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# $(\alpha, \beta)$ - CUT OF INTUITIONISTIC FUZZY BI-IDEALS OF NEAR-SUBTRACTION SEMIGROUPS

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## Abstract

In this paper, we investigate some algebraic nature of intuitionistic fuzzy bi-ideals of near-subtraction semi group and some interesting properties of $(\alpha, \beta)$ - cuts of intuitionistic fuzzy bi-ideals of near-subtraction semi group are discussed.

## 1. Introduction

The fundamental concept of fuzzy set was first initiated by Zadeh [14]. The concept of intuitionistic fuzzy set was introduced by Atanassov [2] as a generalization of the notion of fuzzy set. Narmada et al. [8] introduced the intuitionistic fuzzy bi-ideals in near-rings. The notation of intuitionistic fuzzy R-subgroups of a near-ring is given by Y. B. Jun, Y. H. Yon and K. H. Kim [13]. Sharma [11] studied intuitionistic fuzzy subgroups of a group with the help of their $(\alpha, \beta)$ - cut sets. Mahalakshmi et al. [6] studied the notation of near subtraction semi groups. In this paper we study the properties of intuitionistic fuzzy bi-ideals of near-subtraction semi group with the help of their $(\alpha, \beta)$ - cut sets.

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## 2. Preliminaries

**Definition 2.1.** A non empty set $X$ together with two binary operations "$-$" and "$\cdot$" is said to be a near subtraction semi group (right) if it satisfies the following conditions:

(i) $(X, -)$ is a subtraction algebra.

(ii) $(X, \bullet)$ is a semi group.

(iii) $(x - y)z = xz - yz$

for every $x, y, z \in X$.

It is clear that $0_X = 0$, for all $x \in X$. Similarly we can define a left near-subtraction semi group. Here after a near-subtraction semi group means only a right near-subtraction semi group.

**Definition 2.2.** A nonempty subset $S$ of a subtraction semi group $X$ is said to be a sub algebra of $X$, if $x - y \in S$, for all $x, y \in S$.

**Definition 2.3.** A fuzzy sub algebra $\mu$ of $X$ is called a fuzzy bi-ideal of $X$ if for all $x, y, z \in X$

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\, y\, z) \geq \min\{\mu(x), \mu(z)\}$

**Definition 2.4.** An intuitionistic fuzzy set (IFS) $A$ is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x))/x \in X\}$ where the function $\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership and the degree of non membership of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. We use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x))/x \in X\}$.

**Definition 2.5.** An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in $X$ is called an intuitionistic fuzzy bi-ideal of $X$ if for all $x, y, z \in X$,

(i) $\mu_A(x - y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu_A(xyz) \geq \min\{\mu(x), \mu(z)\}$

(iii) $\lambda_A(x - y) \leq \max\{\lambda(x), \lambda(y)\}$

(iv) $\lambda_A(x\, y\, z) \leq \max\{\lambda(x), \lambda(z)\}$

**Theorem 2.6.** $A = (\mu_A, \lambda_A)$ *be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group* $X$, *then* $\mu_A(x) \leq \mu_A(0)$ *and* $\lambda_A(x) \geq \lambda_A(0)$ *for all* $x \in X$.

**Definition 2.7.** Let $X$ and $Y$ be two near subtraction semi groups. A map $f : X \to Y$ is called near subtraction semi group homomorphism. If it satisfies the following conditions:

(i) $f(x - y) = f(x) - f(y)$

(ii) $f(xy) = f(x)f(y)$ for all $x, y \in X$.

**Definition 2.8.** Let $A$ be an intuitionistic fuzzy set of a universe set $X$. Then $(\alpha, \beta)$-cut of $A$ is a crisp set $C_{(\alpha, \beta)}(A)$ of the IFS $A$ is given by $C_{(\alpha, \beta)}(A) = \{x : x \in X$ such that $\mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta\}$ where $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

**Proposition 2.9** [11]. *If A and B be two IFS's of a universe set X, then the following hold*

(i) $C_{(\alpha, \beta)}(A) \subseteq C_{(\delta, \theta)}(A)$ if $\alpha \geq \delta$ *and* $\beta \geq \theta$

(ii) $C_{(1-\beta, \beta)}(A) \subseteq C_{(\alpha, \beta)}(A) \subseteq C_{(\alpha, 1-\alpha)}(A)$

(iii) $A \subseteq B \Rightarrow C_{(\alpha, \beta)}(A) \subseteq C_{(\alpha, \beta)}(B)$

(iv) $C_{(\alpha, \beta)}(A \cap B) = C_{(\alpha, \beta)}(A) \cap C_{(\alpha, \beta)}(B)$

(v) $C_{(\alpha, \beta)}(A \cup B) = C_{(\alpha, \beta)}(A) \cup C_{(\alpha, \beta)}(B)$ *equality hold if* $\alpha + \beta \leq 1$.

(vi) $C_{(\alpha, \beta)}(\cap A_i) = \cap C_{(\alpha, \beta)}(A_i)$

(vii) $C_{(0, 1)}(A) = X$.

**Proposition 2.10** [11]. *Let map* $f : X \to X'$ *be a mapping. Then the following hold*:

(i) $f(C_{(\alpha, \beta)}(A)) \subseteq C_{(\delta, \theta)}(f(A))$ *for all* $A \in IFS(X)$

(ii) $f^{-1}(C_{(\alpha, \beta)}(B)) \subseteq C_{(\delta, \theta)}(f^{-1}(B))$ *for all* $A \in IFS(X)$

## 3. Main Results

**Proposition 3.1.** *Let* $A = (\mu_A, \lambda_A)$ *be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group X, then* $C_{(\alpha, \beta)}(A)$ *is a bi-ideal of X if* $\mu_A(0) \geq \alpha, \lambda_A(0) \leq \beta$.

**Proof.** Let $A = (\mu_A, \lambda_A)$ in $X$ is called an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$. Let $\mu_A(0) \geq \alpha, \lambda_A(0) \leq \beta\}$. Clearly $C_{(\alpha, \beta)}(A)$ is non-empty.

Let $x, y \in C_{(\alpha, \beta)}(A)$. Then $\mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta$ and $\mu_A(y) \geq \alpha$, $\lambda_A(y) \leq \beta$ which implies that $\min\{\mu_A(x), \mu_A(y)\} \geq \alpha$ and $\max\{\lambda_A(x), \lambda_A(y)\} \leq \beta$ which implies that $\mu_A(x - y) \geq \alpha$ and $\lambda_A(x - y) \leq \beta$ and so $x - y \in C_{(\alpha, \beta)}(A)$.

Let $x, z \in C_{(\alpha, \beta)}(A)$. Then $\mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta$ and $\mu_A(z) \geq \alpha$, $\lambda_A(z) \leq \beta$ which implies that $\min\{\mu_A(x), \mu_A(z)\} \geq \alpha$ and $\max\{\lambda_A(x), \lambda_A(z) \leq \beta$ Now $\mu_A(x\,y\,z) \geq \min\{\mu_A(x), \mu_A(z)\}\} \geq \alpha$ and $\lambda_A(x\,y\,z) \leq \max\{\lambda_A(x), \lambda_A(z)\} \leq \beta$ which implies that $\mu_A(x\,y\,z) \geq \alpha$ and $\lambda_A(x\,y\,z) \geq \beta$ and so $x\,y\,z \in C_{(\alpha,\beta)}(A)$. Hence $C_{(\alpha,\beta)}(A)$ is a bi-ideal of $X$.

**Proposition 3.2.** *Let* $A = (\mu_A, \lambda_A)$ *be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group X, then A is intuitionistic fuzzy bi-ideal of X if and only if* $C_{(\alpha,\beta)}(A)$ *is a bi-ideal of X, for all* $\alpha, \beta \in [0, 1]$ *with* $\alpha + \beta \leq 1$ *and* $\mu_A(0) \geq \alpha$ *and* $\lambda_A(0) \leq \beta$.

**Proof.** Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$, then $C_{(\alpha, \beta)}(A)$ is a bi-ideal of $X$, for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\mu_A(0) \geq \alpha$ and $\lambda_A(0) \leq \beta$ follows from proposition 3.1.

Conversely, let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$, then $C_{(\alpha, \beta)}(A)$ is a bi-ideal of $X$, for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\mu_A(0) \geq \alpha$ and $\lambda_A(0) \leq \beta$. Suppose that $x, y \in X$ and $\mu_A(x - y) < \min\{\mu_A(x), \mu_A(y)\}$ and $\lambda_A(x - y) > \max\{\lambda_A(x), \lambda_A(y)\}$. Choose $\alpha, \beta$ such that $\mu_A(x - y) < \alpha < \min\{\mu_A(x), \mu_A(y)\}$ and $\lambda_A(x - y) > \beta > \max\{\lambda_A(x), \lambda_A(y)\}$. Then we get $x, y \in C_{(\alpha, \beta)}(A)$. But $x - y \notin C_{(\alpha, \beta)}(A)$, which is a contraction. Hence $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(z)\}$ and $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(z)\}$. A similar argument shows that $\mu_A(x\,y\,z) \geq \min\{\mu_A(x), \mu_A(z)\}\} \geq \alpha$ and $\lambda_A(x\,y\,z) \leq \max\{\lambda_A(x), \lambda_A(z)\} \geq \beta$ for all $x, y, z \in X$. Hence $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$.

**Proposition 3.3.** *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$, then the set $M = \{x \in X : \mu_A(x) = \mu_A(0), \lambda_A(x) = \lambda_A(0)\}$ is a bi-ideal in near-subtraction semi group $X$.*

**Proof.** Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$. To show that $M = \{x \in X : \mu_A(x) = \mu_A(x), \lambda_A(0) = \lambda_A(0)\}$ is a bi-ideal in near-subtraction semi group $X$.

Let $x, y \in M$, then $\mu_A(x) = \mu_A(0), \lambda_A(x) = \lambda_A(0)$ and $\mu_A(y) = \mu_A(0)$, $\lambda_A(y) = \lambda_A(0)$. Since $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$, we get $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\} = \mu_A(0)$. By Theorem 2.6, we get $\mu_A(x - y) = \mu_A(0)$. Also $\lambda_A(x - y) \leq \max\{\lambda(x), \lambda(y)\} = \lambda_A(0)$. By Theorem 2.6, we get $\lambda_A(x - y) = \lambda_A(0)$. Hence $\mu_A(x - y) = \mu_A(0)$, $\lambda_A(x - y) = \lambda_A(0)$ and so $x - y \in M$. Thus $M$ is a subgroup of $X$.

Let $x, y \in M$ and $y \in X$. Then $\mu_A(z) = \lambda_A(0)$. Since $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$, we get $\mu_A(x\,y\,z) \geq \min\{\mu_A(x), \mu_A(z)\} = \mu_A(0)$. By Theorem 2.6, we get $\mu_A(x\,y\,z) = \mu_A(0)$. Also $\lambda_A(x\,y\,z) \leq \max\{\lambda(x), \lambda(z)\} = \lambda_A(0)$. By Theorem 2.6, we get $\lambda_A(x\,y\,z) = \lambda_A(0)$. Hence $\mu_A(x\,y\,z) = \mu_A(0), \lambda_A(x\,y\,z) = \lambda_A(0)$ and so $x\,y\,z \in M$. Thus $M$ is a bi-ideal of $X$.

**Corollary 3.4.** *Let X be a near-subtraction semi group. Then the intuitionistic fuzzy set* $A = \{< x, \mu_A(x), \lambda_A(x) >, x \in X : \mu_A(x) = \mu_A(0), \lambda_A(x) = \lambda_A(0)\}$ *of X an intuitionistic fuzzy bi-ideal of a near-subtraction semi group X.*

**Proof.** Taking $\alpha = \mu_A(0), \beta = \lambda_A(0)$, then $C_{(\alpha, \beta)}(A) = M$. Therefore by theorem 3.2, $A$ is an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$.

**Theorem 3.5.** *If A and B be two intuitionistic fuzzy bi-ideal of a near-subtraction semi group X, then* $A \cap B$ *is also an intuitionistic fuzzy bi-ideal of a near-subtraction semi group X.*

**Proof.** Since $A$ and $B$ be two intuitionistic fuzzy bi-ideals of a near-subtraction semi group $X$, by Theorem 3.1 $C_{(\alpha, \beta)}(A)$ and $C_{(\alpha, \beta)}(B)$ are bi-ideals in a near-subtraction semi group $X$. Since intersection of two bi-ideals in the near-subtraction semi group $X$ is a bi-ideal in $X$, $C_{(\alpha, \beta)}(A) \cap C_{(\alpha, \beta)}(B)$ is a bi-ideal of $X$. Therefore $C_{(\alpha, \beta)}(A \cap B)$ is a bi-ideal of $X$ (by proposition 2.9 (iv)). Hence $A \cap B$ is an intuitionistic fuzzy bi-ideal of a near-subtraction semi group $X$. By the Theorem 3.2, the converse is true.

**Corollary 3.6.** *Intersection of family of intuitionistic fuzzy bi-ideal of a near-subtraction semi group X is an intuitionistic fuzzy bi-ideal in X.*

**Theorem 3.7.** *Let X and* $X'$ *be two near-subtraction semi groups of X and let* $: X \to X'$ *be a near-subtraction semi group homomorphism. If* $A = (\mu_A, \lambda_A)$ *be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group* $X'$*, then the pre-image* $f^{-1}(A)$ *of A under f is an intuitionistic fuzzy bi-ideal of X.*

**Proof.** Since $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy bi-ideal of a near-subtraction semi group of $X$, for all $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ and $\mu_A(0) \geq \alpha$ and $\lambda_A(0) \leq \beta$ (by proposition 3.1). Therefore $f^{-1}(C_{(\alpha, \beta)}(A))$ is a bi-ideal of $X$. But $f^{-1}(C_{(\alpha, \beta)}(A)) = C_{(\alpha, \beta)}(f^{-1}(A))$ (by proposition 2.9 (ii)) which implies that $C_{(\alpha, \beta)}(f^{-1}(A))$ is a bi-ideal of $X$ and by theorem 3.2 $f^{-1}(A)$ of $A$ is an intuitionistic fuzzy bi-ideal of a near-subtraction semigroup of $X$.

## References


[1] J. C. Abbott, Sets, Lattices and Boolean Algebras, Allyn and Bacon, Inc., Boston, Mass. 1969.

[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Set and Syst. 20 (1986), 87-96.

[3] Gunter Pilz Near rings, The theory and its applications, North Holland Publishing Company, Amsterdam, New York, Oxford, 1983.

[4] Y. B. Jun and H. S. Kim, On ideals in subtraction algebras, Sci. Math. Jpn. 65(1) (2007), 129-134.

[5] O. Kazanci, S. Yamak and S. Yilmaz, On Intuitionistic $Q$-fuzzy $R$-subgroups of near-rings, International Math. Forum 2(59) (2007), 2859-2910.

[6] V. Mahalakshmi, S. Maharasi and S. Jayalakshmi, Bi-ideals of near subtraction semi group, Indian Advances in Algebra 6(1) (2013), 35-48.

[7] T. Manikandan, Fuzzy bi-ideals of near-rings, J. Fuzzy Math. 17(3) (2009), 659-671.

[8] S. Narmada and V. Mahesh Kumar, On intuitionistic fuzzy bi-ideals and regularity in near-rings, International Journal of Algebra 5(10) (2011), 483-490.

[9] O. Ratnabala Devi, On the intuitionistic $Q$-fuzzy ideals of near-rings, NIFS 15(3) (2009), 25-32.

[10] P. K. Sharma, Homomorphism of intuitionistic fuzzy groups, International Mathematical Forum 6(64) (2011), 3139-3178.

[11] P. K. Sharma, $(\alpha, \beta)$ - cut of intuitionistic fuzzy groups, International Mathematical Forum 6(53) (2011), 2605-2614.

[12] T. Tamizh Chelvam and N. Ganesan, On bi-ideals of near-rings, Indian J. Pure and Appl. Math. 18(11) (1987), 1002-1005.

[13] Y. H. Yon, Y. B. Jun and K. H. Kim, Intuitionistic fuzzy $R$-subgroups of near-rings, Soochow Journal of Mathematics 27(3) (2001), 243-253.

[14] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.