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ALGORITHMIC APPROACH TO SOLVE MIXED INTEGER PROGRAMMING PROBLEM USING SEQUENTIAL LINEAR PROGRAMMING METHOD – A FUZZY ENVIRONMENT

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Abstract

In this work, we propose a new approach for finding the solution of Mixed integer programming problem using Fuzzy sequential linear programming method, an algorithmic method is introduced. Computational results are presented and termination criteria for the solution scheme are discussed.

1. Introduction

Sequential Linear Programming was originally proposed by Griffith and Stewart [6]. Bhavikatti [3] suggested several improvements to the method to

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ensure that the method can be used almost as a black box for practical problems. Sequential Linear Programming [1, 5, 7, 8] is one of the dominant methods for solving nonlinear optimization problems. The Sequential Linear Programming [4] consists in linearizing the constraints and the objective function in the neighbourhood of a design vector and solving the resulting linear programming problem to get a new design vector. The linearization of non-linear equations and the solution of linear programming problem is sustained in a sequence till optimal solution is reached [2].

Fuzzy sets have been introduced by Lotfi. A. Zadeh (1965) [10] as a mathematical way of representing impreciseness or vagueness in everyday life. Fuzzy set theory has been practically applied to many disciplines such as control theory and operational research, mathematical modelling and industrial applications. The concept of fuzzy optimization in wide-ranging was first proposed by Tanaka et al, 1974 [9]. Zimmerman, 1978 [11], Proposed the first formatting of fuzzy linear programming.

The paper is organized as follows: Basic concepts of fuzzy sets and hexagonal fuzzy numbers are given in Section 2, and Section 3 deals an Algorithm to Solve Fuzzy Mixed Integer Programming Problem Using Fuzzy Sequential Linear Programming Method. In section 4 proposed algorithms is given to solve MIFSLPP. Finally, in Section 5, the efficiency of the proposed method is explained by means of an example.

2. Preliminaries

Definition 2.1. A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$ [10]. In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A and the second element $\mu_A(x)$ belongs to the interval [0, 1] called membership function.

Definition 2.2. A fuzzy set $\widetilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number:

(i) $\widetilde{A}$ must be a normal fuzzy set;

(ii) $\alpha_{\widetilde{A}}$ must be a closed interval for every $\alpha \in [0, 1]$; and

(iii) the support of $\widetilde{A}$ must be bounded.

Definition 2.3. A fuzzy number $\widetilde{A}_H$ is a hexagonal fuzzy numbers denoted by $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\widetilde{A}_H}(x)$ is given below

$$\mu_{\widetilde{A}_H}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_2 - a_1}\right) & a_1 \le x \le a_2 \\ 1 + \frac{1}{2}\left(\frac{x - a_3}{a_3 - a_2}\right) & a_2 \le x \le a_3 \\ 1 & a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_5 - a_4}\right) & a_4 \le x \le a_5 \\ \frac{1}{2} - \frac{1}{2}\left(\frac{x - a_6}{a_6 - a_5}\right) & a_5 \le x \le a_6 \\ 0 & x > a_6 \end{cases}$$

Operations of Hexagonal Fuzzy numbers 2.4.

Following are the three operations that can be performed on hexagonal fuzzy numbers, suppose $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$, and $\widetilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy Numbers then

Addition: $\widetilde{A}_H + \widetilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$

Subtraction:

$\widetilde{A}_H - \widetilde{B}_H = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$

Multiplication:

$\widetilde{A}_H * \widetilde{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$

Ranking Function 2.5. An efficient approach for comparing the fuzzy numbers is by the use of an efficient ranking function $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.

(i) $\widetilde{A} > \widetilde{B}$ if and only if $\Re(\widetilde{A}) > \Re(\widetilde{B})$

(ii) $\widetilde{A} < \widetilde{B}$ if and only if $\Re(\widetilde{A}) < \Re(\widetilde{B})$

(iii) $\widetilde{A} \approx \widetilde{B}$ if and only if $\Re(\widetilde{A}) = \Re(\widetilde{B})$

i.e. $R(\widetilde{A}_H) = \left(\frac{2a_1 + 10a_2 + 15a_3 + 15a_4 + 10a_5 + 2a_6}{54}\right)\left(\frac{19w}{54}\right)$

where $(a_1, a_2, a_3, a_4, a_5, a_6)$, are hexagonal fuzzy numbers.

Definition 2.6. When $w = 1$ the hexagonal fuzzy number is a normal fuzzy number.

3. An Algorithm to Solve Fuzzy Mixed Integer Programming Problem Using Fuzzy Sequential Linear Programming Method

Step 1. Assume an initial point $\widetilde{x}^{(0)}$, two convergence parameters $\in$ and δ. Set an iteration Counter $t = 0$.

Step 2. Calculate $\nabla f(\widetilde{x}^{(t)})$. If $\| \nabla f(\widetilde{x}^{(t)}) \| \leq \in$. Terminate; Else go to step 3.

Step 3. Frame the Fuzzy Sequential Linear Programming Problem as Maximize or Minimize $f(\widetilde{x}^{(t)}) + \nabla f(\widetilde{x}^{(t)})(\widetilde{x} - \widetilde{x}^{(t)})$ Subject to constraints

$g_j(\widetilde{x}^{(t)}) + \nabla g_j(\widetilde{x}^{(t)})(\widetilde{x} - \widetilde{x}^{(t)}) \geq 0; j = 1, 2, \ldots, J$

$g_j(\widetilde{x}^{(t)}) + \nabla g_j(\widetilde{x}^{(t)})(\widetilde{x} - \widetilde{x}^{(t)}) \leq 0; j = 1, 2, \ldots, J$

$h_k(\widetilde{x}^{(t)}) + \nabla h_k(\widetilde{x}^{(t)})(\widetilde{x} - \widetilde{x}^{(t)}) = 0; k = 1, 2, \ldots, K$

$\widetilde{x}^{(t)} \leq \widetilde{x}_i \leq \widetilde{x}^{(t)}$.

Step 4. Formulate the given Fuzzy mixed integer programming problem.

Step 5. Drop all the integer variables in the fuzzy objective function and in the constraint set. Apply Sequential linear programming solution mixer integer programming Problem.

Step 6. Does the linear programming have a solution? If yes, stopping at the satisfied criteria and get the results, if No, apply sub routine solution to determine new variable cost substituting for fixed costs.

Step 7. Therefore, the criteria $\| \tilde{x}^{(t)} - \tilde{x}^{(t)} \| \leq \in_1 \| \tilde{x}^{(t)} \|$ and $\| f(\tilde{x}^{(t+1)} - f(\tilde{x}^{(t)})) \| \leq \in_2 \| f(\tilde{x}^{(t)}) \|$ is satisfied and print the result.

4. Numerical Example

Minimize $f(\tilde{x}) = -\tilde{x}_2^4 \tilde{x}_2$ Subject to

$$\tilde{x}_1 + \tilde{x}_2 \leq 5$$

$$\tilde{x}_1 + 2\tilde{x}_2 \leq 6$$

Non-negative restriction $\tilde{x}_1 + \tilde{x}_2 \geq 0$

Solution.

The Formulated Fuzzy Sequential Linear Programming Problem is converted in to Hexagonal Fuzzy Sequential Linear Programming Problem as

Minimize $f(\tilde{x}) = -(0, 0.5, 1, 1.3, 1.5, 2; 1)\tilde{x}_2^4 \tilde{x}_2$

Subject to

$$(0, 0.5, 1, 1.3, 1.5, 2; 1) + \tilde{x}_1 + (0, 0.5, 1, 1.3, 1.5, 2; 1)\tilde{x}_2 \leq (3, 4, 5, 5.5, 6, 7; 1)$$

$$(0, 0.5, 1, 1.3, 1.5, 2; 1)\tilde{x}_1 + (0, 1, 2, 2.5, 3, 4; 1)\tilde{x}_2 \leq (4, 5, 6, 6.5, 7, 8; 1)$$

Non-negative restriction

$\tilde{x}_1, \tilde{x}_2 \geq (0, 0, 0, 0, 0, 0)$.

By using Ranking function, we obtained

$\Re(0, 0.5, 1, 1.3, 1.5, 2; 1) = 0.35$ $\Re(0, 1, 2, 2.5, 3, 4; 1) = 0.70$

$\Re(3, 4, 5, 5.5, 6, 7; 1) = 1.76$ $\Re(4, 5, 6, 6.5, 7, 8; 1) = 2.11$

Therefore

Minimize $f(\tilde{x}) = -0.35\tilde{x}_1^4 \tilde{x}_2^2$ Subject to

$$0.35\tilde{x}_1 + 0.35\tilde{x}_2 \leq 1.76$$

$0.35\tilde{x}_1 + 0.70\tilde{x}_2 \leq 2.11$

Non-negative restriction $\tilde{x}_1, \tilde{x}_2 \geq 0$

Let us calculate fuzzy cost and fuzzy constraints gradients functions at the point $t = 0$ and $\tilde{x}^{(t)}$ Choose $\in_1 = \in_2 = 0.01. \tilde{x}^{(0)} = (1, 1)$

Therefore, the criteria $\| \tilde{x}^{(t+1)} - \tilde{x}^{(t)} \| \leq \in_1 \| \tilde{x}^{(t)} \|$ and $\| f(\tilde{x}^{(t+1)}) - f(\tilde{x}^{(t)}) \| \leq \in_2 \| f(\tilde{x}^{(t)}) \|$ is satisfied.

The optimal solution is $\tilde{x}_1 = 6.01$ and $\tilde{x}_2 = -1.00$.

5. Conclusion

The proposed Fuzzy Sequential Linear Programming algorithm offers a suitable and well-organized method to solve the mixed integer programming problems. It provides not only the optimal solution but also near optimal solutions. So, it covers the way for the performance analysis of the solutions obtained. The optimal solutions obtained from the proposed algorithm are more approximate than many other solutions obtained from the existing methods.

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