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α-MULTI FUZZY SUB ALGEBRA AND α-MULTI ANTI FUZZY SUB ALGEBRA OF BG-ALGEBRA

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Abstract

In this paper, the notions of α-multi fuzzy sub algebra, α-multi fuzzy normal sub algebra, α-multi anti fuzzy sub algebra and α-multi anti fuzzy normal sub algebra of BG-algebra are defined by combining the concepts of multi fuzzy sets and α-fuzzy sets. And also some of their related properties are investigated under cartesian product and homomorphism.

1. Introduction

The notion of a fuzzy subset was initially introduced by Zadeh [18] in 1965, for representing uncertainty. In 2000, S. Sabu and T. V. Ramakrishnan [13, 14] proposed the theory of multi-fuzzy sets in terms of multi-dimensional membership functions and investigated some properties of multilevel fuzziness. Theory of multi-fuzzy set is an extension of theory of fuzzy sets. Complete characterization of many real life problems can be done by multi-fuzzy membership functions of the objects involved in the problem. P. K. Sharma [15, 16, 17] defined α-fuzzy set and gave the notion of α-fuzzy subgroups in 2013. Y. Imai and K. Iseki introduced two classes of abstract

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algebras: BCK algebras and BCI-algebras [4, 5, 6]. It is shown that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. J. Neggers and H. S. Kim [12] introduced a new notion, called a B-algebra. In 2005, C. B. Kim and H. S. Kim [7] introduced the notion of a BG-algebra which is a generalization of B-algebras. With these ideas, fuzzy sub algebras of BG-algebra were developed by S. S. Ahn and H. D. Lee [1]. R. Muthuraj and S. Devi [9, 10, 11] introduced the concept of multi-fuzzy sub algebra and multi anti fuzzy sub algebra of BG-algebra in 2016. In this paper, we define α-multi fuzzy sub algebra and α-multi anti fuzzy sub algebra of BG-algebra and discuss their related properties. Also the homomorphic image and pre-image of α-multi fuzzy sub algebra are obtained.

2. Preliminaries

In this section, the basic definitions of a BG-algebra, multi-fuzzy sets are recalled.

Definition 2.1 [7]**.** A BG-algebra is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms:

1. $x * x = 0$
2. $x * 0 = x$,
3. $(x * y) * (0 * y) = x$ for all $x, y \in X$.

Example 2.2 [7]**.** Let $X = \{0, 1, 2\}$ be a set with the following table:

Table 2.1.

$*$	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then $(X; *, 0)$ is a BG-algebra.

Definition 2.3 [13]**.** Let X be a non-empty set and let $\{L_i : i \in P\}$ be a family of complete lattices. A multi-fuzzy set A in X is a set of ordered sequences:

$$A = \{\langle x, \mu_1(x), \mu_2(x), \ldots, \mu_i(x), \ldots \rangle : x \in X\}, \text{ where } \mu_i \in L_i^x, \text{ for } i \in P.$$

Remarks 2.4 [13]**.**

(i) If the sequences of the membership functions have only k-terms (finite number of terms), k is called the dimension of A.

(ii) In this paper $L_i = [0, 1]$ (for $i = 1, 2, \ldots, k$).

(iii) The multi-membership function μ_A is a function from X to I^k such that for all x in X, $\mu_A(x) = \langle \mu_1(x), \mu_2(x), \ldots, \mu_k(x) \rangle$.

(iv) For the sake of simplicity, we denote the multi-fuzzy set $A = \{(x, \mu_1(x), \mu_2(x), \ldots, \mu_k(x)) : x \in X\}$ as $A = (\mu_1, \mu_2, \ldots, \mu_k)$.

Definition 2.5 [13]**.** Let k be a positive integer and let μ and ν in $M^k FS(X)$, that is $\mu = (\mu_1, \mu_2, \ldots, \mu_k) = \{\langle x, \mu_1(x), \mu_2(x), \ldots, \mu_k(x) \rangle : x \in X\}$ and $\nu = (\nu_1, \nu_2, \ldots, \nu_k) = \{\langle x, \nu_1(x), \nu_2(x), \ldots, \nu_k(x) \rangle : x \in X\}$ then we have the following relations and operations:

(i) $\mu \subseteq \nu$ if and only if $\mu_i \leq \nu_i$, for all $i = 1, 2, \ldots, k$;

(ii) $\mu = \nu$ if and only if $\mu_i = \nu_i$, for all $i = 1, 2, \ldots, k$;

(iii) $\mu \cup \nu = (\mu_1 \cup \nu_1, \ldots, \mu_k \cup \nu_k) = \{\langle x, \max(\mu_1(x), \nu_1(x)), \ldots, \max(\mu_k(x), \nu_k(x)) \rangle : x \in X\}$;

(iv) $\mu \cap \nu = (\mu_1 \cap \nu_1, \ldots, \mu_k \cap \nu_k) = \{\langle x, \min(\mu_1(x), \nu_1(x)), \min(\mu_k(x), \nu_k(x)) \rangle : x \in X\}$.

Definition 2.6 [9]**.** Let A be a multi-fuzzy set in a BG-algebra X. Then A is called a multi-fuzzy sub algebra of X if $A(x * y) \geq \min\{A(x), A(y)\}$, $\forall\ x, y \in X$.

Definition 2.7 [10]**.** Let A be a multi-fuzzy set in a BG-algebra X. Then A is called a multi anti fuzzy sub algebra of X if $A(x * y) \leq \max\{A(x), A(y)\}$, $\forall\ x, y \in X$.

Definition 2.8 [15]**.** Let A be a fuzzy subset of a group G. Let $\alpha \in [0, 1]$. Then the fuzzy set A^{α} of G is called the α-fuzzy subset of G (w. r. to fuzzy set A) and is defined as $A^{\alpha}(x) = \min\{A(x), \alpha\}$, for all $x \in G$.

Remark 2.9 [15]**.** Clearly $A^1 = A$ and $A^0 = 0$.

Remark 2.10 [15]**.** (i) Let A and B be two fuzzy subsets of X. Then $(A \cap B)^{\alpha} = A^{\alpha} \cap B^{\alpha}$

(ii) Let $f : X \to Y$ be a mapping and A and B be two fuzzy subsets of X and Y respectively, then $f^{-1}(B^{\alpha}) = (f^{-1}(B))^{\alpha}$ and $f(A^{\alpha}) = (f(A))^{\alpha}$.

Definition 2.11 [16]**.** Let A be a fuzzy subset of a group G. Let $\alpha \in [0, 1]$. Then the fuzzy set A_{α} of G is called the α-anti fuzzy subset of G (w. r. t fuzzy set A) and is defined as $A_{\alpha}(x) = \max\{A(x), 1 - \alpha\}$, for all $x \in G$.

Definition 2.12 [7]**.** Let X and Y be BG-algebras. A mapping $\varphi : X \to Y$ is called a BG-homomorphism if $\varphi(x * y) = \varphi(x) * \varphi(y)$ for any $x, y \in X$.

3. α-Multi Fuzzy BG-Sub Algebra

In this section, we define α-multi fuzzy sub algebra and α-multi fuzzy normal sub algebra of BG-algebra and discussed some of its properties.

Definition 3.1. Let A be a multi-fuzzy subset in X. Let $\alpha \in [0, 1]$. Then the multi fuzzy set A^{α} of X is called α-multi fuzzy subset of X and is defined as $A^{\alpha}(x) = \min\{A(x), \alpha\} = \{\langle x, \min(\mu_1(x), \alpha)), \ldots \min(\mu_k(x), \alpha)\rangle : x \in X\}$.

Definition 3.2. Let A be a multi-fuzzy subset of a BG-algebra X. Let $a \in [0, 1]$.

Then A is called α-multi fuzzy subalgebra of X if A^{α} is a multi-fuzzy subalgebra of X i.e., if A^{α} satisfies the following condition: $A^{\alpha}(x * y) \geq \min\{A^{\alpha}(x), A^{\alpha}(y)\}$ for all $x, y \in X$.

Theorem 3.3. *If A is a multi-fuzzy subalgebra of X then A is also α-multi fuzzy sub algebra of X.*

Proof. Let $x, y \in X$. Then $A^{\alpha}(x * y) = \min\{A(x * y), \alpha\}$

$\geq \min\{\min\{A(x), A(y)\}, \alpha\}$, since A is a multi-fuzzy sub algebra of X.

$= \min\{\min\{A(x), \alpha\}, \min\{A(y), \alpha\}\}$

$= \min\{A^{\alpha}(x), A^{\alpha}(y)\}, \forall x, y \in X$.

$\Rightarrow A^{\alpha}(x * y) \geq \min\{A^{\alpha}(x), A^{\alpha}(y)\}$.

Hence A is α-multi fuzzy sub algebra of X.

Remark 3.4. The converse of the above theorem is not true.

Example. Consider a BG-algebra $X = \{0, 1, 2\}$ as in example 2.1.

Define a multi-fuzzy set A as $A(0) = (0.6, 0.5, 0.4)$, $A(1) = (0.8, 0.7, 0.6)$, and $A(2) = (0.4, 0.3, 0.2)$.

Since $A(0) \ngeq \min\{A(1), A(1)\}$, A is not a multi-fuzzy sub algebra of X. Let $\alpha = 0.1 \in [0, 1]$. Then $A(x) > \alpha$ for all x in X.

Therefore $A^{\alpha}(x * y) \geq \min\{A^{\alpha}(x), A^{\alpha}(y)\}$ for all x, y in X.

Hence A is α-multi fuzzy sub algebra of X.

Theorem 3.5. *The intersection of two α-multi fuzzy sub algebra of X is also α-multi fuzzy sub algebra of X.*

Proof. Let A and B be two α-multi fuzzy sub algebras of X.

Let $x, y \in X$.

$$
\begin{aligned}
\text{Then } (A \cap B)^{\alpha}(x * y) &= (A^{\alpha} \cap B^{\alpha})(x * y) \\
&= \min\{A^{\alpha}(x * y), B^{\alpha}(x * y)\} \\
&\geq \min\{\min\{A^{\alpha}(x), A^{\alpha}(y)\}, \min\{B^{\alpha}(x), B^{\alpha}(y)\}\} \\
&= \min\{\min\{A^{\alpha}(x), B^{\alpha}(x)\}, \min\{A^{\alpha}(y), B^{\alpha}(y)\}\} \\
&= \min\{(A^{\alpha} \cap B^{\alpha})(x), (A^{\alpha} \cap B^{\alpha})(y)\}
\end{aligned}
$$

$$= \min \{(A \cap B)^{\alpha}(x), (A \cap B)^{\alpha}(y)\}$$

$$\Rightarrow (A \cap B)^{\alpha}(x * y) \geq \min \{(A \cap B)^{\alpha}(x), (A \cap B)^{\alpha}(y)\}.$$

Remark 3.6. Union of two α-multi fuzzy sub algebras of a BG-algebra X need not be α-multi fuzzy sub algebra of X which is shown in the following example.

Example. Let $X = \{0, 1, 2, 3, 4, 5\}$ be a BG-algebra with the following cayley table:

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	5	3	4
2	2	1	0	4	5	3
3	3	4	5	0	1	2
4	4	5	3	2	0	1
5	5	3	4	1	2	0

Define the two multi fuzzy sets A and B by $A(0) = A(3) = (0.8, 0.7)$ and $A(1) = A(2) = A(4) = A(5) = (0.3, 0.2)$ $B(0) = B(4) = (0.6, 0.5)$ and $B(1) = B(2) = B(3) = B(5) = (0.4, 0.3)$. Take $\alpha = 1$. Clearly A and B are 1-multi fuzzy sub algebra of X. Now, $(A \cup B)(x) = \max\{A(x), B(x)\}$. Therefore $(A \cup B)(0) = (A \cup B)(3) = (0.8, 0.7)$, $(A \cup B)(1) = (A \cup B)(2) = (A \cup B)(5) = (0.4, 0.3)$ and $(A \cup B)(4) = (0.6, 0.5)$. Since $(A \cup B)(3 * 4) = (A \cup B)(1) = (0.4, 0.3)$ $\ngeq (0.6, 0.5) = \min \{(A \cup B)(3), (A \cup B)(4)\}$, $A \cup B$ is not a 1-multi fuzzy sub algebra of X. Hence union of two α-multi fuzzy sub algebra need not be a α-multi fuzzy sub algebra of X.

Definition 3.7. Let A and B be two α-multi fuzzy subsets in X. Then their Cartesian product $A^{\alpha} \times B^{\alpha}$ is defined as $(A^{\alpha} \times B^{\alpha})(x_1, x_2) = \min \{A^{\alpha}(x_1), B^{\alpha}(x_2)\}$ where $(x_1, x_2) \in X \times X$.

Theorem 3.8. *The Cartesian product of two α-multi fuzzy sub algebra of X is also α-multi fuzzy sub algebra of X.*

Proof. Let $(x_1, x_2), (y_1, y_2) \in X \times X$. Then

$$(A^\alpha \times B^\alpha)(x_1, x_2) * (y_1, y_2) = (A^\alpha \times B^\alpha)(x_1 * y_1, x_2 * y_2)$$

$$= \min\{A^\alpha(x_1 * y_1), B^\alpha(x_2 * y_2)\}$$

$$\geq \min\{\min\{A^\alpha(x_1), A^\alpha(y_1)\}, \min\{B^\alpha(x_2), B^\alpha(y_2)\}\}$$

$$= \min\{\min\{A^\alpha(x_1), B^\alpha(x_2)\}, \min\{A^\alpha(y_1), B^\alpha(y_2)\}\}$$

$$= \min\{(A^\alpha \times B^\alpha)(x_1, x_2), (A^\alpha \times B^\alpha)(y_1, y_2)\}.$$

Hence $A^\alpha \times B^\alpha$ is α-multi fuzzy sub algebra of X.

Definition 3.9. Let A be a multi-fuzzy subset of X. Then A is called α-multi fuzzy normal sub algebra of X if A^α is multi-fuzzy normal sub algebra of X i.e., A^α satisfies the following condition:

$$A^\alpha((x * a) * (y * b)) \geq \min\{A^\alpha(x * y), A^\alpha(a * b)\} \text{ for every } x, y \in X.$$

Theorem 3.10. *If A is a multi fuzzy normal sub algebra of X, then A is also α-multi fuzzy normal sub algebra of X.*

Proof. Let $x, y, a, b \in X$.

Since A is a multi fuzzy normal sub algebra of X, $A((x * a) * (y * b)) \geq \min\{A(x * y), A(a * b)\}$

$$A^\alpha((x * a) * (y * b)) = \min\{A((x * a) * (y * b)), \alpha\}$$

$$\geq \min\{\min\{A(x * y), A(a * b)\}, \alpha\}$$

$$= \min\{\min\{A(x * y), \alpha\}, \min\{A(a * b), \alpha\}\}$$

$$= \min\{A^\alpha(x * y), A^\alpha(a * b)\}, \forall x, y \in X.$$

Hence A is α-multi fuzzy normal sub algebra of X.

4. α-Multi Anti Fuzzy BG-Sub Algebra

In this section, we define α-multi anti fuzzy sub algebra and α-multi anti fuzzy normal sub algebra of BG-algebra and discussed some of its properties.

Definition 4.1. Let A be a multi fuzzy subset in X. Let $\alpha \in [0, 1]$. Then the multi fuzzy set A_α of X is called α-multi anti fuzzy subset of X and is defined as

$$A_\alpha(x) = \max\{A(x), 1-\alpha\} = \{\langle x, \max(\mu_1(x), 1-\alpha), \ldots \max(\mu_k(x), 1-\alpha)\rangle : x \in X\}.$$

Definition 4.2. Let A be a multi-fuzzy subset of a BG-algebra X. Let $\alpha \in [0, 1]$. Then A is called α-multi anti fuzzy sub algebra of X if A_α is a multi anti fuzzy sub algebra of X i.e., if A_α satisfies the following condition:

$$A_\alpha(x * y) \le \max\{A_\alpha(x), A_\alpha(y)\} \text{ for all } x, y \in X.$$

Theorem 4.3. *If A is a multi anti fuzzy sub algebra of X then A is also α-multi anti fuzzy sub algebra of X.*

Proof. Let $x, y \in X$.

Then $A_\alpha(x * y) = \max\{A(x * y), 1-\alpha\} \le \max\{\max\{A(x), A(y)\}, 1-\alpha\}$, since A is a multi anti fuzzy sub algebra of $X = \max\{\max\{A(x), 1-\alpha\}, \max\{A(y), 1-\alpha\}\} = \max\{A_\alpha(x), A_\alpha(y)\}$, $\forall\, x, y \in X \Rightarrow A_\alpha(x * y) \le \max\{A_\alpha(x), A_\alpha(y)\}$. Hence A is α-multi anti fuzzy sub algebra of X.

Remark 4.4. The converse of the above theorem is not true.

Example. Consider a BG-algebra $X = \{0, 1, 2\}$ as in example 2.1. Define a multi-fuzzy set A as $A(0) = (0.6, 0.5, 0.4)$, $A(1) = (0.4, 0.3, .2)$, and $A(2) = (0.8, 0.7, 0.6)$. Since $A(0) \not\le \max\{A(1), A(1)\}$, A is not a multi anti fuzzy sub algebra of X. Let $\alpha = 0.05$. So that $A_\alpha(x) = \max\{A(x), 1-\alpha\} = 1-\alpha$. Then $A(x) < 1-\alpha$, for all x in X. Therefore $A_\alpha(x * y) \le \max\{A_\alpha(x), A_\alpha(y)\}$ for all x in X. Hence A is α-multi anti fuzzy sub algebra of X.

Theorem 4.5. *The union of two α-multi anti fuzzy sub algebra of X is also α-multi anti fuzzy sub algebra of X.*

Proof. Let A and B be two α-multi anti fuzzy sub algebras of X. Let $x, y \in X$. Then

$$(A \cup B)_\alpha (x * y) = (A_\alpha \cup B_\alpha)(x * y)$$

$$= \max \{A_\alpha (x * y), B_\alpha (x * y)\}$$

$$\leq \max \{\max \{A_\alpha (x), A_\alpha (y)\}, \max \{B_\alpha (x), B_\alpha (y)\}\}$$

$$= \max \{\max \{A_\alpha (x), B_\alpha (x)\}, \max (A_\alpha (y), A_\alpha (y)\}\}$$

$$= \max \{A_\alpha \cup B_\alpha)(x), (A_\alpha \cup B_\alpha)(y)\}$$

$$= \max \{(A \cup B)_\alpha (x), (A \cup B)_\alpha (y)\}, \forall\ x, y \in X$$

$$\Rightarrow (A \cup B)_\alpha (x * y) \leq \max \{(A \cup B)_\alpha (x), (A \cup B)_\alpha (y)\}.$$

Remark 4.6. Intersection of two α-multi anti fuzzy sub algebras of a BG-algebra X need not be α-multi anti fuzzy sub algebra of X.

Example. Consider a BG-algebra X as in the example 3.6 and define two multi fuzzy sets $A(0) = A(3) = (0.3, 0.2)$ and $A(1) = A(2) = A(4) = A(5) = (0.8, 0.7)$ $B(0) = B(4) = (0.4, 0.3)$ and $B(1) = B(2) = B(3) = B(5) = (0.6, 0.5)$. Take $\alpha = 0.95$. Clearly A and B are (0.95)-multi anti fuzzy sub algebra of X. Now, $(A \cap B)(x) = \min \{A(x), B(x)\}$. Therefore $(A \cap B)(0) = (A \cap B)(3) = (0.3, 0.2)$, $(A \cap B)(1) = (A \cap B)(2) = (A \cap B)(5) = (0.6, 0.5)$ and $(A \cap B)(4) = (0.4, 0.3)$.

Since $(A \cap B)(3 * 4) = (A \cap B)(1) = (0.6, 0.5) \nleq (0.4, 0.3) = \max \{(A \cup B)(3), (A \cup B)(4)\}$, $A \cap B$ is not a (0.95)-multi anti fuzzy sub algebra of X. Hence intersection of two α-multi anti fuzzy sub algebra of x need not be α-multi anti fuzzy sub algebra of X.

Definition 4.7. Let A and B two α-multi anti fuzzy sub algebra in X. Then their cartesian product $A_\alpha \times B_\alpha$ is defined as $(A_\alpha \times B_\alpha)(x_1, x_2) = \max \{A_\alpha (x_1), B_\alpha (x_2)\}$, where $(x_1, x_2) \in X \times X$.

Theorem 4.8. *The Cartesian product of two α-multi anti fuzzy sub algebra of X is also α-multi anti fuzzy sub algebra of X.*

Proof. Let $(x_1, x_2), (y_1, y_2) \in X \times X$.

$$(A_\alpha \times B_\alpha)((x_1, x_2) * (y_1, y_2)) = (A_\alpha \times B_\alpha)(x_1 * y_1, x_2 * y_2)$$

$$= \max\{A_\alpha(x_1 * y_1), B_\alpha(x_2 * y_2)\}$$

$$\leq \max\{\max\{A_\alpha(x_1), A_\alpha(y_1)\}, \max\{B_\alpha(x_2), B_\alpha(y_2)\}\}$$

$$= \max\{\max\{A_\alpha(x_1), B_\alpha(x_2)\}, \max\{A_\alpha(y_1), B_\alpha(y_2)\}\}$$

$$= \max\{(A_\alpha \times B_\alpha)(x_1, x_2), (A_\alpha \times B_\alpha)(y_1, y_2)\}.$$

Definition 4.9. Let A be a multi-fuzzy subset of X. Then A is called α-multi anti fuzzy normal sub algebra of X if A_α is multi anti fuzzy normal sub algebra of X i.e., A_α satisfies the following condition: $A_\alpha((x * a) * (y * b)) \leq \max\{A_\alpha(x * y), A_\alpha(a * b)\}$ for every $x, y \in X$.

Theorem 4.10. *If A is a multi anti fuzzy normal sub algebra of X, then A is also α-multi anti fuzzy normal sub algebra of X.*

Proof. Let $x, y, a, b \in X$. Then

$$A_\alpha((x * a) * (y * b)) = \max\{A((x * a) * (y * b)), 1 - \alpha\}$$

$$\leq \max\{\max\{A(x * y), A(a * b)\}, 1 - \alpha\}$$

$$= \max\{\max\{A(x * y), 1 - \alpha\}, \max\{A(a * b), 1 - \alpha\}\}$$

$$= \max\{A_\alpha(x * y), A_\alpha(a * b)\}, \forall\ x, y \in X.$$

Hence A is α-multi anti fuzzy normal subalgebra of X.

5. Homomorphism of α-multi Fuzzy sub Algebra of BG-Algebra

Theorem 5.1. *Let $f : X \to Y$ be a homomorphism of a BG-algebra X into a BG-algebra Y. Let B be α-multi fuzzy sub algebra of Y. Then $f^{-1}(B)$ is α-multi fuzzy sub algebra of X.*

Proof. Let $x_1, x_2 \in X$

$$(f^{-1}(B))^\alpha(x_1 * x_2) = f^{-1}(B^\alpha)(x_1 * x_2)$$

$$= B^\alpha(f(x_1 * x_2))$$

$$= B^\alpha(f(x_1) * f(x_2))$$

$$\geq \min\{B^{\alpha}(f(x_1)), B^{\alpha}(f(x_2))\}$$

$$= \min\{f^{-1}(B^{\alpha})(x_1), f^{-1}(B^{\alpha})(x_2)\}$$

$$= \min\{(f^{-1}(B))^{\alpha}(x_1), (f^{-1}(B))^{\alpha}(x_2))\}.$$

Theorem 5.2. *Let $f : X \to Y$ be a homomorphism of a BG-algebra X into a BG-algebra Y. Let B be α-multi fuzzy normal sub algebra of Y. Then $f^{-1}(B)$ is α-multi fuzzy normal sub algebra of X.*

Theorem 5.3. *Let $f : X \to Y$ be a bijective homomorphism of a BG-algebra X onto a BG-algebra Y. Let A be α-multi fuzzy sub algebra of X. Then $f(A)$ is α-multi fuzzy subalgebra of Y.*

Proof. Let $y_1, y_2 \in Y$. Then there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$

$$(f(A))^{\alpha}(y_1 * y_2) = \min\{f(A)(y_1 * y_2), \alpha\}$$

$$= \min\{f(A)(f(x_1) * f(x_2)), \alpha\}$$

$$= \min\{f(A)(f(x_1 * x_2)), \alpha\}$$

$$= \min\{A(x_1 * x_2), \alpha\} = A^{\alpha}(x_1 * x_2)$$

$$\geq \min\{A^{\alpha}(x_1), A^{\alpha}(x_2)\}$$

for all $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$

$$= \min\{\min\{A^{\alpha}(x_1) \mid f(x_1) = y_1\}, \min\{A^{\alpha}(x_2) \mid f(x_2) = y_2\}\}$$

$$= \min\{f(A^{\alpha})(y_1), f(A^{\alpha})(y_2)\}$$

$$= \min\{f(A)^{\alpha}(y_1), f(A)^{\alpha}(y_2)\}.$$

Theorem 5.4. *Let $f : X \to Y$ be a bijective homomorphism of a BG-algebra X onto a BG-algebra Y. Let A be α-multi fuzzy normal sub algebra of X. Then $f(A)$ is α-multi fuzzy normal sub algebra of Y.*

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