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MATHEMATICAL MODEL TO SPECULATE ZOMBIES ATTACK USING FRACTIONAL DERIVATIVES

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Abstract

The paper aims to extend the model of the zombies attack to the mathematical model of fractional order using Atangana-Baleanu derivative operator. A detailed proof for the existence, uniqueness of the solution for the fractional mathematical model is presented. A numerical approach is used to find the solution of the stated model and the results are represented graphically.

1. Introduction

A Zombie is a revenant created by human imagination. They are commonly featured in movies and video games and described as being bought through an epidemic.

Zombies are subservient either to a sorcerer or to a devastate hunger for human flesh. They are portrayed as lumbering or crumbling and sometimes they show characteristics of superhuman such as increase in strength and power. Zombies kill and infect other human beings by biting-who then themselves become zombies. Since zombies in most of the cases are

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considered deceased so it is impossible to kill them by conventional methods such as stabbing, gunshots, etc. therefore removing the head and destroying the connection of body and brain is the only way to stop them.

Apart from an imagination and a fictional story, zombies can be visualised as a viral disease that can become a challenge for human race. Its spread will not be the same as that of other viral disease.

As we know that the mathematical models are capable of decision making, saving lives, assisting in policy, and many more. These are helpful in understanding the conditions needed to sustain lives and provide us ways to study and predict the behavior of the spread. The concept of derivatives and integrals pays a lot in the formulation of these mathematical models. In this work, we will study the SIZR mathematical model for the outbreak of zombies attack. A SIZR model determines the number of people infected with a transmissible infection in a closed population over a while. These models are acquired in such a way that they involve equations that correlate number of susceptible people $S(t)$, number of people infected $I(t)$, number of people who have transformed to zombie, and who have removed $R(t)$. Classifying the total population $N(t)$ into different groups of individuals as susceptible (S), infectious (W), zombies (Z), Removed (R) quarantined (Q), we develop a model that involves equations that associate with the group of individuals those are susceptible (S), infectious (W), zombies (Z), removed (R), and quarantined (Q) The extended SIR mathematical model for Zombies infection is given as [17]

$$\frac{dS}{dt} = N - \tilde{\beta}SZ - \rho S, \quad S(0) = S_0$$

$$\frac{dI}{dt} = \tilde{\beta}SZ - \delta I - \rho I - \chi I \quad I(0) = I_0$$

$$\frac{dZ}{dt} = \delta I + \eta R - \tilde{\alpha}SZ - \sigma Z \quad Z(0) = Z_0$$

$$\frac{dR}{dt} = \rho S + \rho I + \tilde{\alpha}SZ - \tilde{\alpha}SZ - \eta R + rQ \quad R(0) = R_0$$

$$\frac{dQ}{dt} = \chi I + \sigma Z - rQ, \quad Q(0) = Q_0$$

Where, N is the birth rate, $\tilde{\beta}$ is the transmission rate, ρ is the natural death rate, η rate by which the removed individuals resurrect and become a zombie, $\tilde{\alpha}$ is the rate by which zombies move to removed class by destroying their brains, χ is the rate at which infected population enter the quarantined area, and σ is the rate at which the zombies population enter the quarantined area.

In the current work, we will study the mathematical model of fractional order for the zombies infection. The biological systems have memory or after effects. Therefore, the mathematical modeling of such biological systems using the theory of fractional order derivatives is more useful. Thus, fractional derivatives efficiently study the behavior of these biological models. Growth and progress of fractional calculus ([1]-[5]) have many developments in many field of physics, biology, chemistry, biochemistry, medicine, and many more. Recent developments in the field of fractional calculus can be seen in ([6]-[12]). There exist many fractional derivative operators, some with the singular kernel such as Riemann-Liouville, Liouville-Caputo, etc. ([1]-[4]). some with non-singular kernel such as M. Caputo and M. Fabrizio [13]. For fractional derivatives with non-singular kernel see ([14]-[16]). It was revealed by researchers that the power-law kernel as waiting time and scaling invariant for mean square displacement and while dealing with fluid flow in a heterogeneous medium these properties are very essential. Hence the fractional derivative operator with the non-singular kernel rebuilds the Gaussian to non-Gaussian as density distribution, the stretched exponential to power as waiting time. The main aim for the choice of including the Atangana-Baleanu fractional derivative operator is to include into the mathematical formulation of the dynamical system the effect of non-local fading memory. For detailed study see ([14]-[16]). In this work, we will use the Atangana-Baleanu derivative operator ([16]) to study the fractional model of zombies infection, which is given as

$$ {}_{0}^{\mathcal{ABC}}\kappa_{t}^{\zeta} S(t) = N - \tilde{\beta} SZ - \rho S \quad S(0) = S_0 $$

$$ {}_{0}^{\mathcal{ABC}}\kappa_{t}^{\zeta} I(t) = \tilde{\beta} SZ - \delta I - \rho I - \chi I, \quad I(0) = I_0 $$

$$ {}_{0}^{\mathcal{ABC}}\kappa_{t}^{\zeta} Z(t) = \delta I + \eta R - \tilde{\alpha} SZ - \sigma Z, \quad Z(0) = Z_0 $$

$$ {}_{0}^{\mathcal{ABC}}\kappa_{t}^{\zeta} R(t) = \rho S + \rho I + \tilde{\alpha} SZ - \tilde{\alpha} SZ - \eta R + rQ, \quad R(0) = R_0 $$

$$ {}_{0}^{ABC}\kappa_t^{\zeta} Q(t) = \chi I + \sigma Z - rQ, \quad Q(0) = Q_0 $$

here ${}_{0}^{ABC}\kappa_t^{\zeta}$ is Atangana-Baleanu derivative in Caputo sense defined as.

Definition 1 [16]. Let $\mathcal{F}$ be defined on $H^1[a, b]$, let $0 < \zeta < 1$, then the Atangana-Baleanu fractional derivative of order ζ in Caputo sense is given as

$$ {}_{0}^{ABC}\kappa_t^{\zeta}(\mathcal{F}(t)) = \frac{K(\zeta)}{1-\zeta}\int_0^t F'(\tau)\, E_{\zeta}\left[-\zeta\frac{(t-\tau)}{1-\zeta}\right]d\tau. \tag{1} $$

Where, E_{ζ} is the Mittag-Leffler function and $K(\zeta)$ is the normalization function such that $K(0) = K(1) = 1$.

Definition 2 [16]. Let $\mathcal{F}$ be defined on $H^1[a, b]$, the fractional integral of Atangana-Baleanu fractional derivative of order ζ is given as

$$ \mathcal{I}_t^{\mathcal{AB}^{\zeta}}(\mathcal{F}(t)) = \frac{1-\zeta}{K(\zeta)}g(t) + \frac{\zeta}{K(\zeta)\Gamma(\zeta)}\int_0^t \mathcal{F}(\eta)(t-\eta)^{\zeta-1}d\eta. \tag{2} $$

Theorem 1 [16]. *The fractional differential equation*

$$ \mathcal{I}_t^{\mathcal{AB}^{\delta}}(\mathcal{F}(t)) = v(t), $$

has a unique solution, given as

$$ \mathcal{F}(t) = \frac{1-\zeta}{G(\zeta)}v(t) + \frac{\zeta}{G(\zeta)\Gamma(\zeta)}\int_0^t v(\eta)(t-\eta)^{\zeta-1}d\eta. $$

1.1 Existence and Uniqueness of the Solution

Theorem 2 [11]. *Following functions*

$$ \begin{aligned} H_1(t, S) &= N - \tilde{\beta}SZ - \rho S, \\ H_2(t, I) &= \tilde{\beta}SZ - \delta I - \rho I - \chi I \\ H_3(t, Z) &= \delta I + \eta R - \tilde{\alpha}SZ - \sigma Z, \\ H_4(t, R) &= \rho S + \rho I + \tilde{\alpha}SZ - \tilde{\alpha}SZ - \eta R + rQ, \\ H_5(t, Q) &= \chi I + \sigma Z - rQ \end{aligned} \tag{1} $$

fulfil the Lipschitz condition and the contractions hold if following hold:

(i) $0 < b_1 < 1$

(ii) $0 < b_2 < 1$

(iii) $0 < b_3 < 1$

(iv) $0 < b_4 < 1$

(v) $0 < b_5 < 1$

Proof. Consider

$$H_1(t, S) = N - \tilde{\beta} SZ - \rho S$$

Let S_1 and S_2 be two functions, then

$$\| H_1(t, S_1) - H_1(t, S_2) \| = \| \tilde{\beta}(S_1 - S_2)Z - \rho(S_1 - S_2) \|$$

$$(\tilde{\beta} \| Z \| + \rho) \| S_1 - S_2 \| \quad (2)$$

Let $e_1 = \sup_t \| S(t) \|$, $e_2 = \sup_t \| I(t) \|$, $e_3 = \sup_t \| Z(t) \|$, $e_4 = \sup_t \| R(t) \|$, $e_5 = \sup_t \| Q(t) \|$, then

$$\| H_1(t, S_1) - H_1(t, S_2) \| \leq b_1 \| S_1(t) - S_2(t) \|.$$

where

$$b_1 = \tilde{\beta} e_3 + \rho$$

Hence, this proves Lipschitz's condition for $G_1(t, S)$ and if $0 < b_1 < 1$, then this proves contraction for $H_1(t, S)$. We can similarly prove the result for $H_2(t, I)$, $H_3(t, Z)$, $H_4(t, R)$, $H_5(t, Q)$.

Theorem 3. *The fractional extended SIR mathematical model for zombie infection*

$${}_0^{ABC}\kappa_t^{\zeta} S(t) = N - \tilde{\beta} SZ - \rho S \quad S(0) = S_0$$

$${}_0^{ABC}\kappa_t^{\zeta} I(t) = \tilde{\beta} SZ - \delta I - \rho I - \chi I, \quad I(0) = I_0$$

$$ {}_0^{ABC}\kappa_t^{\zeta} Z(t) = \delta I + \eta R - \tilde{\alpha} SZ - \sigma Z, \quad Z(0) = Z_0 $$

$$ {}_0^{ABC}\kappa_t^{\zeta} R(t) = \rho S + \rho I + \tilde{\alpha} SZ - \tilde{\alpha} SZ - \eta R + rQ, \quad R(0) = R_0 $$

$$ {}_0^{ABC}\kappa_t^{\zeta} Q(t) = \chi I + \sigma Z - rQ, \quad Q(0) = Q_0 $$

gives a unique solution under the constraints, we can search for a $t_{\max}$ *which satisfies*

$$ \frac{1-\zeta}{K(\zeta)} b_i + \frac{t_{\max}^{\zeta}}{K(\zeta)\Gamma(\zeta)} b_i < 1, \; for \; i = 1, 2, 3, 4, 5. \tag{3} $$

Proof. Consider □

$$ {}_0^{ABC}\kappa_t^{\zeta} S(t) = N - \tilde{\beta} SZ - \rho S \quad S(0) = S_0 $$

Let

$$ H_1(t, S) = N - \tilde{\beta} SZ - \rho S $$

Then equation (1.4) can be written as

$$ {}_0^{ABC}\kappa_t^{\zeta} S(t) = H_1(t, S). \tag{4} $$

Using theorem 1, we get

$$ S(t) = S_0 + \frac{1-\zeta}{K(\zeta)} H_1(t, S(t)) + \frac{\zeta}{K(\zeta)\Gamma(\zeta)} \int_0^t (t-\mu)^{\zeta-1} H_1(\mu, S(\mu)) d\mu. \tag{5} $$

Let $Z = (0, T)$ and define an operator $X : \mathcal{C}(Z, \mathbb{R}^5) \to \mathcal{C}(J, \mathbb{R}^5)$ such that

$$ X[S(t)] = S_0 + \frac{1-\zeta}{K(\zeta)} H_1(t, S(t)) + \frac{\zeta}{K(\zeta)\Gamma(\zeta)} \int_0^t (t-\mu)^{\zeta-1} G_1(\mu, S(\mu)) d\mu. \tag{6} $$

So equation (1.5) can be seen as $X[S(t)] = S(t)$. Define the supremum norm on Z as $\| S \| = Sup_{t \in Z} | S(t) |$. Then $\mathcal{C}(Z, \mathbb{R}^5)$ and $\| \cdot \|$ defines a Banach Space.

Consider

$$X[S_1(t)] - X[S_2(t)] = \frac{1-\zeta}{K(\zeta)}(H_1(t, S_1(t)) - H_2(t, S_1(t)))$$

$$+ \frac{\zeta}{K(\zeta)\Gamma(\zeta)}\int_0^t (t-\mu)^{\mu-1}(L_1(\mu, S_1(\mu)) - H_1(\mu, S_1(\mu)))d\rho. \tag{7}$$

Taking modulus on equation (1.7) and then using the triangle inequality we get

$$| X[S_1(t)] - X[S_2(t)] | \leq \frac{1-\zeta}{K(\zeta)} | (H_1(t, S_1(t)) - H_2(t, S_1(t))) |$$

$$+ \frac{\zeta}{K(\zeta)\Gamma(\zeta)}\int_0^t | (t-\mu)^{\zeta-1}(H_1(\mu, S_1(\mu)) - H_1(\mu, S_1(\mu)))d\mu |. \tag{8}$$

As the function $H_1(t, S(t))$ agrees with the Lipschitz condition, we have

$$| X(S_1) - X(S_2) | \leq \left(\frac{1-\mu}{K(\mu)} b_1 + \frac{t_{\max}^{\zeta}}{K(\zeta)\Gamma(\zeta)} b_1\right) | S_1 - S_2 |. \tag{9}$$

Also equation (1.9) will be a contraction only if

$$\frac{1-\zeta}{K(\zeta)} b_1 + \frac{t_{\max}^{\zeta}}{K(\zeta)\Gamma(\zeta)} b_1 < 1. \tag{10}$$

Using the Banach Fixed Point theorem, we finally govern the existence of a solution which is unique as well for the fractional mathematical extended SIR model to speculate the zombie infection in the sense of Atangana-Baleanu derivative operator.

1.2 Numerical Solution Using Predictor-Corrector Method

We now apply the Predictor-Corrector method in Atangana-Baleanu sense for finding the solution of the given model. For Predictor-Corrector method see [18]. We assume

$$W(t) = S(t), I(t), Z(t), R(t), Q(t)$$

and

$$W_0(t) = S_0(t), I_0(t), Z_0(t), R_0(t), Q_0(t)$$

Consider

$$ {}_{0}^{\mathcal{ABC}}\kappa_{t}^{\zeta}W(t) = P(t, W(t)), t \geq 0, W(0) = W_0. \tag{11}$$

The above equation now reduces to the fractional voltera equation

$$W(t) = W_0(t) + (1-\zeta)P(t_{i+1}, W_{i+1})$$

$$+ \frac{\zeta}{\Gamma(\zeta)}\int_0^{t_{i+1}} (t_{i+1} - \mu)^{\zeta-1} P(\mu, W(\mu))d\mu. \tag{12}$$

Now following the method given in [?] for $\mu \in [0, 1]$, $0 \leq t \leq T$ and setting $\| = \frac{T}{N}$ and $t_n = n \|$ for $n = 0, 1, 2, \ldots, N$, where N is positive integer, the Predictor-Corrector formula [?] of the above problem is given as

$$W_{i+1} = W_0 + \frac{\zeta k^{\zeta}}{\Gamma(\zeta+2)}\left(p_{i+1,i+1}P(t_{i+1}, W_{i+1}^M) + \sum_{j=0}^{i} p_{i+1}, jP(t_j, W_j) \right) \tag{13}$$

where

$$p_{i+1,j} = \begin{cases} i^{\zeta+1} - (i-\zeta)(i+1)^{\zeta} & \text{if } j = 0 \\ (i-j+2)^{\zeta+1} + (i-j)^{\zeta+1} - 2(i-j+1)^{\zeta+1} & \text{if } 1 \leq j \leq i \\ 1, & j = i+1 \end{cases}$$

and

$$b_{i+1,i+1} = 1 + \frac{(1-\zeta)\Gamma(\zeta+2)}{\zeta h^{\zeta}}$$

The Predictor-Corrector formula is given as

$$W_{i+1}^M = W_0 + \frac{k^{\zeta}}{\Gamma(\zeta)}\sum_{j=0}^{i} a_{i+1,j}P(t_j, W_j) \tag{14}$$

where

$$a_{i+1,j} = \begin{cases} -(i-j)^{\zeta} + (i-j+1)^{\zeta} & \text{if } j = 0, \ldots, i-1 \\ 1 + \frac{(1-\zeta)\Gamma(\zeta)}{h^{\zeta}} & \text{if } i = j \end{cases}$$

Using the above method, the numerical solution for the zombie infection model is given as

$$S_{i+1} = S_0 + \frac{\zeta k^{\zeta}}{\Gamma(\zeta+2)}\left(p_{i+1,i+1}H_1(t_{i+1}, S_{i+1}^M) + \sum_{j=0}^{i} p_{i+1},\ jH_1(t_j, S_j)\right)$$

$$I_{i+1} = I_0 + \frac{\zeta k^{\zeta}}{\Gamma(\zeta+2)}\left(p_{i+1,i+1}H_2(t_{i+1}, I_{i+1}^M) + \sum_{j=0}^{i} p_{i+1},\ jH_2(t_j, I_j)\right)$$

$$Z_{i+1} = Z_0 + \frac{\zeta k^{\zeta}}{\Gamma(\zeta+2)}\left(p_{i+1,i+1}H_3(t_{i+1}, Z_{i+1}^M) + \sum_{j=0}^{i} p_{i+1},\ jH_3(t_j, Z_j)\right) \tag{15}$$

$$R_{i+1} = R_0 + \frac{\zeta k^{\zeta}}{\Gamma(\zeta+2)}\left(p_{i+1,i+1}H_4(t_{i+1}, R_{i+1}^M) + \sum_{j=0}^{i} p_{i+1},\ jH_4(t_j, R_j)\right)$$

$$Q_{i+1} = Q_0 + \frac{\zeta k^{\zeta}}{\Gamma(\zeta+2)}\left(p_{i+1,i+1}H_5(t_{i+1}, Q_{i+1}^M) + \sum_{j=0}^{i} p_{i+1},\ jH_5(t_j, Q_j)\right)$$

where,

$$S_{i+1}^M = S_0 + \frac{k^{\nu}}{\Gamma(\nu)}\sum_{j=0}^{i} a_{i+1,\,j}H_1(t_j, S_j)$$

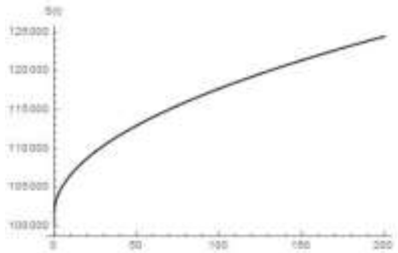
$$I_{i+1}^M = I_0 + \frac{k^{\nu}}{\Gamma(\nu)}\sum_{j=0}^{i} a_{i+1,\,j}H_2(t_j, I_j)$$

$$Z_{i+1}^M = Z_0 + \frac{k^{\nu}}{\Gamma(\nu)}\sum_{j=0}^{i} a_{i+1,\,j}H_3(t_j, Z_j)$$

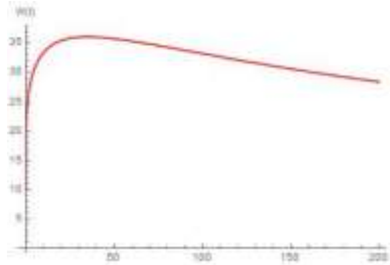
$$R_{i+1}^M = R_0 + \frac{k^{\nu}}{\Gamma(\nu)}\sum_{j=0}^{i} a_{i+1,\,j}H_4(t_j, R_j)$$

$$Q_{i+1}^M = Q_0 + \frac{k^{\nu}}{\Gamma(\nu)}\sum_{j=0}^{i} a_{i+1,\,j}H_5(t_j, Q_j) \tag{16}$$

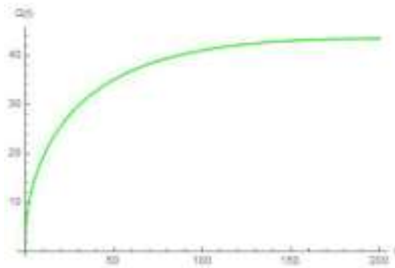
1.3 Graphical Representation



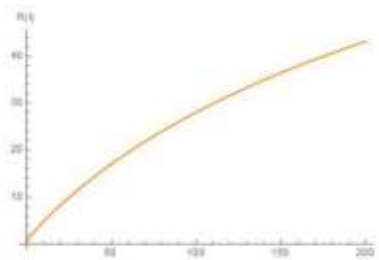
(a) Susceptible



(b) Infected

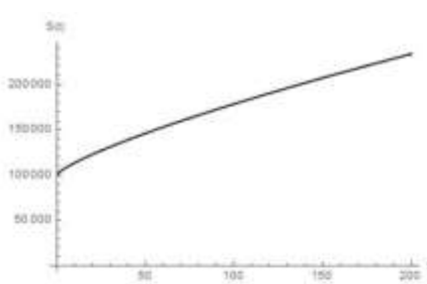


(c) Quarantined

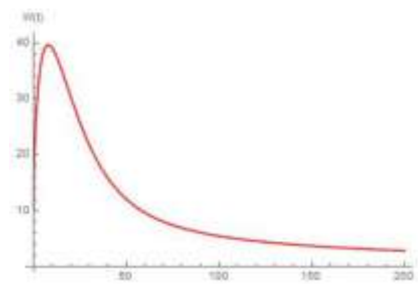


(d) Recovered

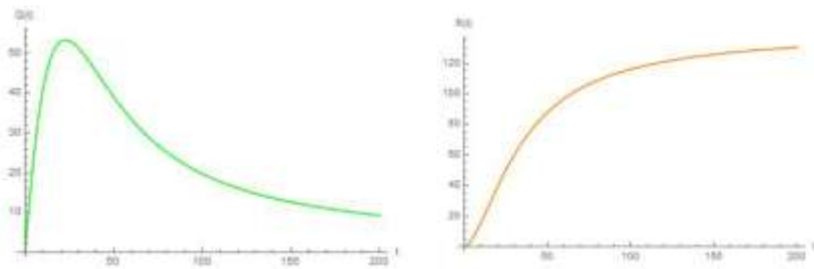
Figure 1. The behaviour of the solution for the fractional order is $\zeta = 0.5$.



(a) Susceptible



(b) Infected



(c) Quarantined (d) Recovered

Figure 2. The behaviour of the solution for the fractional order is $\zeta = 0.98$.

2. Conclusion

In this paper, we have extended the classical mathematical model of zombies attack with the effect of quarantine to the fractional ordered mathematical model using Atangana-Baleanu fractional derivative operator. The existence and the uniqueness of the solution is presented. Graphical representations shows the behavior of different class of zombies population with the change in the fractional order of ζ.

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