Advances and Applications in Mathematical Sciences
Volume 21, Issue 5, March 2022, Pages 2911-2934

A STUDY ON DOMINATION IN PRODUCT PICTURE FUZZY GRAPH AND ITS APPLICATION

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Abstract

Fuzzy graph algorithms can be used to model and solve a wide range of practical issues. Fuzzy graph theory, in general, has a wide range of applications in a wide range of domains. Because ambiguous information is a common real-life problem that is frequently uncertain, an expert must model these challenges using a fuzzy graph. A useful mathematical model for dealing with uncertain real-world circumstances is the picture fuzzy set (PFS). The PFS is a variant of the traditional fuzzy set. It can be particularly effective in ambiguous settings that require more yes, no, abstain, and rejection responses. This research introduces the concept of a product picture fuzzy graph (PPFG). Some varieties of PPFGs are discussed, including strong PPFG and complete PPFG, as well as their features. One of the most extensively utilized notions in numerous areas is dominance in fuzzy graphs theory. Many current research investigations are attempting to uncover new uses for dominance in their sector if there is enough interest. As a result, in this paper, we introduce various types of dominating sets in PPFGs, such as the fixed vertex dominating set (FVDS), fixed edge dominating set (FEDS), total fixed edge dominating set (TFEDS), and fixed edge restrained dominating set (FERDS), and try to represent their properties using examples. Finally, we give a medical example to demonstrate the importance of FVD in PPFGs.

2020 Mathematics Subject Classification: 05C72.
Keywords: Product picture fuzzy graph, Fixed vertex dominating set, Fixed edge dominating set, Total fixed edge dominating set, Fixed edge restrained dominating set.
Received November 11, 2021; Accepted December 22, 2021

1. Introduction

Zadeh [1] introduced fuzzy set theory and related fuzzy logic as a technique of dealing with and addressing a wide range of situations in which variables, parameters, and relationships are only approximated, necessitating the employment of approximate reasoning systems. This is true of practically all nontrivial occurrences, processes, and systems that exist in reality, and standard binary logic mathematics cannot sufficiently characterize them. We summarise Gorzalczany's work on IVFSs [2] and Roy et al., [3] works on fuzzy relations because interval-valued fuzzy sets are commonly employed. IFGs were first proposed by R. Parvathi et al. [4]. IFG operations were investigated by R. Parvathi et al. [5]. In IFGs, A. N. Gani [6] developed the concepts of degree, order, and size. S. Samanta and M. Pal [7] have also expressed many fuzzy graphs. H. Rashmanlou and M. Pal [8] advised irregular IVFGs. Cuong [9] proposed a PFS ranking method as well as a set of PFS attributes. Many picture fuzzy relation compositions were proposed by Phong et al., [10]. Singh [11] campaigned for PFSs.

S. Sahoo and M. Pal [12] pioneered the concept of goods on IFGs. Son [13] talked about image fuzzy clustering. Dinesh [14] looked at the topic of fuzzy incidence graphs (FIGs). Peng and Dai [15] suggested and implemented an algorithmic technique for PFS in a decision-making situation. Wei [16] has presented a strategy for determining decisions. Kalaiarasi et al., [17] have also articulated fuzzy strong graphs. S. Sahoo et al., [18] proposed the concept of multiple IFGs. S. Mathew and J. N. Mordeson [19] proposed concepts in FIGs. Wei [20] talked about how to measure photo fuzzy sets. Mean operators and their applications have been extended by Wang et al., [21]. Cen Zuo, Anita Pal, and Arindam Dey [22] discussed PFGs. S. Sahoo et al., [23] presented a fuzzy graph with application.

Ore and Berge were the first to introduce dominance. Irfan Nazeer et al., [24], developed the new graph's product. Notions of PFG were addressed by Rukhshanda Anjum [25] et al. Bipolar PFGs have been created by Waheed Ahmad Khan, Babir Ali, and Abdelghani Taouti [26]. Haynes and Hedetniemi [27] looked into dominance in graphs further. Somasundaram and Somasundaram [28] have gained supremacy in fuzzy graphs by utilising effective edges. In fuzzy graphs, Xavior et al., [29] suggested dominance. Pradip Debnath [30] has also shown dominance in IVFGs. For fuzzy graphs, Revathi and Harinarayaman [31] developed an equitable domination number. Sunitha and Manjusha [32] have also declared that they have a stronghold. A. Nagoorgani and V. T. Chandrasekaran [33] have also demonstrated dominance in a fuzzy graph. Sarala and Kavitha [34] have also expressed (1, 2)-domination for fuzzy graphs. Dharmalingam and Nithya [35] have also provided dominance values for fuzzy graphs. O. T. Manjusha et al. [36] have studied paired domination. Bozhenyuk et al. [37] have discussed the dominant set. In FIGs, Irfan Nazeer et al. [38] have achieved dominance.

A. N. Shain and MMQ Shubatah [39] advocated the inverted dominating set of IVFGs. A new path graph definition was proposed by Tushar et al. [40]. A. Nagoor Gani et al. [41] addressed domination in PFG using fuzzy graphs. A. M. Ismayil and HS Begum [42] have both accurately depicted split dominance. In ambiguous graphs, Yongsheng Rao et al. [43] established dominance. In an uncertain context, a fuzzy graph was utilized to model a decision-making dilemma. Because uncertainties are effectively expressed using picture fuzzy sets, PFG would be an attractive study area for modeling uncertain optimization challenges. This prompted us to present the concept of a PPFG and its associated operations. The definition of PPFG is proposed in this paper. We also go through the many varieties of PPFG, such as regular PPFG, strong PPFG, and complete PPFG. Many emergency patients have died in the past as a result of transportation delays to the hospital. As a result, we present an example from the transportation sector to show how dominance in PPFGs is important.

2. Preliminaries

Definition 1 [22]**.** Let A_P be a PFS. A_P in X_P is denoted by $A_P = \{x_P, \mu_{A_P}(x_P), \eta_{A_P}(x_P), \gamma_{A_P}(x_P) / x_P \in X_P\}$, where $\mu_{A_P}(x_P), \eta_{A_P}(x_P)$ and $\gamma_{A_P}(x_P)$ follow the condition $0 \le \mu_{A_P}(x_P) + \eta_{A_P}(x_P) + \gamma_{A_P}(x_P) \le 1$. The $\mu_{A_P}(x_P), \eta_{A_P}(x_P), \gamma_{A_p}(x_P) \in [0, 1]$, denote respectively the positive membership degree, neutral membership degree and negative membership degree of the element x_P in the set A_P. For each PFS A_P in X_P, the refusal membership degree is described as $\pi_{A_P}(x_P) = 1 - \{\mu_{A_P}(x_P) + \eta_{A_P}(x_P) + \gamma_{A_P}(x_P)\}$.

Definition 2 [22]**.** Assume $G^* = (V, E)$ is a graph. A pair $G_P = (A_P, B_P)$ is called a PFG on G^* where $A_P = \{\mu_{A_P}, \eta_{A_P}, \gamma_{A_P}\}$ is a PFS on V_P and $B_P = \{\mu_{B_P}, \eta_{B_P}, \gamma_{B_P}\}$ is PFS on $E_P \subseteq V_P \times V_P$ such that for each edge $u_P v_P \in E_P \cdot \mu_{B_P}(u_P v_P) \leq \min(\mu_{A_P}(u_P), \mu_{A_P}(v_P)), \eta_{B_P}(u_P v_P) \leq \min(\eta_{A_P}(u_P), \eta_{A_P}(v_P)), \gamma_{B_P}(u_P v_P) \geq \max(\gamma_{A_P}(u_P), \gamma_{A_P}(v_P))$.

Example 1. Consider a PFG G_P as in Figure 1, such that $V_P = \{u_P, v_P, w_P\} E_P = \{u_P v_P, v_P w_P, w_P u_P\}$.

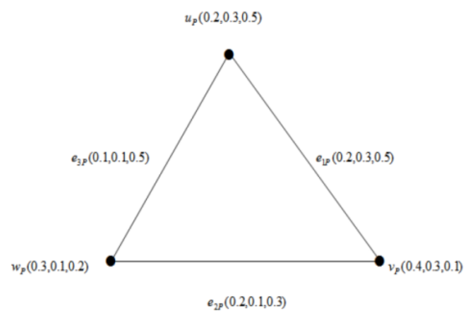

Figure 1. PFG G_P.

Definition 3 [22]**.** A PFG $G_P = (A_P, B_P)$ is said to be strong PFG if $\mu_{B_P}(u_P v_P) = \min(\mu_{A_P}(u_P), \mu_{A_p}(v_P)), \eta_{B_P}(u_P v_P) = \min(\eta_{A_P}(u_P), \eta_{A_P}(v_P)), \gamma_{B_P}(u_P v_P) = \max(\gamma_{A_P}(u_P), \gamma_{A_P}(v_P)) \forall u_P v_P \in V_P$.

Definition 4 [22]**.** A PFG $G_P = (A_P, B_P)$ is defined as complete PFG if $\mu_{B_P}(u_P v_P) = \min(\mu_{A_P}(u_P), \mu_{A_p}(v_P)), \eta_{B_P}(u_P v_P) = \min(\eta_{A_P}(u_P), \eta_{A_P}(v_P)), \gamma_{B_P}(u_P v_P) = \max(\gamma_{A_P}(u_P), \gamma_{A_P}(v_P)) \forall u_P v_P \in V_P$.

Definition 5 [41]**.** If $\mu_{B_P}(u_P v_P) \geq \mu^{\infty}_{B_P}(u_P v_P), \eta_{B_P}(u_P v_P) \geq \eta^{\infty}_{B_P}(u_P v_P)$ and $\gamma_{B_P}(u_P v_P) \geq \gamma^{\infty}_{B_P}(u_P v_P)$ for every $u_P v_P \in V_P$, an edge $(u_P v_P)$ is called a strong edge, where $\mu^{\infty}_{B_P}(u_P v_P), \eta^{\infty}_{B_P}(u_P v_P)$ and $\gamma^{\infty}_{B_P}(u_P v_P)$ are the strength of the connectedness between u_P and v_P in the PFG produced from G_P by removing the edge $(u_P v_P)$.

Example 2. In figure 1, e_{1P} and e_{2P} are strong edges.

Definition 6 [41]**.** Let $G_P = (A_P, B_P)$ be a PFG. Let $u_P v_P \in V_P$. Then u_P dominates v_P in G_P if there exists a strong edge between them.

Definition 7 [41]**.** A dominating set D_P of the PFG is said to be minimal PFDS if there is no proper subset of D_P is a PFDS.

Definition 8 [41]**.** The domination number of G_P, represented by $\gamma_P(G_P)$, is the lowest cardinality among all PFDSs.

Example 3. In figure 1, $\{u_P, v_P\}$ and $\{w_P, v_P\}$ are minimal PFDSs and $\gamma_P(G_P) = 0.7$.

3. Product Picture Fuzzy Graphs

Notation	-	Meaning
PFS	-	Picture Fuzzy Set
PFG	-	Picture Fuzzy Graph
PPFS	-	Product Picture Fuzzy Set
PPFG	-	Product Picture Fuzzy Graph
EIS	-	Edge Independent Set
ECS	-	Edge Covering Set
PFDS	-	Picture Fuzzy Dominating Set
FVDS	-	Fixed Vertex Dominating Set
FEDS	-	Fixed Edge Dominating Set
FEDN	-	Fixed Edge Domination Number
IE	-	Isolated Edge
TFEDS	-	Total Fixed Edge Dominating Set
FERDS	-	Fixed Edge Restrained Dominating Set

ERIS	-	Edge Restrained Independent Set
IVFG	-	Interval
FIG	-	Fuzzy Incidence Graph
IFG	-	Intuitionistic Fuzzy Graph

Definition 9. Let R_{PP} be a PPFS. R_{PP} in X_{PP} is defined by $R_{PP} = \{x_{PP}, \mu_{R_{PP}}(x_{PP}), \eta_{R_{PP}}(x_{PP}), \gamma_{R_{PP}}(x_{PP}) / x_{PP} \in X_{PP}\}$, where $\mu_{R_{PP}}(x_{PP}), \eta_{R_{PP}}(x_{PP})$ and $\gamma_{R_{pp}}(x_{PP})$ follow the condition $0 \leq \mu_{R_{PP}}(x_{PP}) \times \eta_{R_{PP}}(x_{R_{PP}}) \times \gamma_{R_{PP}}(x_{PP}) \leq 1$. The $\mu_{R_{PP}}(x_{PP}), \eta_{R_{PP}}(x_{PP}), \gamma_{R_{PP}}(x_{PP}) \in [0, 1]$, denote respectively the positive membership degree, neutral membership degree and negative membership degree of the element x_{pp} in the set R_{PP}. For each PPFS R_{PP} in X_{PP}, the refusal membership degree is described as $\pi_{R_{PP}}(x_{PP}) = 1 - \{\mu_{R_{PP}}(x_{PP}) \times \eta_{R_{PP}}(x_{PP}) \times \gamma_{R_{PP}}(x_{PP})\}$.

Definition 10. Assume $G^*_{PP} = (V_{PP}, E_{PP})$ is a graph. A pair $\xi_{PP} = (R_{PP}, S_{PP})$ is referred a PPFG on G^*_{PP} where $R_{PP} = \{\mu_{R_{PP}}, \eta_{R_{PP}}, \gamma_{R_{PP}}\}$ is a PPFS on V_{PP} and $S_{PP} = \{\mu_{S_{PP}}, \eta_{S_{PP}}, \gamma_{S_{PP}}\}$ is a PPFS on $E_{PP} \subseteq V_{PP} \times V_{PP}$ such that for each edge $f_{PP}h_{PP} \in E_{PP}$.

$$\mu_{S_{PP}}(f_{PP}h_{PP}) \leq \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}), \eta_{S_{PP}}(f_{PP}h_{pp}) \leq \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}), \gamma_{S_{PP}}(f_{PP}h_{pp}) \geq \gamma_{R_{PP}}(f_{pp}) \times \gamma_{R_{PP}}(h_{PP}).$$

Example 4. Consider a PPFG ξ_{PP} as in Figure 2, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}, j_{PP}\}$ $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}j_{PP}, j_{PP}f_{PP}\}$.

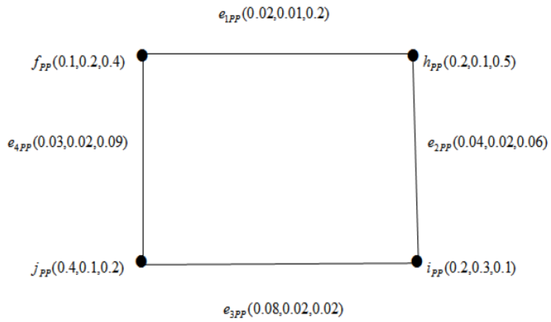

Figure 2. PPFG ξ_{PP}.

Note 1. There is no edge between f_{PP} and h_{PP} when $\mu_{S_{PP}}(f_{PP}h_{PP}) = \eta_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{S_{PP}}(f_{PP}h_{PP}) = 0$

Remark 1. A PPFG is not necessarily a PFG.

Example 5. In Figure 2, it is easy to show that PPFG but not PFG.

Definition 11. A PPFG $\xi_{PP} = (R_{PP}, S_{PP})$ is said to be strong PPFG if $\mu_{S_{PP}}(f_{PP}h_{PP}) = \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP})$, $\eta_{S_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP})$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}) \forall f_{PP}h_{PP} \in E_{PP}$.

Example 6. Consider a strong PPFG ξ_{PP} as in Figure 3, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}, j_{PP}\}$ $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}j_{PP}, j_{PP}f_{PP}, f_{PP}i_{PP}\}$.

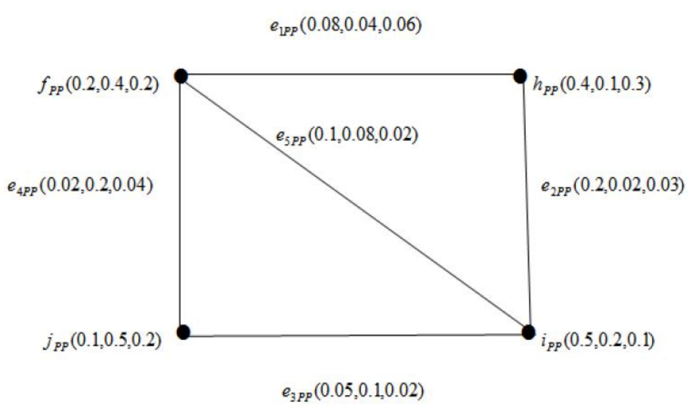

Figure 3. Strong PPFG ξ_{PP}.

Definition 12. A PPFG $\xi_{PP} = (R_{PP}, S_{PP})$ is defined as complete PPFG if $\mu_{S_{PP}}(f_{PP}h_{PP}) = \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP})$, $\eta_{S_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP})$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}) \forall f_{PP}h_{PP} \in E_{PP}$.

Example 7. Consider a complete PPFG ξ_{PP} as in Figure 4, such that $V_{PP} = \{f_{PP}, h_{PP}, i_{PP}, j_{PP}\}$ $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}j_{PP}, j_{PP}f_{PP}, f_{PP}i_{PP}, h_{PP}j_{PP}\}$.

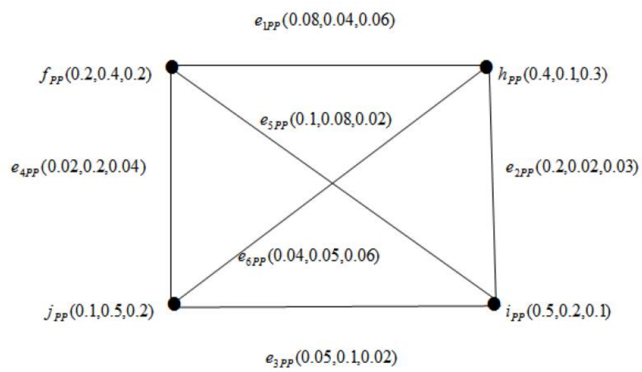

Figure 4. Complete PPFG ξ_{PP}.

Remark 2. Every complete PPFG is a strong PPFG but not conversely.

Example 8. In Figure 3, it is simple to demonstrate that ξ_{PP} is a strong PPFG but not a complete PPFG.

Definition 13. Let $\xi_{PP} = (R_{PP}, S_{PP})$ be a PPFG

(i) ξ_{PP} cardinality is determined by

$$| \xi_{PP} | = \sum_{f_{iPP} \in V_{PP}} \frac{1 + \mu_{R_{PP}}(f_{ipp}) - \eta_{R_{PP}}(f_{iPP}) - \gamma_{R_{PP}}(f_{iPP})}{2}$$

$$\sum_{f_{iPP},\, h_{iPP} \in E_{PP}} \frac{1 + \mu_{S_{PP}}(f_{iPP}, h_{iPP}) - \eta_{S_{PP}}(f_{iPP}, h_{iPP}) - \gamma_{S_{PP}}(f_{iPP}, h_{iPP})}{2}$$

(ii) ξ_{PP} vertex cardinality is determined by

$$|V_{PP}| = \sum_{f_{iPP} \in V_{PP}} \frac{1 + \mu_{R_{PP}}(f_{iPP}) - \eta_{R_{PP}}(f_{iPP}) - \gamma_{R_{PP}}(f_{iPP})}{2} \forall f_{iPP} \in V_{PP},$$ is

referred the order of a PPFG ξ_{PP}, and it is denoted by $p(\xi_{PP})$

(iii) ξ_{PP} edge cardinality is specified by

$$|E_{PP}| = \sum_{f_{iPP},\, h_{iPP} \in E_{PP}} \frac{1 + \mu_{S_{PP}}(f_{iPP}, h_{iPP}) - \eta_{S_{PP}}(f_{iPP}, h_{iPP}) - \gamma_{S_{PP}}(f_{iPP}, h_{iPP})}{2}$$

$\forall f_{iPP}, h_{iPP} \in E_{PP}$, is referred the size of a PPFG ξ_{PP}, and it is denoted by $q(\xi_{PP})$.

Example 9. In Figure 2, $|V_{PP}| = 0.25 + 0.3 + 0.4 + 0.55 = 1.5$

$$|E_{PP}| = 0.405 + 0.48 + 0.52 + 0.46 = 1.865$$

$$|\xi_{PP}| = 0.5 + 1.865 = 3.365.$$

Definition 14. An edge f_{PP}, h_{PP} in a PPFG $\xi_{PP} = (R_{PP}, S_{PP})$ is called the strong edge if $\mu_{S_{PP}}(f_{PP}, h_{PP}) \geq \mu^{\infty}_{S_{PP}}(f_{PP}, h_{PP})$, $\eta_{S_{PP}}(f_{PP}, h_{PP}) \geq \eta^{\infty}_{S_{PP}}(f_{PP}, h_{PP})$, $\gamma_{S_{PP}}(f_{PP}, h_{PP}) \leq \gamma^{\infty}_{S_{PP}}(f_{PP}, h_{PP})$.

Example 10. In figure 2, $e_{2PP}, e_{3PP}, e_{4PP}$ are strong edges

4. Fixed Vertex Domination in PPFG

Definition 15. In a PPFG, two vertices f_{PP} and h_{PP} are considered to be neighbors if one of the following conditions holds.

(i) $\mu_{S_{PP}}(f_{PP}h_{PP}) > 0$, $\eta_{S_{PP}}(f_{PP}h_{PP}) > 0$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$

(ii) $\mu_{S_{PP}}(f_{PP}h_{PP}) = 0$, $\eta_{S_{PP}}(f_{PP}h_{PP}) \geq 0$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$

(iii) $\mu_{S_{PP}}(f_{PP}h_{PP}) > 0$, $\eta_{S_{PP}}(f_{PP}h_{PP}) = 0$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$

(iv) $\mu_{S_{PP}}(f_{PP}h_{PP}) \geq 0$, $\eta_{S_{PP}}(f_{PP}h_{PP}) > 0$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) > 0$ $\forall f_{PP}h_{PP} \in V_{PP}$.

Definition 16. In a PPFG ξ_{PP}, the two vertices f_{PP} and h_{PP} are considered to be strong neighbors if $\mu_{S_{PP}}(f_{PP}, h_{PP}) = \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP})$, $\eta_{S_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP})$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP})$.

Definition 17. Let ξ_{PP} be a PPFG and f_{PP} and h_{PP} are neighbors of ξ_{PP}. We say that f_{PP} dominates h_{PP} if they are strong neighbors. An vertex subset M_{PP} of V_{PP} in a PPFG ξ_{PP} is called an fixed vertex dominating set, if for each vertex $V_{PP} - M_{PP}$ is dominates exactly one vertex in M_{PP}. An FVDS M_{PP} of a PPFG ξ_{PP} is said to be a minimal FVDS if for each edge $f_{PP} \in M_{PP}$, $M_{PP} - \{f_{PP}\}$ is not an FVDS. An FVDN of ξ_{PP} is the smallest cardinality between all minimal FVDSs, and it is described by $\gamma_{VPP}(\xi_{PP})$ or simply γ_{VPP}.

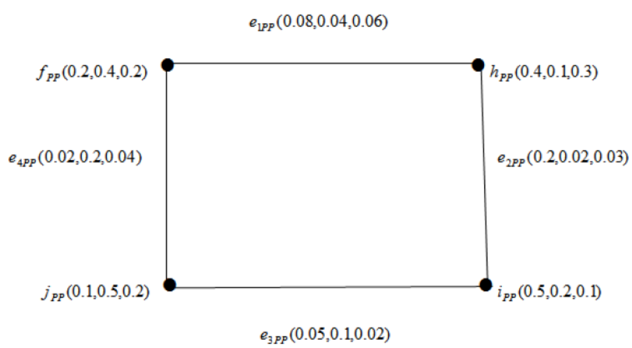

Figure 5. PPFG ξ_{PP} with FVDS.

Example 11. Consider the PPFG ξ_{PP} as in figure 5, $D_{11} = \{f_{PP}, h_{PP}\}$, $D_{22} = \{h_{PP}, i_{PP}\}$, $D_{33} = \{i_{PP}, j_{PP}\}$, $D_{44} = \{f_{PP}, j_{PP}\}$ is a fixed vertex dominating sets and $\gamma_{VPP}(\xi_{PP}) = 0.5$.

5. Fixed Edge Domination in PPFG

Definition 18. If two edges e_{1PP} and e_{2PP} in a PPFG ξ_{PP} are neighbors, they are said to be adjacent.

Definition 19. An edge subset L_{PP} of E_{PP} in a PPFG ξ_{PP} is referred an edge independent set (EIS) if $\mu_{S_{PP}}(f_{PP}h_{PP}) < \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP})$, $\eta_{S_{PP}}(f_{PP}h_{PP}) < \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP})$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) > \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}$ $(h_{PP}) \forall\ f_{PP}h_{PP} \in L_{PP}$. The EIN is the highest cardinality among all maximal EIS in ξ_{PP}, and it is indicated by $\beta_{IPP}(\xi_{PP})$ or β_{IPP}.

Example 12.

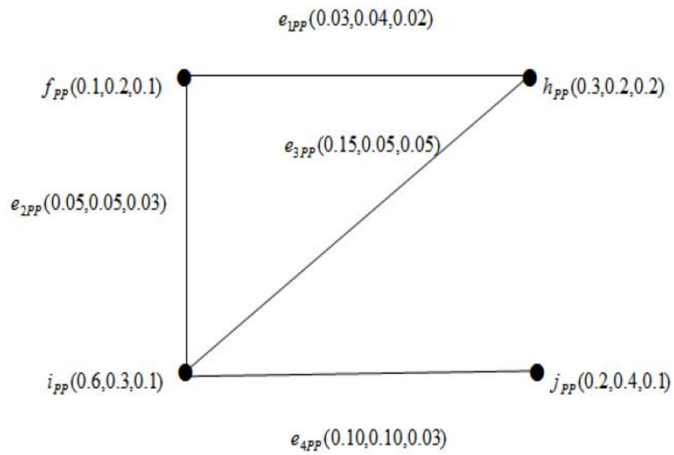

Figure 6. PPFG ξ_{PP} with EISs.

In figure 6, $\{e_{2PP}, e_{3PP}\}, \{e_{2PP}, e_{4PP}\}, \{e_{3PP}, e_{4PP}\}, \{e_{2PP}, e_{3PP}, e_{4PP}\}$ are EISs in ξ_{PP} and $\beta_{IPP}(\xi_{PP}) = 1.495$.

Definition 20. If an edge e_{1PP} and a vertex k_{PP} in a PPFG ξ_{PP} are incident, they are said to cover each other.

Definition 21. An edge subset ξ_{PP} of E_{PP} in a PPFG ξ_{PP}, which covers all nodes in ξ_{PP}, is termed a edge cover set (ECS) of ξ_{PP}. The edge cover number (ECN) of ξ_{PP} is the lowest cardinality among all ECS, and it is denoted by $\alpha_{CPP}(\xi_{PP})$ or simply α_{CPP}.

Example 13. Consider the PPFG ξ_{PP} in figure 2

Here $\{e_{1PP}, e_{3PP}\}$ and $\{e_{2PP}, e_{4PP}\}$ are ECSs and $\alpha_{CPP}(\xi_{PP}) = 0.925$.

Theorem 1. *An edge subset* $L_{PP} \subseteq E_{PP}$ *in a PPFG* ξ_{PP} *is an EIS in* ξ_{PP} *if* $E_{PP} - L_{PP}$ *is an ECS of* ξ_{PP}.

Proof. L_{PP} is an EIS if and only if no two of its edges are adjacent, if and only if each of its edges is incident with at least one vertex of $E_{PP} - L_{PP}$, and if and only if $E_{PP} - L_{PP}$ is an ECS of ξ_{PP}.

Example 14.

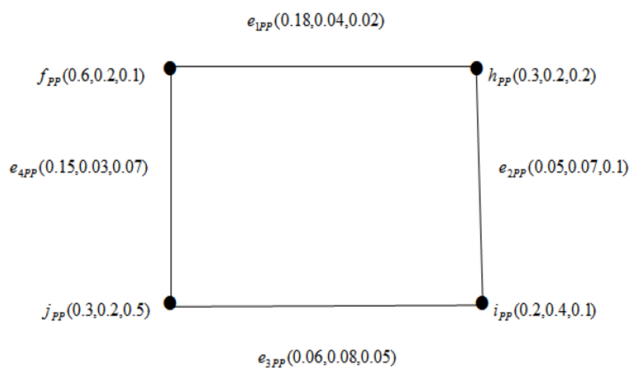

Figure 7. PPFG ξ_{PP} with EIS and ECS.

Consider the PPFG ξ_{PP} as in figure 7. It is easy to show that $L_{PP} = \{e_{2PP}, e_{4PP}\}$ is an EIS and $E_{PP} - L_{PP} = \{e_{1PP}, e_{3PP}\}$ is an ECS.

Definition 22. An edge $f_{PP}h_{PP}$ in a PPFG ξ_{PP} is labeled an effective edge if $\mu_{S_{PP}}(f_{PP}h_{PP}) = \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP})$, $\eta_{S_{PP}}(f_{PP}h_{PP}) = \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP})$, $\gamma_{S_{PP}}(f_{PP}h_{PP}) = \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP})$.

Example 15. Consider a PPFG ξ_{PP} as in Figure 8, such that $V_{PP} = \{f_{PP}h_{PP}, i_{PP}\}$ $E_{PP} = \{f_{PP}h_{PP}, h_{PP}i_{PP}, i_{PP}f_{PP}\}$.

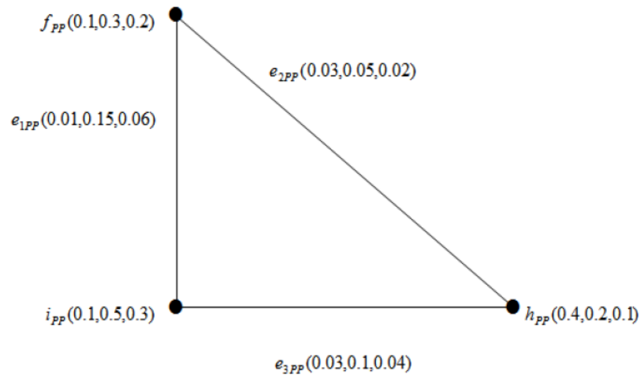

Figure 8. PPFG ξ_{PP} with effective edge.

Here e_{1PP} is an effective edge.

$\mu_{S_{PP}}(f_{PP}i_{PP}) = 0.1 \times 0.1 = 0.01$, $\eta_{S_{PP}}(f_{PP}i_{PP}) = 0.3 \times 0.5 = 0.15$,
$\gamma_{S_{PP}}(f_{PP}h_{PP}) = 0.2 \times 0.3 = 0.06$.

Definition 23. An edge e_{PP} of a PPFG ξ_{PP} is said to be an isolated edge (IE) if no effective edge are incident with the vertices of e_{PP}. As a result, no other edge in ξ_{PP} is dominated by an IE.

Example 16. In figure 7, the edges e_{1PP} and e_{3PP} are IEs.

Theorem 2. *For any PPFG* $\xi_{PP} = (R_{PP}, S_{PP})$ *with* IEs, $\alpha_{CPP} = (\xi_{PP}) + \beta_{IPP}(\xi_{PP}) = q_{PP}$.

Proof. Let L_{PP} be an EIS in ξ_{PP} and M_{PP} be an ECS in ξ_{PP} so that $|L_{PP}| = \beta_{IPP}(\xi_{PP})$ and $|M_{PP}| = \alpha_{CPP}(\xi_{PP})$. Then, by theorem 1, $E_{PP} - L_{PP}$ is an ECS of ξ_{PP}. Therefore, $|M_{PP}| \le |E_{PP} - L_{PP}|$ and $\alpha_{CPP}(\xi_{PP}) \le q_{PP} - \beta_{IPP}(\xi_{PP})$ or $\alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) \le q_{PP}$ (1)

Furthermore, by theorem 1, $E_{PP} - M_{PP}$ is an EIS in ξ_{PP}, so $|L_{PP}| \ge |E_{PP} - M_{PP}|$. Therefore, $\beta_{IPP}(\xi_{PP}) \ge q_{PP} - \alpha_{CPP}(\xi_{PP})$ or $\alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) \ge q_{PP}$ (2)

From (1) and (2), we obtain $\alpha_{CPP}(\xi_{PP}) + \beta_{IPP}(\xi_{PP}) = q_{PP}$.

Example 17. In figure 7, $\alpha_{CPP}(\xi_{PP}) = 1.025, \beta_{IPP}(\xi_{PP}) = 0.965$ and $q_{PP} = 1.99$.

Definition 24. Let e_{PP} be any edge in a PPFG ξ_{PP}. Then, $N(e_{PP}) = \{m_{PP} \in E_{PP} : m_{PP}$ is an effective edge incident with the nodes of $e_{PP}\}$ and is called the open degree neighborhood set of e_{PP}. $N[e_{PP}] = N(e_{PP}) \cup \{e_{PP}\}$ is named the closed neighborhood set of e_{PP}.

Definition 25. Let e_{PP} be any edge in a PPFG ξ_{PP}. Then, $d_N(e_{PP}) = \sum_{l_{PP} \in N(e_{PP})} | l_{PP} |$ is termed the edge neighborhood degree of e_{PP}. The minimum edge neighborhood degree of a PPFG ξ_{PP} is $\delta_N(\xi_{PP}) = \min\{d_N(e_{PP}) / e_{PP} \in E_{PP}\}$ The maximum edge neighborhood degree of a PPFG ξ_{PP} is $\Delta_N(\xi_{PP}) = \max\{d_N(e_{PP}) / e_{PP} \in E_{PP}\}$.

Example 18. Consider the PPFG ξ_{PP} as in figure 3. It is obvious that $N(e_{1PP}) = \{e_{2PP}, e_{4PP}, e_{5PP}\}$ and $d_N(e_{1PP}) = 1.465$.

Definition 26. Two edges e_{1PP} and e_{2PP} in a PPFG ξ_{PP}, are said to be strong neighbor if they are effective edges.

Definition 27. Let ξ_{PP} be a PPFG and e_{1PP} and e_{2PP} be two edges of ξ_{PP}. We say that e_{1PP} dominates e_{2PP} if e_{1PP} is effective edge and they are adjacent. An edge subset L_{PP} of E_{PP} in a PPFG ξ_{PP} is named a fixed edge dominating set (FEDS) if, for each edge $E_{PP} - L_{PP}$ is dominates exactly one edge in L_{PP}. An FEDS L_{PP} of a PPFG ξ_{PP} is said to be a minimal FEDS if for each edge $e_{PP} \in L_{PP}, L_{PP} - \{e_{PP}\}$ is not an FEDS. An FEDN of ξ_{PP} is the least cardinality between all minimal FEDSs and is denoted by $\gamma_{EPP}(\xi_{PP})$ or simply γ_{EPP}. An FEDS L_{PP} of a PPFG ξ_{PP} is said to be independent if $\mu_{S_{PP}}(f_{PP}h_{PP}) < \mu_{R_{PP}}(f_{PP}) \times \mu_{R_{PP}}(h_{PP}), \eta_{S_{PP}}(f_{PP}h_{PP}) < \eta_{R_{PP}}(f_{PP}) \times \eta_{R_{PP}}(h_{PP}), \gamma_{S_{PP}}(f_{PP}h_{PP}) > \gamma_{R_{PP}}(f_{PP}) \times \gamma_{R_{PP}}(h_{PP}) \; \forall \; f_{PP}h_{PP} \in L_{PP}$.

Example 19.

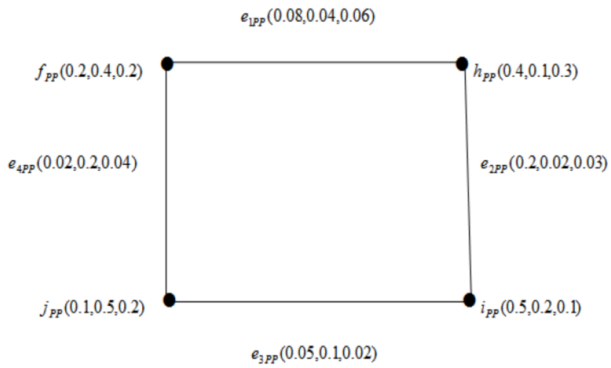

Figure 9. PPFG ξ_{PP} with FEDSs.

Consider the PPFG ξ_{PP} as in figure 9, $D_{11} = \{e_{1PP}, e_{2PP}\}$, $D_{22} = \{e_{2PP}, e_{3PP}\}$, $D_{33} = \{e_{3PP}, e_{4PP}\}$ and $D_{44} = \{e_{1PP}, e_{4PP}\}$ are FEDSs and $\gamma_{EPP}(\xi_{PP}) = 0.88$.

Theorem 3. *For any PPFG without IEs,* $\gamma_{EPP}(\xi_{PP}) \leq \frac{q_{PP}}{2}$.

Proof. Any PPFG without IEs has two disjoint FEDSs and hence $\gamma_{EPP}(\xi_{PP}) \leq \frac{q_{PP}}{2}$.

Example 20. Consider the PPFG ξ_{PP} as in figure 9 with $q_{PP} = 1.92$ and $\gamma_{EPP}(\xi_{PP}) = 0.88 < \frac{q_{PP}}{2} 0.96$.

Theorem 4. *An EIS* L_{PP} *of a PPFG* ξ_{PP} *is a maximal EIS iff it is an EIS and FEDS.*

Proof. Let L_{PP} be a maximal EIS in a PPFG ξ_{PP} and, hence for each edge $e_{PP} \in E_{PP} - L_{PP}$, the set $L_{PP} \cup \{e_{PP}\}$ is not independent. For each edge $e_{PP} \in E_{PP} - L_{PP}$ is dominated exactly one edge in L_{PP}. Hence, L_{PP} is an FEDS. Therefore, L_{PP} is both an FEDS and EIS.

Conversely, assume L_{PP} is both independent and an FEDS. Suppose that L_{PP} is not a maximal EIS, then there exist an edge $e_{PP} \in E_{PP} - L_{PP}$, and the set $L_{PP} \cup \{e_{PP}\}$ is independent. If $L_{PP} \cup \{e_{PP}\}$ is independent, then no effective edge in L_{PP} is strong neighbor to e_{PP}. Therefore, L_{PP} cannot be an FEDS, which is a contradiction. Thus, ξ_{PP} is a maximal EIS.

Example 21. Consider the PPFG ξ_{PP} as in figure 10, $\{e_{2PP}, e_{3PP}\}$ is a maximal EIS that is both an EIS and FEDS.

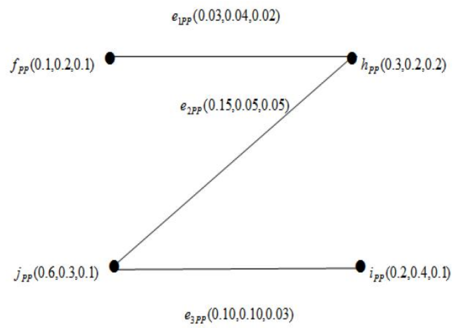

Figure 10. PPFG ξ_{PP} with EIS and FEDS.

Theorem 5. *Every maximal EIS L_{PP} in a PPFG ξ_{PP} is a minimal FEDS.*

Proof. Let L_{PP} be a maximal EIS in a PPFG ξ_{PP}. By theorem 4, L_{PP} is an FEDS. Assume L_{PP} is not a minimal FEDS. There exist at least one edge $e_{PP} \in L_{PP}$ for which $L_{PP} - \{e_{PP}\}$ is an FEDS. However, if $E_{PP} - \{L_{PP} - \{e_{PP}\}\}$ dominates $L_{PP} - \{e_{PP}\}$, then at least one edge in $E_{PP} - \{L_{PP} - \{e_{PP}\}\}$ must be strong neighbor to e_{PP}. This contradicts the fact that L_{PP} is an EIS in ξ_{PP}. Hence, L_{PP} must be a minimal FEDS.

6. Total Fixed Edge Domination in PPFG

Definition 28. Let $\xi_{PP} = (R_{PP}, S_{PP})$ be a PPFG without IEs. An edge subset L_{PP} of E_{PP} is said to be TFEDS if for each edge $e_{PP} \in E_{PP}$, there exist an edge $g_{PP} \in L_{PP}$, $g_{PP} \neq e_{PP}$, so that e_{PP} dominates exactly one edge in L_{PP} and the corresponding vertex for each edge in L_{PP} have same degree.

Definition 29. The TFEDN of ξ_{PP} is represented by TPP $\gamma_{TPP}(\xi_{PP})$ and is the smallest cardinality among all TFEDSs.

Theorem 6. *Let $\xi_{PP} = (R_{PP}, S_{PP})$ be any PPFG without IEs. Then, for each minimal TFEDS L_{PP}, $E_{PP} - L_{PP}$ is also an TFEDS.*

Proof. Let e_{PP} be any edge in L_{PP}. Since ξ_{PP} has no IEs, there is an edge $g_{PP} \in N(e_{PP})$ and $g_{PP} \in E_{PP} - L_{PP}$. Hence, each element of $E_{PP} - L_{PP}$ is dominated exactly one edge in L_{PP} and the corresponding vertex for each edge in L_{PP} have same degree. Thus $E_{PP} - L_{PP}$ is an TFEDS in ξ_{PP}.

7. Fixed Edge Restrained Domination in PPFG

Definition 30. Let $\xi_{PP} = (R_{PP}, S_{PP})$ be a PPFG. An edge subset $L_{PP} \subseteq E_{PP}$ is called FERDS if

(i) Each edge in $E_{PP} - L_{PP}$ is dominates exactly one edge in L_{PP}.

(ii) In L_{PP}, all of the edges have the equal degree.

Example 22. In figure 11, $D_{11} = \{e_{1PP}, e_{2PP}\}$, $D_{22} = \{e_{2PP}, e_{3PP}\}$, $D_{33} = \{e_{3PP}, e_{4PP}\}$ and $D_{44} = \{e_{1PP}, e_{4PP}\}$ are FERDSs.

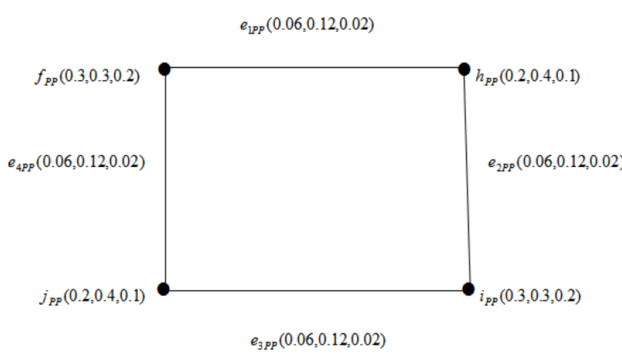

Figure 11. PPFG ξ_{PP} with FERDS.

Definition 31. An edge independent set L_{PP} of a PPFG ξ_{PP} is labeled an ERIS if all the edges of L_{PP} have the equal degrees. L_{PP} is a maximal ERIS if for every $f_{PP} \in V_{PP} - L_{PP}$, and the set $L_{PP} \cup \{f_{PP}\}$ is not an ERIS.

Example 23. Consider the PPFG ξ_{PP} as in figure 12, $D_{11} = \{e_{1PP}, e_{4PP}\}$ is a ERIS.

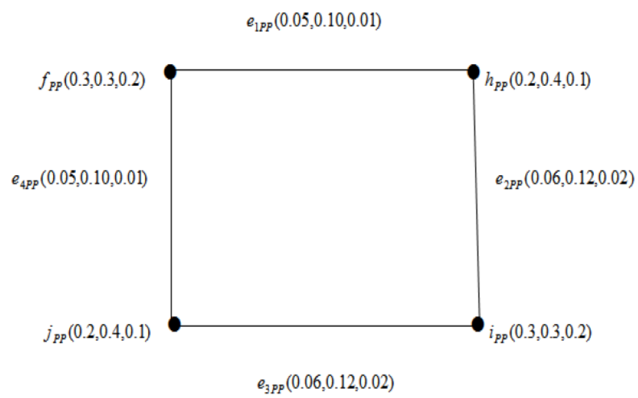

Figure 12. PPFG ξ_{PP} with ERIS.

Theorem 7. *An ERIS is a maximal ERIS of a PPFG* ξ_{PP} iff *it is an ERIS and FERDS.*

Proof. Let L_{PP} be a maximal ERIS in a PPFG ξ_{PP}, then for each $t_{PP} \in E_{PP} - L_{PP}$, the set $L_{PP} \cup \{t_{PP}\}$ is not an independent set, that is for every $t_{PP} \in E_{PP} - L_{PP}$, there exist a edge $n_{PP} \in L_{PP}$ so that t_{PP} dominates n_{PP}. Therefore, L_{PP} is a FERDS of ξ_{PP} and also an ERIS of ξ_{PP}. Therefore, L_{PP} is an ERIS and FERDS.

Conversely, assume that L_{PP} is both an ERIS and FERDS of ξ_{PP}. We have to prove that L_{PP} is a maximal ERIS. Suppose that L_{PP} is not a maximal independent set. Then, there exist a edge $t_{PP} \notin L_{PP}$ so that $L_{PP} \cup \{t_{PP}\}$ is an independent set, there is no edge in L_{PP} strong neighbor to t_{PP}, and hence, t_{PP} is not dominated any edge in L_{PP}. Thus, L_{PP} cannot be a FERDS of ξ_{PP}, which is a contradiction. Therefore, L_{PP} is a maximal ERIS.

8. Application

Many emergency accident patients have died in the past as a result of transportation delays to the hospital. One of the elements driving this delay is traffic congestion in cities. As a result, we attempted to find the closest hospitals in our study based on distance, traffic load, and patient suggestions. We evaluate five hospitals located in diverse locations along the by-pass road for this purpose. The hospital is depicted as A_{22}, A_{33}, A_{44}, A_{55}, A_{66}. In this PPFG, one vertex (A_{11}) represents the accident site, while the other vertices correspond to hospitals located throughout the bypass road.

The vertex $A_{22}(0.2, 0.1, 0.1)$ indicates that it has 20% of the essential amenities for treating the patient, but only 10% of the required equipment and only 10% of patient referrals to the proper hospital. The edge $A_{11}A_{22}$ denotes a 4% distance between the accident site and the hospital, a 4% low traffic load on the patient's ambulance transport route to the hospital, and a 2% heavy traffic load on the patient's ambulance transport route to the hospital. The FVDSs for figure 13 are as follows.

$D_{1PP} = \{A_{11}A_{44}\}$, $D_{2PP} = \{A_{11}A_{33}\}$, $D_{3PP} = \{A_{11}A_{55}\}$, $D_{4PP} = \{A_{11}A_{66}\}$ $D_{5PP} = \{A_{11}A_{22}A_{33}\}$.

After calculating the cardinality of $D_{1PP}, D_{2PP}, D_{3PP}, D_{4PP}$ and D_{5PP} we obtain $|D_{1PP}| = 0.5$, $|D_{2PP}| = 0.6$, $|D_{3PP}| = 0.9$, $|D_{4PP}| = 0.8$, $|D_{5PP}| = 1$.

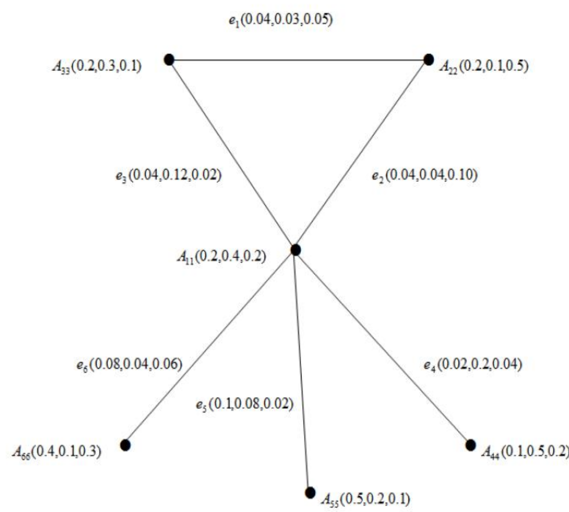

Figure 13. PPFG ξ_{PP}.

Because D_{1PP} has the smallest size among the other FVDS, we conclude that it is the best option because it allows the ambulance to travel from the accident scene to the hospital A_{44} with more free space, allowing it to transport patients to their desired location faster, saving our lives, time, and money. Second, hospital A_{44} offers a tremendous medical services than some other hospitals. As a result, the government should invest more money on improving intercity routes and traffic control so that ambulances can deliver patients to specialist hospitals swiftly.

9. Analytical Comparison

Our investigation will be fruitful in fully comprehending the additional properties of FVD in PPFG. We applied the model to FVD in PPFG (figure 13) and domination in PFG (example 24) and obtained the following results. In figure 13, $\gamma_{VPP}(\xi_{PP}) = 0.5$ and example 24, $\gamma_P(G_P) = 0.6$. Here $\gamma_{VPP} < \gamma_P$. As a result of this explanation, the current model is useful for estimating the best hospital in approximate. However, our method is effective in determining the best hospital in accurate. As a result, our proposed strategy outperforms the current method.

Example 24. Let G_P be a PFG with $A_{11} = (0.2, 0.4, 0.2)$, $A_{22} = (0.2, 0.1, 0.5)$, $A_{33} = (0.2, 0.3, 01)$, $A_{44} = (0.1, 0.5, 0.2)$, $A_{55} = (0.5, 0.2, 0.1)$, $A_{66} = (0.4, 0.1, 0.3)$, $e_{1P} = (0.2, 0.1, 0.5)$, $e_{2P} = (0.2, 0.1, 0.5)$, $e_{3P} = (0.2, 0.3, 0.2)$, $e_{4P} = (0.1, 0.4, 0.2)$, $e_{5P} = (0.2, 0.2, 0.2)$, $e_{6P} = (0.2, 0.1, 0.3)$ and the PFDSs are $D_{1P} = \{A_{11}A_{22}\}$, $D_{2P} = \{A_{11}A_{33}\}$, $D_{3P} = \{A_{22}A_{33}\}$ with $\gamma_P(G_P) = 0.6$.

10. Conclusion

In a range of domains, a fuzzy graph is a useful tool for replicating a variety of uncertain real-world decision-making difficulties. A direct extension of fuzzy set and PFS is the PPFS. We also go through some of the many forms of PPFG, such as strong PPFG and complete PPFG. When compared to traditional fuzzy graph models, the PPFG can boost flexibility, efficiency, precision, and comparability when modeling complicated real-world settings. One of the most commonly discussed subjects in numerous sciences, artificial intelligence, and other fields is dominance in fuzzy graphs. As a result, we describe numerous types of DSs in PPFGs in this study, such as FVDS, FEDS, TFEDS, and FERDS. We also establish the relationship between EISs and ECSs by presenting the attributes of each through numerous examples. Finally, we discussed how dominance can be used in the transportation system. The concept of a PPFG can be used to database systems, transportation networks, and image processing, among other things. The examination of new concepts of product picture bridges, product picture cycles, and product picture competition graphs, as well as their applications in medical sciences, will be the focus of future research.

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