Advances and Applications in Mathematical Sciences
Volume 21, Issue 3, January 2022, Pages 1373-1388

RELIABILITY UNDER PREVENTIVE MAINTENANCE

P. MALLIGA, C. MAHENDRAN, P. PRIYA and P. SUGANYA

Assistant Professor
Department of Mathematics
Dhanalakshmi Srinivasan College of Arts
and Science for Women (Autonomous)
Perambalur, India
E-mail: malligakavinesh@gmail.com
n.j.malini@gmail.com
priyaperiyasamy13@gmail.com

Abstract

The core analysis of a Reliability Centered Maintenance (RCM) process for an offshore wind turbine. The aim is to provide an engineering guide which can improve the maintenance of the system, and consequently increases its availability and the production of energy. The initial investigations have been carried out using a database for an onshore 5 MW wind turbine; the data has then been converted using a proper conversion factor, so that it can be used for a 10 MW offshore turbine case. The reliability and availability of the entire offshore wind turbine have been calculated through Reliability Prediction and a Reliability Block Diagram (RBD). In addition, a failure mode analysis is done using FMECA, in order to identify the most important failure modes in a risk priority order, and to the note the effect of propagation of each functional failure. The maintenance part of the RCM analysis has also been studied, to facilitate the creation of an optimum packaging of preventive maintenance tasks, which can help to avoid the functional failures of items throughout the system. Although the main target of the RCM is to reduce the downtime of the wind turbine, a reduction in Life Cycle Costs can be also accomplished through this process.

Introduction

Probability theory a branch of mathematics concerted with the analysis of random phenomena. The actual outcome considered to be determined by change. The word probability relative frequencies coins, cards, dice and

2020 Mathematics Subject Classification: 60.
Keywords: Point availability, Uptime, downtime, Preventive maintenance.
Received October 30, 2021; Accepted November 10, 2021

wheels provides examples. It was developed in the early 19th century as the study of population, economics and moral action.

Reliability characteristics, such as probability of survival, mean time to failure, availability, mean down time frequency of failure are some of the measure of system effectiveness. If a component is put in to operation at some specified time and observed until it fails, then the time to fails or life length say $T(\geq 0)$ is a continuous random variable with some probability density function (pdf) $f(t)$. We introduce the concept to maintain ability and availability of the system.

Definition 1.1. Repair Maintenance. It comes under reactive maintenance. It is performed once the failure has occurred. This is usually a very expensive operation. When a failure occurs, it usually causes an emergency interruption of service and this must be repaired or replaced immediately.

Definition 1.2. Primary Maintenance. This type of maintenance is essentially the replacement of failed part with a good one. This purpose of this is to get the system up and running again as soon as possible.

Definition 1.3. Preventive Maintenance. It is scheduled downtime, usually inspect periodically and then to repair or replace as the need require (such as cleaning, adjustment, alignment, painting, waxing etc.,). It is to eliminate the need for radical treatment. This type of maintenance may be either time based or condition based.

Definition 1.4 Availability. Availability of an system is defined as the probability that the system is operating successfully at any time. The system is in operation (i) means either the system has no failure (ii) or if the system has failure, it has repaired.

Generally we define, availability (steady-state) $= \dfrac{\text{uptime}}{\text{uptime} + \text{downtime}}$

Where the up-time is the actual period for which the system is available for use. The down-time includes, active repair time, administrative and other delays related to the repair. The denominator is equal to the total time for which the system is required to function.

There are three different types of steady state availability depending on the definitions of uptime and downtime.

i. Inherent availability

$$A_i = \underset{t \to \infty}{\text{It}} A(T) = \frac{MTBF}{MTBF + MTTR},$$

where $A(T)$ is the availability at time t, called point availability A_i is based on the failure distribution and repair time distribution.

ii. Achieved availability

$$A_a = \frac{MTBM}{MTBM + M},$$

where MTBM is the mean time between maintenance and M is the mean active maintenance time including preventive maintenance but not including supply or maintenance delay times.

(i) Operational availability

$$A_0 = \frac{MTBM}{MTBM + MDT},$$

where MDT is the mean maintenance down time which include active maintenance time, logistic delay time and administrative delay time.

Theorem 2.1. Reliability under Preventive Maintenance. *The increase in reliability can be achieved through a preventive maintenance and* $R(t)$ *be the reliability of a system without maintenance and* $R_m(t)$ *be the reliability of a system with preventive maintenance. Let* T *be the interval of time between preventive maintenance.*

Since $R_m(t) = P$ (the maintained system does not fail before t)

$\therefore R_m(t) = R(t) 0 \leq t < T$

$= R(t)$ for $t = T$

After the first maintenance at T, the system becomes as a new one.

Therefore, in $T \leq t < 2T$, (i.e., time between first maintenance and second maintenance).

$R_m(t) = P$ (the system does not fail up to T and it survives for a time $(t - T)$ without failure)

$= R(T)\ R(t - T)$ for $T \leq t < 2T$

Similarly in $2T \leq t < 3T$,

$$R_m(t) = (R(T))^2 R(t - 2T)$$

Continuing in this manner we get,

$R_m(t) = (R(T))^2 R(t - nT)$ for $T \leq t < (n+1)T$ where $n = 0, 1, 2, 3$

MTTF of a system with preventive maintenance

$$MTTF = \int_0^{\infty} R_m(t)\, dt$$

$$= \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} R_m(t)\, dt$$

(divide the range into intervals of length T)

$$MTTF = \sum_{n=0}^{\infty} \int_{nT}^{(n+1)T} (R(T))^n R(t - nT) dt$$

Put $t - nT = t_1$

$$MTTF = \sum_{n=0}^{\infty} R((T))^n R(t - nT) dt$$

Since $\sum_{n=0}^{\infty} (R(T))^n$ is an infinite geometric series having as its sum $\frac{1}{1 - R(T)}$ we get,

$$MTTF = \frac{\int_0^T R(t)\, dt}{1 - R(T)}$$

Note.

Under a constant failure rate, preventive maintenance has no effect

For constant failure rate $R(t) = e^{-\lambda t}$

$$R_m(t) = (R(T))^n R(t - nT)$$

$$= (e^{-\lambda T})^n e^{-\lambda(t-nT)}$$

$$= e^{-\lambda nT} e^{-\lambda t} e\lambda nT$$

$$= e^{-\lambda t}$$

$$R_m(t) = R(t)$$

This shows that preventive maintenance has no effect for a constant failure rate modal (or) we can restate as the preventive maintenance does not improve the reliability of the system under a constant failure rate model.

Theorem 2.2. Maintainability-Repair Maintenance-Maintainability Function. *Maintainability function is an indice associated with an equipment under repair. (or) we can define it as a measure of how fast a system may be repaired following failure. It is the probability that the failed system will be repaid within time. It T is a random variable representing the repair time, then maintainability is defined mathematically as the cumulative distribution function of the random variable T. It is denoted by $M(t)$.*

i.e., $M(t) = P(T \leq t) = \int_0^t g(t)\, dt$ (1)

where $g(t)$ id the p.d.f of T.

The expected value of repair time called the mean time to repair T and is denoted by MTTR.

i.e., $MTTR = E(T) = \int_0^t g(t)\, dt$

Assume that the system has failed at t and it will be repaired between t and $t + \Delta t$. Then the conditional probability that the system will be repaired given that it has failed and the repair starts immediately is $\mu(t)\Delta t$ then $\mu(t)$ is called repair rate which denotes the number of repairs in unit time.

i.e., $\dfrac{P(t \le T \le t + \Delta t)}{P(T > t)} = \mu(t)\Delta t$

$$\frac{g(t)\Delta t}{1 - M(t)} = \mu(t)\Delta t$$

$$\therefore \frac{g(t)}{1 - M(t)} = \mu(t) \quad (2)$$

From (1), $g(t) = M'(t)$ (3)

Substituting in (2)

$$\mu(t) = \frac{M'(t)}{1 - M(t)}$$

$$\int_0^t \mu(t)\, dt = \int_0^t \frac{M'(t)}{1 - M(t)}\, dt \quad (4)$$

$$\int_0^t \mu(t)\, dt = [\, - \log(1 - M(t))]_0^t$$

$$= -\log(1 - M(t))$$

$$= \log(1 - M(t))^{-1} \quad \because M(0) = 0$$

$$\frac{1}{1 - M(t)} = e^{\int_0^t \mu(t)\, dt}$$

$$1 - M(t) = e^{-\int_0^t \mu(t)\, dt}$$

i.e., $M(t) = 1 - e^{-\int_0^t \mu(t) dt}$ is called the maintainability equation (5)

$$M'(t) = \mu(t) e^{-\int_0^t \mu(t)\, dt}$$

i.e., $g(t) = \mu(t) e^{-\int_0^t \mu(t)\, dt}$ using (3)

Note. The random variable T follows an exponential distribution if and only if repair rate is constant and mean time to repair is $\frac{1}{\mu}$ for constant repair rate.

Let the repair rate $\mu(t) = \mu$ be a constant

We have $g(t) = \mu(t)\, e^{-\int_0^t \mu(t)\, dt}$

$$= \mu e^{-\int_0^t \mu dt}$$

$g(t) = \mu e^{-\mu t},\ t > 0$

Which is an exponential distribution with parameter.

Conversely, if $g(t) = \mu e^{-\mu t},\ t > 0$

Then $M(t) = 1 - e^{-\int_0^t \mu(t)\, dt}$

$$= 1 - e^{-\mu(t)}$$

$$\therefore \mu(t) = \frac{M'(t)}{1 - M(t)}$$

$$= \frac{\mu e^{-\mu(t)}}{1 - (1 - e^{-\mu(t)})}$$

$= \mu$ which is a constant

Now,

$$MTTR = \int_0^\infty t\ g(t)\, dt$$

$$= \int_0^\infty t\ \mu\ e^{-\mu(t)} dt$$

$$= \frac{1}{\mu}$$

Theorem 2.3. Availability Function By Markov Modal-A Single

System. *Availability function also can be computed using Markov modal. We assume that the failure rate* λ *and repair rate* μ *are constants.* C*onsider the transition for the availability of a single system with repair and repair starts as soon as system fails. Then the system will be in one of the two possible*: *state* 0 *denotes that the system is in operation and state* 1 *denoted that the system is under repair.*

The probability that the system will be in state 0 at time $(t + \Delta t)$ is,

$$P_0(t + \Delta t) = P_0(t)\,(1 - \lambda \Delta t) + P_1(t)\,\mu\,\Delta t \tag{1}$$

The probability that the system will be in state 1 at time $(t + \Delta t)$ is,

$$P_1(t + \Delta t) = P_1(t)\,(1 - \mu \Delta t) + P_0(t)\,\mu\,\Delta t \tag{2}$$

(1) and (2) can be written as

$$\frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = -P_0(t)\lambda + P_1(t)\mu$$

$$\frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} = -P_0(t)\lambda - P_1(t)\mu$$

$$\text{i.e., } \frac{dP_0}{dt} + P_0(t)\lambda = P_1(t)\mu \tag{3}$$

and

$$\frac{dP_1}{dt} + P_1(t)\lambda = P_0(t)\lambda \tag{4}$$

At time $t = 0,\ P_0(0) = 1$ and $P_1(0) = 0$

Solving equation (3) and (4) using the condition (5) we get,

$$P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}$$

and

$$P_1(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}$$

Since state 0 is the availability state, by definition

$$A(t) = P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}$$

The steady state availability of a single system it's

$$A = A(\infty) = Lt_{t \to \infty} A(t) = Lt_{t \to \infty} P_0(t)$$

$$= \frac{\mu}{\lambda + \mu}$$

$$A = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}}$$

(where $\frac{1}{\lambda}$ is the mean time failures and $\frac{1}{\lambda}$ is the mean time to repair).

i.e., $A + \frac{MTTF}{MTTF + MTTR}$

Note.

In steady state, as $t \to \infty$, $P_0(t)$ becomes

Availability (which is also called point availability) and also as $t \to \infty$, $P_1(t)$ becomes availability,

In that case $P_1 = \frac{\lambda}{\lambda + \mu}$

i.e., $P_1 = \frac{\frac{1}{\mu}}{\frac{1}{\lambda} + \frac{1}{\mu}}$

$$P_1 = \frac{MTTR}{MTTF + MTTR}$$

System Availability

i. For n independent system in series, each system has availability $A_t(t)$, then the system availability is given by

$$A_s(t) = A_1(t) A_2(t) \ldots A_n(t)$$

$$= \prod_{k=1}^{n} A_k(t)$$

ii. For n system in parallel, the system availability is given by

$$A_s(t) = 1 - (1 - A_1(t))(1 - A_2(t)), \ldots, (1 - A_n(t))$$

$$= 1 - \prod_{k=1}^{n} (1 - A_k(t))$$

iii. When m out of n identical system are required for the system to function successfully, the availability of the system is

$$A_s(t) = A_i^n + \sum_{i=1}^{n-m} \frac{n!\, A_i^{(n-1)}(i - A_i)^i}{(n-i)!\, i!}$$

Where A_i is the availability of each system.

iv. The steady state availability of the standby system is given by

$$A = A_s(\infty) = \frac{\lambda_1 \mu + \mu^2}{\mu^2 + \lambda_1 \mu + \lambda_1 \lambda_2}$$

Where λ_1 and λ_2 are failure rates and μ is a repair rate. In this we always assume that the standby system does not fail in standby mode, but the repair of the standby unit is also permitted.

Problem 3.1. The time to repair a power generator is best described by the following probability density function $\frac{t^2}{333}$, $1 \le t \le 10$ hrs

(i) Determine the probability that a repair will be completed in 6 hours.

(ii) What is mean time to repair ?

(iii) Compute the repair rate.

Proof.

Here $g(t) = \frac{t^2}{333}$, $1 \le t \le 10$

(i) If T is the time to repair, $P(T < 6) = P(1 \le T < 6)$

$$= \int_1^6 g(t)\, dt$$

$$= \int_1^6 \frac{t^2}{333}\, dt$$

$$= \frac{t^2}{333}\left(\frac{t^3}{3}\right)_1^6$$

$$= \frac{215}{999}$$

$$= 0.2152$$

(ii) Mean time to repair $= MTTR = \int_0^\infty t\ g(t)\ dt$

$$= \int_1^{10} \frac{t^3}{333}\, dt$$

$$= \frac{t^2}{333}\left(\frac{t^4}{3}\right)_1^{10}$$

$$= 7.5068 \text{ hrs}$$

(iii) Repair rate $= \mu(t) = \frac{g(t)}{1 - m(t)}$

$$= \frac{\left(\frac{t^2}{333}\right)}{\int_t^{10} \frac{t^2}{333}\, dt}$$

$$= \frac{3t^2}{1000 - t^3} \text{ per hours.}$$

Problem 3.2. Y. Bull, a reliability engineer with the Maken Brake company, has determined that the hazard rate function for its milling a machine is the following $\lambda(t) = 0.0004521 t^{0.8}$, $t \geq 0$

Where t is measured in years. Determine which of the following options will provide the greatest reliability over machine's 20 years operating life.

Option A. Do nothing-operate the machine until it fails.

Option B. An annual preventive maintenance program (assume no maintenance-induced failure).

Option C. operate a second machine in parallel with the first (active redundant).

Proof.

$$R_A(t) = e^{-\int_0^t \lambda(t)\,dt}$$

$$= \exp\left[-\int_0^t 0.0004521 t^{0.8} dt\right]$$

$$= \exp\left[\frac{0.0004521}{1.8}(20)^{1.8}\right]$$

$$= 0.9465.$$

Option B

$$R_B(t) = (R(t))^n R(t - nT).$$

Since annual preventive maintenance program for 20 years, there will be maintenance in (0, 20). Here $t = 20$, $T = 1$, $n = 2$

$$R_B(t) = (R(1))^{19} R(20 - 19)$$

$$= (R(1))^{19} R(1)$$

$$= (R(1))^{20}$$

$$= \left[\exp \frac{-0.0004521}{1.8}\right]^{20}$$

$$= 0.9950$$

Option C

$$R_C(t) = 1 - (1 - R(t))^n$$

$$R_C(20) = 1 - (1 - R(20))^2$$

$$= 1 - (1 - 0.9465)^n$$

$$= 0.9971$$

By option A, reliability $= 0.9465$

Option B, reliability $= 0.9950$

Option C, reliability $= 0.9971.$

Hence option C gives the greatest reliability.

Problem 3.3. A washing machine has a time to failure distribution that is log normal with a space parameter $s = 0.56$ and a scale parameter $t_n = 40$ hours. Repair is normally distribution with mean of 3 hours and a standard deviation of 2 hours. Find the steady state availability of the machine.

Proof.

For log normal distribution MTTF $= t_n e^{\left(\frac{s^2}{2}\right)}$

$$= 40e^{\left(\frac{(0.56)^2}{2}\right)}$$

$$= 46.7905 \text{ hrs}$$

Mean of the normal repair distribution $= 3$ hrs

Steady state availability $= A = A(\infty)$

$$= \frac{MTTF}{MTTF + MTTR}$$

$$= \frac{46.7905}{46.7905 + 3}$$

$$= 0.9397$$

Problem 3.4. A system has a mean time between failures of 100 hrs and a mean to repair of 15 hrs. What is the inherent availability ?

Proof.

$$\text{Inherent availability } = A_i = \frac{MTBF}{MTBT + MTTR}$$

$$= \frac{100}{100+15}$$

$$= 0.8695$$

$$\simeq 87\%$$

Problem 3.5. An equipment is required to meet an inherent availability requirement 0.985 and a mean time between failures of 100 hrs. What is the permissible mean time to repair ?

Proof.

$$A_i = \frac{MTBF}{MTBT + MTTR}$$

Given $A_i = 0.985$, $MTBF = 100$ hrs, MTTR = ?

$$\therefore 0.985 = \frac{100}{100 + MTTR}$$

$$MTTR = \frac{100(1 - 0.985)}{0.985}$$

$$= 15228$$

Problem 3.6. An component in a system has a constant failure rate of 0.1 per day. Once it has failed, the mean time repair is 2.5 days (which is constant).

(i) What are the point availability at the end of 3 days and the steady state availability ?

(ii) If two component operate in series, compute the availability at the end of 3 days.

(iii) If they operate in parallel, determine the steady state availability of the system.

(iv) If one component operates in a standby mode with no failure in standby – state availability ?

Proof. Failure rate $= \lambda = 0.1$ per days

Mean time to repair $= \frac{1}{\mu} = 2.5$ days. i,e ., $\mu = 0.4$ per day

(i) Point availability $A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t}$

$$\therefore A(3) = \frac{0.4}{0.5} + \frac{0.1}{0.5} e^{-(0.5)(3)}$$

$$= 0.8 + 0.2\, e^{-1.5}$$

$$= 0.8446$$

(ii) $A_s(t) = \prod_{k=1}^{2} A_k(t)$

$$A_s(3) = (A(3))^2$$

$$= (0.8446)^2$$

$$= 0.7133.$$

(iii) Steady state availability for parallel,

$$A = A_s(\infty)$$

$$= 1 - (1 - A(\infty))^2$$

$$A = 1 - (1 - 0.8)^2$$

$$A = 0.96.$$

(iv) The steady state availability of standby system is

$$A = A_s(\infty)$$

$$= \frac{\lambda_2\mu + \mu^2}{\mu^2 + \lambda_1\mu + \lambda_1\lambda_2}$$

$$= \frac{(0.1)(0.4) + (0.4)^2}{(0.4)^2 + (0.1)(0.4) + (0.1) + (0.1)}$$

$$A = 0.9524.$$

Conclusion

The paper concludes that there is a system to develop reliability of system. If a system is put into operation at specified time, but it is impossible to construct a good quality system from poor quality system element. It the beam develop a break it attains a state of failure. Maintainability provides a measure of the reliability of a system where a certain amount of failure can be allowed, while the availability provides a prediction of probability that the complex system will be ready for use at any moment in time.

References

[1] P. Kandasamy, K. Thilagavathi and K. Gunavathi, Probability Statistics and Queueing theory, S. Chand and Company LTD, Ram Nagar, New Delhi.

[2] S. Osaki, Stochastic Modals in Reliability and Maintenance, Springer Publication, (2002).

[3] UK defence standard The Management of Reliability, Maintainability, HMSO.

[4] British standard, BS 5760: Reliability of system, Equipments and Components, British standards Institution, London.

[5] S. P. Eugene Xavier, Probability theory and Random Processes, S. Chand. and Company LTD, Ram Nagar, New Delhi.