^{ht} = \sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{C}_{ij}^{ht} \widetilde{X}_{ij}^{ht}$

$$\text{Subject to } \sum_{i=1}^{n} \tilde{X}_{ij}^{ht} \approx \tilde{1}^{ht},\ j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} \tilde{X}_{ij}^{ht} \approx \tilde{1}^{ht},\ i = 1, 2, \ldots, n$$

$$\tilde{X}_{ij}^{ht} \approx \tilde{0}^{ht},\ j = 1, 2, \ldots, n.$$

$$\text{Where } \tilde{X}_{ij}^{ht} \approx \begin{cases} \tilde{1}^{\mathrm{ht}}, \text{ if the } \mathrm{i}^{\mathrm{th}} \text{personisassigned to } \mathrm{j}^{\mathrm{th}} \text{ job} \\ \tilde{0}^{\mathrm{ht}}, \text{ if the } \mathrm{i}^{\mathrm{th}} \text{personisassigned to } \mathrm{j}^{\mathrm{th}} \text{ job.} \end{cases}$$

The Triangular Hesitant Fuzzy Assignment problem can be stated in the Form of $(n \times n)$ Fuzzy cost matrix $\tilde{C}_{ij}^{ht}$ be the corresponding Triangular Hesitant Fuzzy cost of assigning the i^{th} person to the j^{th} job as given in the table:

Hesitant fuzzy cost matrix of Hesitant Fuzzy Assignment problem

		Jobs						
Persons		1	2	3	...	j	...	n
	1	$\tilde{C}_{11}^{ht}$	$\tilde{C}_{12}^{ht}$	$\tilde{C}_{13}^{ht}$	...	$\tilde{C}_{1j}^{ht}$	...	$\tilde{C}_{1n}^{ht}$
	2	$\tilde{C}_{21}^{ht}$	$\tilde{C}_{22}^{ht}$	$\tilde{C}_{23}^{ht}$	...	$\tilde{C}_{2j}^{ht}$	...	$\tilde{C}_{2n}^{ht}$
	...	...	...	...	...	...	...	...
	I	$\tilde{C}_{11}^{ht}$	$\tilde{C}_{12}^{ht}$	$\tilde{C}_{13}^{ht}$	...	$\tilde{C}_{ij}^{ht}$	...	$\tilde{C}_{in}^{ht}$
	...	...	...	...	...	...	...	...
	N	$\tilde{C}_{n1}^{ht}$	$\tilde{C}_{n2}^{ht}$	$\tilde{C}_{n3}^{ht}$	...	$\tilde{C}_{nj}^{ht}$	...	$\tilde{C}_{nn}^{ht}$

4.3. Algorithm of Hesitant Fuzzy Assignment Problem

Step 1. Find the minimum (maximum) element of each row in the Triangular Hesitant Assignment matrix $(say\, \tilde{a}_{i}^{ht})$ in a minimization (maximization) case, then reduce the minimum (maximum) element of each row from all the elements in the row of the given assignment matrix.

Step 2. Reduce the minimum (maximum) element of each column from all the elements in the column of the resulting Assignment matrix obtained in step 1 to the minimum (maximum) element of each column.

Step 3. To reduce the above column matrix, use the score function.

Step 4. Reduce to zero by subtracting the minimum number from all of the rows and columns. Draw the fewest number of lines necessary to cover all of the matrix's zeros. The full assignment is not possible if the number of drawn lines is less than n, but it is possible if the number of drawn lines is exactly equal to n.

Select the smallest (largest) element $(say\, \tilde{d}_{ij}^{ht})$ from those that do not lie on any of the lines in the above matrix if a complete Assignment problem is not possible in step 4. Each part of the uncovered rows or columns, which $\tilde{d}_{ij}^{ht}$ lies on, was then subtracted by $\tilde{d}_{ij}^{ht}$. This operation adds a new row or column to this row or column.

Step 5. Select the smallest (largest) element $(say\, \tilde{d}_{ij}^{ht})$ from those that do not lie on any of the lines in the above matrix if a complete Assignment problem is not possible in step 4. Each part of the uncovered rows or columns, which $\tilde{d}_{ij}^{ht}$ lies on, was then subtracted by $\tilde{d}_{ij}^{ht}$. This operation adds a new row or column to this row or column.

If this new matrix still does not yield a complete optimal assignment, repeat steps 4 and 5 iteratively. The best assignment can be obtained by repeating the same process.

5. Numerical Example

In this section, we present the hesitant triangular fuzzy number in assignment problem using score function of some numerical examples provided using the operations.

Example 5.1. Let us consider the hesitant triangular fuzzy matrix

$$\begin{bmatrix} (0.1, 0.2, 0.3) & (0.2, 0.4, 0.6) & (0.7, 0.8, 0.9) & (0.3, 0.4, 0.5) \\ (0.6, 0.7, 0.8) & (0.1, 0.2, 0.3) & (0.2, 0.3, 0.4) & (0.5, 0.7, 0.9) \\ (0.2, 0.4, 0.6) & (0.7, 0.8, 0.9) & (0.6, 0.7, 0.8) & (0.1, 0.3, .5) \\ (0.3, 0.4, 0.5) & (0.3, 0.5, 0.7) & (0.1, 0.2, 0.3) & (0.5, 0.6, 0.7) \end{bmatrix}$$

Step 1. Reduce the minimum element of each row in all the elements in the row of the assignment matrix.

$$\begin{bmatrix} (0.17, 0.04, 0.23) & (0.28, 0.28, 0.56) & (0.43, 0.76, 0.89) & (0.05, 0.28, 0.45) \\ (0.28, 0.64, 0.78) & (0.17, 0.04, 0.23) & (0.12, 0.16, 0.34) & (0.05, 0.64, 0.89) \\ (0.28, 0.22, 0.56) & (0.43, 0.74, 0.89) & (0.28, 0.61, 0.78) & (0.35, 0.09, 0.45) \\ (0.05, 0.28, 0.45) & (0.19, 0.4, 0.67) & (0.17, 0.04, 0.23) & 0.15, 0.52, 0.23 \end{bmatrix}$$

Step 2. Reduce the minimum element of each column in all the elements in the column of the reducing assignment matrix.

$$\begin{bmatrix} (0.0209, 0.0016, 0.0991) & (0.1232, 0.2512, 0.4852) & (0.0773, 0.7504, 0.8713) & (0.3775, 0.0784, 0.4225) \\ (0.2816, 0.6256, 0.7426) & (0.0209, 0.0016, 0.0991) & (0.1792, 0.1264, 0.2278) & (0.7955, 0.5392, 0.8845) \\ (0.1576, 0.1888, 0.4852) & (0.0773, 0.7296, 0.8713) & (0.2816, 0.5944, 0.7426) & (0.0575, 0.1648, 0.4225) \\ (0.3775, 0.2512, 0.3565) & (0.3527, 0.0016, 0.6139) & (0.0209, 0.0016, 0.0991) & (0.0455, 0.3856, 0.1915) \end{bmatrix}$$

Step 3. To reduce the above column matrix, use the score function.

$$\begin{bmatrix} 0 & 0.24 & 0.52 & 0.25 \\ 0.5 & 0 & 0.13 & 0.69 \\ 0.23 & 0.51 & 0.49 & 0.17 \\ 0.28 & 0.28 & 0 & 0.16 \end{bmatrix}$$

Step 4. To find an optimal solution.

$$\begin{bmatrix} 0 & 0.08 & 0.52 & 0.09 \\ 0.66 & 0 & 0.29 & 0.69 \\ 0.23 & 0.35 & 0.49 & 0.01 \\ 0.28 & 0.12 & 0 & 0 \end{bmatrix}$$

Step 5. The optimal solution is obtained.

$$\begin{bmatrix} 0 & 0.8 & 0.51 & 0.08 \\ 0.66 & 0 & 0.28 & 0.68 \\ 0.23 & 0.35 & 0.48 & 0 \\ 0.29 & 0.13 & 0 & 0 \end{bmatrix}$$

$$A \to 1,\ B \to 2,\ C \to 4,\ D \to 3.$$

Total Optimal Cost $= (0.1,\ 0.2,\ 0.3) + (0.1,\ 0.2,\ 0.3) + (0.1,\ 0.2,\ 0.3) + (0.1,\ 0.3,\ 0.5) = 0.9$.

Example 5.2. Let us consider the hesitant triangular fuzzy matrix

$$\begin{bmatrix} (0.10,\ 0.12,\ 0.14) & (0.14,\ 0.16,\ 0.18) & (0.12,\ 0.14,\ 0.16) & (0.13,\ 0.15,\ 0.17) \\ (0.8,\ 0.10,\ 0.12) & (0.10,\ 0.12,\ 0.14) & (0.6,\ 0.8,\ 0.10) & (0.14,\ 0.16,\ 0.18) \\ (0.12,\ 0.14,\ 0.16) & (0.15,\ 0.17,\ 0.19) & (0.11,\ 0.13,\ 0.15) & (0.10,\ 0.12,\ 0.14) \\ (0.11,\ 0.13,\ 0.15) & (0.14,\ 0.16,\ 0.18) & (0.10,\ 0.12,\ 0.14) & (0.16,\ 0.18,\ 0.20) \end{bmatrix}$$

Step 1. Reduce the minimum element of each row in all the elements in the row of the assignment matrix.

$$\begin{bmatrix} (0.026,\ 0.014,\ 0.054) & (0.015,\ 0.059,\ 0.098) & (0.021,\ 0.037,\ 0.076) & (0.018,\ 0.048,\ 0.087) \\ (0.776,\ 0.008,\ 0.032) & (0.026,\ 0.014,\ 0.054) & (0.56,\ 0.776,\ 0.01) & (0.015,\ 0.059,\ 0.098) \\ (0.021,\ 0.037,\ 0.076) & (0.011,\ 0.070,\ 0.109) & (0.023,\ 0.026,\ 0.065) & (0.026,\ 0.014,\ 0.054) \\ (0.023,\ 0.026,\ 0.65) & (0.015,\ 0.059,\ 0.098) & (0.026,\ 0.014,\ 0.054) & (0.008,\ 0.082,\ 0.12) \end{bmatrix}$$

Step 2. Reduce the minimum element of each column in all the elements in the column of the reducing assignment matrix.

$$\begin{bmatrix} (0.027,\ 0.0001,\ 0.029) & (0.081,\ 0.046,\ 0.039) & (0.053,\ 0.023,\ 0.052) & (0.67,\ 0.0.35,\ 0.052) \\ (0.769,\ 0.006,\ 0.007) & (0.027,\ 0.0001,\ 0.029) & (0.556,\ 0.773,\ 0.016) & (0.081,\ 0.046,\ 0.039) \\ (0.053,\ 0.023,\ 0.052) & (0.097,\ 0.015,\ 0.096) & (0.040,\ 0.012,\ 0.041) & (0.27,\ 0.001,\ 0.029) \\ (0.040,\ 0.012,\ 0.041) & (0.081,\ 0.046,\ 0.039) & (0.027,\ 0.0001,\ 0.029) & (0.111,\ 0.069,\ 0.097) \end{bmatrix}$$

Step 3. To reduce the above column matrix, use the score function.

$$\begin{bmatrix} 0.019 & 0.055 & 0.043 & 0.051 \\ 0.261 & 0.019 & 0.448 & 0.055 \\ 0.043 & 0.069 & 0.031 & 0.019 \\ 0.031 & 0.055 & 0.019 & 0.092 \end{bmatrix}$$

Step 4. The optimal solution is obtained.

$$\begin{bmatrix} 0 & 0.036 & 0.024 & 0.032 \\ 0.542 & 0 & 0.429 & 0.036 \\ 0.024 & 0.05 & 0.012 & 0 \\ 0.012 & 0.036 & 0 & 0.073 \end{bmatrix}$$

$$A \to 1,\ B \to 2,\ C \to 4,\ D \to 3.$$

Total Optimal Cost

$= (0.10, 0.12, 0.14) + (0.10, 0.12, 0.14) + (0.10, 0.12, 0.14) + (0.10, 0.12, 0.14)$

$= 0.48.$

6. Conclusion

In this paper, the concept of hesitant fuzzy number in assignment problem using score function and numerical examples are verified.

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