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ON PRIME HYPERIDEALS IN TERNARY SEMIHYPERRING

T. ANURADHA and V. LAKSHMI PRASANNAM

Research Scholar
Department of Mathematics
Krishna University
Machilipatnam, India
E-mail: anuradha1982.t@gmail.com

Department of Mathematics
P. B. Sidhartha College of Arts and Science
Siddhartha Nagar, Vijayawada, India
E-mail: drvlp@rediffmail.com

Abstract

In this paper we introduce and characterize the prime hyperideal and hyperideal, the semiprime hyperideal in ternary semi hyperring. Some properties of them are investigated.

1. Introduction

In 1934, Hyper structure theory was born at the Scandinavian Mathematicians8th congress, where the hyper group notion was introduced by Marty [11] as a generalization of groups. Recently, binary relations were studied by Davvaz and Leoreanu-Fotea [4] on ternary semi hyper groups and some basic properties of compatible relations on them. The main purpose of this paper is to introduce and study prime, semi prime and irreducible hyper ideals in ternary semi hyper rings and investigate some basic properties of them. We introduce the concepts of hyper filters and hyper semi lattice congruence of ternary semi hyper rings. We give some characterizations of hyper filters in ternary semi hyper rings. Some relationships between the

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hyper filters and the prime hyper ideals and hyper semi lattice congruences in ternary semi hyper rings are considered.

2. Preliminaries

Definition 2.1. Let H be a non-empty set and $\circ : H \to \wp*(H)$ be a hyperoperation, where $\wp*(H)$ is the family of all non-empty subsets of H. The couple $(\circ, H)$ is called a hypergroupoid. For any two non-empty subsets A and B of H and $x \in H$ we have

$$A \circ B = \bigcap_{a \in A,\, b \in B} a \circ b,\ A \circ \{x\} = A \circ x \text{ and } \{x\} \circ A = x \circ A.$$

Definition 2.2. A ternary hyper grouped is called the pair $(H, [\ ])$ if H_1, H_2, H_3 are the non-empty subsets of H then we define $[H_1 H_2 H_3] = \bigcup_{h_1 \in H_1,\, h_2 \in H_2,\, h_3 \in H_3} [h_1 h_2 h_3]$.

Definition 2.3. A non-empty set H is called ternary semi hyper ring if for all $h_1, h_2, h_3, h_4, h_5 \in H$ and $(H, \oplus)$ is a commutative semi hyper group and the ternary multiplication [ ] satisfies the following conditions

1. $[[h_1 h_2 h_3] h_4 h_5] = [h_1 [h_2 h_3 h_4] h_5] = [h_1 h_2 [h_3 h_4 h_5]]$
2. $[(h_1 \oplus h_2) h_3 h_4] = [h_1 h_3 h_4] \oplus [h_2 h_3 h_4]$
3. $[h_1 (h_2 \oplus h_3) h_4] = [h_1 h_2 h_4] \oplus [h_1 h_3 h_4]$
4. $[h_1 h_2 (h_3 \oplus h_4)] = [h_1 h_2 h_3] \oplus [h_1 h_2 h_4]$.

Definition 2.4. Let $(H, [\ ])$ be a ternary hypergroupoid. Then,

(1) $(H, [\ ])$ is (1, 3)-commutative if for all $h_1, h_2, h_3 \in H$, $[h_1 h_2 h_3] = [h_3 h_2 h_1]$.

(2) $(H, [\ ])$ is (2, 3)-commutative if for all $h_1, h_2, h_3 \in H$, $[h_1 h_2 h_3] = [h_1 h_3 h_2]$.

(3) $(H, [\ ])$ is (1, 2)-commutative if for all $h_1, h_2, h_3 \in H$, $[h_1 h_2 h_3] = [h_2 h_1 h_3]$.

Definition 2.5. A ternary semi hyper ring is called commutative if $[h_1h_2h_3] = [h_2h_3h_1] = [h_3h_1h_2] = [h_2h_1h_3] = [h_3h_2h_1] = [h_1h_3h_2]\ \forall\, h_1, h_2, h_3 \in H$.

Definition 2.6. Let H be a ternary semi hyper ring and G is a non-empty subset of H then G is a ternary sub semi hyper ring of H if and only if $[GGG] \subseteq G$.

Definition 2.7. A ternary hyper semi ring H is said to have a zero element if there exist an element $0 \in H$ such that for all $h_1, h_2 \in H$, $[0h_1h_2] = [h_1 0 h_2] = [h_1h_2 0] = \{0\}$.

Definition 2.8. An element e of ternary hyper semi ring H is called an identity if $[h_1h_2e] = [h_1eh_1] = [eh_1h_1] = \{h_1\}$ for all $h_1 \in H$ and it is clear that $[eeh_1] = [eh_1e] = [h_1ee] = \{h_1\}$.

Definition 2.9. A non empty additive sub semi hyper group I of a ternary semi hyper ring H is called

1. A left hyper ideal of H if $[HHI] \subseteq I$

2. A lateral hyper ideal of H if $[HIH] \subseteq I$

3. A right hyper ideal of H if $[IHH] \subseteq I$.

If I is both left as well as right hyper ideal of H, then I is called a two sided hyper ideal of H. If I is a left, a lateral, a right hyper ideal of H then I is called a hyper ideal of H.

Example 2.10. In example no 2.5, we can easily observe that $I = \langle 2 \rangle = \{2K / K \in Z\}$ is a hyper ideal of Z.

Remark 2.11. Let $(H, \oplus, [\ ])$ be a ternary semi hyper ring for every element $h \in H$ then the left, lateral, right, two sided and hyper ideal generated by h are respectively shown by

$$L(h) = \langle h \rangle_l = \{h\} \cup [HHh]$$

$$M(h) = \langle h \rangle_m = \{h\} \cup [HhH] \cup [H[HhH]H]$$

$$R(h) = \langle h \rangle_m = \{h\} \cup [hHH]$$

$$T(h) = \langle h \rangle_r = \{h\} \cup [HHh] \cup [hHH] \cup [H[HhH]H]$$

$$J(h) = \langle h \rangle = \{h\} \cup [HHh] \cup [HhH] \cup [H[HhH]H] \cup [hHH]$$

Definition 2.12. Let H be a ternary semi hyper ring and M be a hyper ideal of H then M is known as maximal hyper ideal of H if $M \neq H$ and their does not exist any proper ideal of I of H such that $M \subseteq I$.

Lemma 2.13. *If A, B, C are any three hyperideals of a ternary semi hyperring H, then $[ABC]$ is a hyperideal of H.*

Proof. We have $[HH[ABC]] = [[HHA]BC] \subseteq [ABC]$. Since A is hyperideal of H. Therefore $[ABC]$ is a left hyperideal of H.

Again $[HH[ABC]HH] = [[HHA]B[CHH]] = [ABC]$, since A, C are hyperideals of H and hence $[ABC]$ is a lateral hyperideal of H and $[[ABC]HH] = [AB[CHH]] = [ABC]$, since C is a hyperideal of H and hence $[ABC]$ is a right hyperideal of H. Therefore, $[ABC]$ is a hyperideal of H.

3. On Prime left Hyperideals in Ternary Semi Hyperring

In this section we introduce the concept of prime hyper ideals in ternary semi hyperring and characterised prime hyper ideals.

Definition 3.1. Let H be a ternary semi hyperring. A proper hyperideal P of H is called prime hyperideal of H if $[ABC] \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$ for any three hyperideals A, B, C of H.

Definition 3.2. Let H be a ternary semihyperring. A proper hyperideal P of H is called semiprime hyperideal of H if $[AAA] \subseteq P$ implies $A \subseteq P$ for any hyperideal A of H.

It is clear that every prime hyperideal of a ternary semi hyperring H is also a semiprime hyperideal of H.

Definition 3.3. Let H be a ternary semihyperring. A proper hyperideal P of H is said to be irreducible, if for hyperideals J and K of H, $J \cap K = P$ implies that $P = J$ or $P = K$ or equivalently, $J \cap K = P$ implies that $J \subseteq P$ or $K \subseteq P$.

Example 3.4. Let $H = \{a, b, c, d, e, f\}$ and $[xyz] = (x \cdot y) \cdot z$ for all $x, y, z \in H$, where "." is defined by the table:

$\oplus$	a	b	c	d	e	f
a	a	$\{a, \mathrm{b}\}$	c	$\{c, d\}$	e	$\{e, f\}$
b	b	b	d	d	f	f
c	c	$\{c, d\}$	c	$\{\mathrm{c}, \mathrm{d}\}$	c	$\{c, d\}$
d	d	d	d	d	d	d
e	e	$\{e, f\}$	c	$\{c, d\}$	e	$\{e, f\}$
f	f	f	D	d	f	f

.	a	b	c	d	e	f
a	a	b	c	c	e	e
b	b	b	d	d	f	f
c	c	d	c	d	c	c
d	d	d	d	d	d	d
e	e	f	c	c	e	f
f	f	f	d	d	f	f

Then, H is a ternary semihyperring. Clearly, $I_1 = \{c, d\}, I_2 = \{c, d, e, f\}$ and H are left hyperideals of H. It can be easily verified that I_2 and H are prime left hyperideals. I_1 is irreducible left hyperideal.

Theorem 3.5. *Let H be a ternary semihyperring, P a semiprime hyperideal of H and $h \in H$. Then, $h \in P$ if and only if $[HHhHH] \subseteq P$.*

Proof. Assume that P is a semiprime hyperideal of H.

If $a \in P$, then, $[HHhHH] \subseteq [HHPHH] \subseteq P$.

Conversely, assume that $[HHhHH] \subseteq P$ where P is a semiprime hyperideal of H. Then

$[\langle h\rangle\langle h\rangle\langle h\rangle] = [[\{h\}\cup[HHh]\cup[HhH]\cup[H[HhH]H]\cup[hHH]]]$ $[\{h\}\cup[HHh]\cup[HhH]\cup[H[HhH]H]\cup[hHH]][\{h\}\cup[HHh]\cup[HhH]$ $\cup[H[HhH]H]\cup[hHH]] \subseteq [HHhHH] \subseteq P$. Since P is a semiprime hyperideal, we have $\langle h\rangle \subseteq P$ and hence $h \in P$.

Theorem 3.6. *Let H be a ternary semi hyperring and P a hyperideal of H. The following statements are equivalent*:

(1) *P is a prime hyperideal of H.*

(2) $[HHaHHbHHcHH] \subseteq P$ *implies* $a \in P$ *or* $b \in P$ *or* $c \in P$.

(3) $[\langle a\rangle\langle b\rangle\langle c\rangle] \subseteq P$ *implies* $a \in P$ *or* $b \in P$ *or* $c \in P$.

Proof. (1) $\Rightarrow$ (2). Assume that P is a prime hyperideal of H and $[HHaHHbHHcHH] \subseteq P$. Then, $[HH[aHHbHHc]HH] \subseteq [HHPHH] \subseteq P$. This implies $[[HHa][HHbHH][cHH]] \subseteq P$. Since P is a prime hyperideal of H, so $[HHa] \subseteq P$ or $[HHbHH] \subseteq P$ or $[cHH] \subseteq P$.

Since every prime hyperideal is semiprime, so P is semiprime hyperideal.

Hence, by Theorem 3.5, $a \in P$ or $b \in P$ or $c \in P$.

(2) $\Rightarrow$ (3). $[\langle a\rangle\langle b\rangle\langle c\rangle] \subseteq P$ for some $a, b, c \in P$.

Then, $[HHaHHbHHcHH] = [[aHH][HHbHH][cHH]] \subseteq [\langle a\rangle\langle b\rangle\langle c\rangle] \subseteq P$.

Thus, by (2) we have $a \in P$ or $b \in P$ or $c \in P$.

(3) $\Rightarrow$ (1). Let A, B, C be three hyperideals of H such that $[ABC] \subseteq P$. Let $B \not\subseteq P$ and $C \not\subseteq P$. Let $b \in B$ and $c \in C$ such that $b, c \notin P$.

Then, for every $a \in A$ we have $[\langle a\rangle\langle b\rangle\langle c\rangle] \subseteq [ABC] \subseteq P$.

By (3) we get $a \in P$ or $b \in P$ or $c \in P$, but $b, c \notin P$, so $a \in P$ and hence $A \subseteq P$.

Therefore, P is a prime hyperideal of H.

Corollary 3.7. *Let H be a commutative ternary semihyperring and P a hyperideal of H. Then, P is a prime hyperideal if and only if* $[abc] \subseteq P$ *implies* $a \in P$ *or* $b \in P$ *or* $c \in P$ *for all* $a, b, c \in H$.

It can be easily seen that the above result is also valid for (1, 3)-commutative ternary semi hyperring.

Theorem 3.8. *Any maximal hyperideal of H is a prime hyperideal.*

Proof. Let M be any maximal hyperideal of H. To show that M is a prime let $[HHaHHbHHcHH] \subseteq M$. Suppose that $a, b \notin M$. $[HaHHbH]$ is a hyperideal of H which contains a, b. By M is a hyperideal, $M + [HaHHbH] = H$. As $1 \in H, 1 = m + \sum_{i=1}^{n} s_t a t_t b y_t$, then $1cc = (m + \sum_{i=1}^{n} s_i a t_i x_i b y_i)cc \subseteq M + [HaHHbHHcc] \subseteq M + [HHaHHbHHcHH] \subseteq M.$

Therefore, $c \in M$. Hence M is a prime hyperideal.

Definition 3.9. Let H be a ternary semihyperring. A nonempty subset A of H is called an m-system if for every $a, b, c \in A$ there exist the elements $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$ such that $[h_1 h_2 a h_3 h_4 b h_5 c h_7 h_8] \subseteq A$.

Theorem 3.10. *Let H be a ternary semihyperring and P a proper hyperideal of H. P is a prime hyperideal if and only if its complement* $H \setminus P$ *is an m-system.*

Proof. Let P be a prime hyperideal of H. Assume that $a, b, c \notin P$. Then, $a, b, c \in H \setminus P$. Let assume that $H \setminus P$ is not an m-system. Then, for all $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H, [h_1 h_2 a h_3 h_4 b h_5 h_6 c h_7 h_8] \not\subseteq H \setminus P$. Thus, $[h_1 h_2 a h_3 h_4 b h_5 h_6 c h_7 h_8] \subseteq P$. Since P is prime hyperideal of H, by Theorem 3.6, we get $a \in P$ or $b \in P$ or $c \in P$. It is impossible. Hence, $H \setminus P$ is a m-system.

Conversely, suppose that $H \setminus P$ is an m-system. Let $a, b, c \in H \setminus P$. Then, there exist $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$ such that $[h_1 h_2 a h_3 h_4 b h_5 h_6 c h_7 h_8] \subseteq H \setminus P$. Thus, $[h_1 h_2 a h_3 h_4 b h_5 h_6 c h_7 h_8] \not\subseteq H \setminus P$. Hence, if $a, b, c \not\subseteq P$, then $[HHaHHbHHcHH] \not\subseteq P$. Thus, by Theorem 3.6, P is a prime hyperideal of H.

Definition 3.11. Let H be a ternary semi hyperring and P a hyperideal of H. Then, P is called a completely prime hyperideal of H if $[abc] \in P$ implies $a \in P$ or $b \in P$ or $c \in P$ for every triplet of elements $a, b, c \in H$.

Theorem 3.12. *Every completely prime ideal of a ternary semi hyperring H is a prime hyperideal of H.*

Proof. Suppose that P is a completely prime hyperideal of a ternary semihyperring H.

Let $a, b, c \in H$ and $[\langle a \rangle \langle b \rangle \langle c \rangle] \subseteq P$. Then $[abc] \in P$.

Since P is a completely prime, either $a \in P$ or $b \in P$ or $c \in P$.

Therefore P is a prime hyperideal of H.

The converse in general may not be true.

Theorem 3.13. *In a commutative ternary semi hyperring a hyperideal is completely prime if and only if it is prime hyperideal.*

Proof. Suppose that Q is a prime hyperideal of a ternary semi hyperring H.

Let $x, y, z \in H, [xyz] \subseteq Q$. Now $[xyz] \subseteq Q, Q$ is a hyperideal of $H \Rightarrow [xyzHHHHHHHH] \subseteq Q$, where H is the ternary semi hyperring with ternary multiplication identity.

Since H is commutative, $[[HHx][HHyHH]zHH] = [xyzHHHHHHHH] \subseteq Q$.

By theorem 3.6, either $x \in Q$ or $y \in Q$ or $z \in Q$.

Hence Q is a completely prime hyperideal of H.

Conversely suppose that Q is a completely prime hyperideal of H. By theorem 3.12, Q is a prime hyperideal of H.

Lemma 3.14. *Let H be a ternary semi hyperring. A hyperideal P of a ternary semi hyperring H is completely prime if and only if* $H \setminus P$ *is ternary sub-semi hyperring of H.*

Theorem 3.15. *A hyperideal A of a ternary semi hyperring H is completely prime if and only if* $x_1, x_2, \ldots, x_n \in H$, *n is odd natural number,* $[x_1, x_2, \ldots, x_n] \in A \Rightarrow x_i \in A$ *for some* $i = 1, 2, 3 \ldots, n$.

Proof. Suppose that A is a completely prime hyperideal of H.

Let $x_1, x_2, \ldots, x_n \in H$, where n is odd natural number and $x_1, x_2, \ldots, x_n \in A$.

If $n = 1$ then clearly $x_1 \in A$.

If $n = 3$ then $[x_1x_2x_3] \in A \Rightarrow x_1 \in A$ or $x_2 \in A$ or $x_3 \in A$.

If $n = 5$ then $[[x_1x_2x_3]x_4x_5] \in A \Rightarrow [x_1x_2x_3] \in A$ or $x_4 \in A$ or $x_5 \in A \Rightarrow x_1 \in A$ or $x_2 \in A$ or $x_3 \in A$ or $x_4 \in A$ or $x_5 \in A$.

Therefore by induction of n is an odd natural number, then $x_1, x_2, \ldots, x_n \in A \Rightarrow x_i \in A$ for some $i = 1, 2, 3, \ldots, n$. The converse part is trivial.

Theorem 3.16. *Let H is a ternary semihyperring with identity. Then, every maximal hyperideal of H is a prime hyperideal of H.*

Proof. Let P be a maximal hyperideal of H. Let A, B, C be three hyperideals of H such that $[ABC] \subseteq P$. Suppose that $A, B \not\subseteq P$. Then, $A \cup P = H$ and $B \cup P = H$. Since $e \in H$, we have $e \in A \cup P$ and $e \in B \cup P$. Thus, $e \in A$ or $e \in P$ and $e \in B$ or $e \in P$. Since $e \notin P$, we have $e \in A$ and $e \in B$ implies $A = H$ and $B = H$. Now since $e \in H$, we have $C = [HHC] = [ABC] \subseteq P$ implies $C \subseteq P$. Therefore, P is a prime hyperideal of H.

Theorem 3.17. *Let H be a ternary semi hyperring, P be a m-system and Q be a hyperideal of H such that $P \cap Q = \emptyset$. Then, there exists a maximal hyperideal M of H containing Q and $M \cap Q = \emptyset$. Further, M is also a prime hyperideal of H.*

Proof. Let $\tau_P = \{B : B$ is a prime hyperideal of $H, Q \subseteq B, B \cap P = \varnothing\}$. Since $Q \in \tau_P, \tau_P \neq \varnothing$. τ_P is partially ordered set by set inclusion. Let $\{M_i\}$ be an arbitrary chain in τ_P. Since union of hyperideals is a hyperideal, $\bigcup_{i \in I} M_i$ is a hyperideal of H. Since $Q \subseteq M_i$ for all $i \in I$, we have $Q \subseteq \bigcup_{i \in I} M_i$. Assume that $(\bigcup_{i \in I} M_i) \cap P \neq \varnothing$.

Then, there exist some $a \in H$ such that $a \in (\bigcup_{i \in I} M_i) \cap P$. This implies $a \in \bigcup_{i \in I} M_i$ and $a \in P$. Thus, $a \in M_i$ for some $i \in I$ and $a \in P$.

Thus, $M_i \cap P \neq \varnothing$. It is impossible. Hence, $(\bigcup_{i \in I} M_i) \cap P = \varnothing$. Thus, $\bigcup_{i \in I} M_i$ is an upper bound of $\{M_i\}$. Since $\{M_i\}$ is an arbitrary chain, we have that every chain in τ_P has an upper bound in τ_P. Hence, by Zorn's Lemma the family τ_P contains a maximal element *M*. We will show that *M* is a prime hyperideal of *H*. Let A, B, C be three hyperideals of *H* such that $[ABC] \subseteq M$. Assume that $A \not\subseteq M$, $B \not\subseteq M$ and $C \not\subseteq M$. Then, there exist $a \in A$, $b \in B$ and $c \in C$ such that $a, b, c \notin M$. Now $\langle a \rangle \cup M$, $\langle b \rangle \cup M$ and $\langle c \rangle \cup M$ are hyperideals of H properly containing *M*, so $(\langle a \rangle \cup M) \cap P \neq \varnothing$, $(\langle b \rangle \cup M) \cap P \neq \varnothing$ and $(\langle c \rangle \cup M) \cap P \neq \varnothing$. Let $x \in (\langle a \rangle \cup M) \cap P$, $y \in (\langle b \rangle \cup M) \cap P$ and $z \in (\langle c \rangle \cup M) \cap P$. Since $x, y, z \in P$ and P is m-system, we have $[h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8] \subseteq P$ for some $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$. Also $[h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8] \subseteq [h_1 h_2 (\langle a \rangle \cup M) h_3 h_4 (\langle b \rangle \cup M) h_4 h_5 (\langle c \rangle \cup M) h_7 h_8]$. We have.

Case I. If $x \in \langle a \rangle \subseteq A$, $y \in \langle b \rangle \subseteq B$, $z \in \langle c \rangle \subseteq C$, then $[h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8] \subseteq [[HHA][HHBHH][CHH]] \subseteq [ABC] \subseteq M$.

Case II: If $x \in M$, then $[h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8] \subseteq [[HHM][HHMHH][MHH]] \subseteq [MMM] \subseteq M$.

Similarly, if $y \in M$ or $z \in M$, then $[h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8] \subseteq [HH[HMH]HH] \subseteq [HMH] \subseteq M$ and $[h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8] \subseteq [HHHH[HHM]HHHH] \subseteq [HHMHH] \subseteq M$. Hence, $\varnothing P \cap M \neq \varnothing$, it is impossible. Thus, $A \subseteq M$ or $B \subseteq M$ or $C \subseteq M$. Therefore, *M* is a prime hyperideal of *H*.

4. Semi Prime Hyperideals

In this section, we introduce the concepts of "semi prime hyper ideals in ternary semi hyper tings". We give some characterizations of "prime hyper ideals in ternary semi hyper tings". Some relationships between the "semi prime hyper ideal", "prime hyperideals and completely semiprime hyper ideals in ternary semi hyper tings" are considered.

Theorem 4.1. *Let H be a ternary semi hyperring and P a hyperideal of H. The following statements are equivalent*:

(1) *P is a "semiprime hyperideal" of H.*

(2) $[HHaHHaHHaHH] \subseteq P$ *implies* $a \in P$.

(3) $[\langle a\rangle\langle a\rangle\langle a\rangle] \subseteq P$ *implies* $a \in P$.

Proof. (1) $\Rightarrow$ (2). Assume that P is a semiprime hyperideal of H and $[HHaHHaHHaHH] \subseteq P$. Then, $[HH[aHHaHHaHH]HH] \subseteq [HHPHH] \subseteq P$. This implies $[[HHa][HHaHH][aHH]] \subseteq P$. Since P is a semiprime hyperideal of H, so $[HHa] \subseteq P$ or $[HHaHH] \subseteq P$ or $[aHH] \subseteq P$.

Hence, by Theorem 3.5, $a \in P$.

(2) $\Rightarrow$ (3). $[\langle a\rangle\langle a\rangle\langle a\rangle] \subseteq P$ for some $a \in P$.

Then, $[HHaHHaHHaHH] = [[aHH][HHaHH][aHH]] \subseteq [\langle a\rangle\langle a\rangle\langle a\rangle] \subseteq P$.

Thus, by (2) we have $a \in P$.

(3) $\Rightarrow$ (1). Let A be a hyperideal of H such that $[AAA] \subseteq P$. Let $A \not\subseteq P$. Let $a \in A$ such that $a \notin P$. Then, for every $a \in A$ we have $[\langle a\rangle\langle a\rangle\langle a\rangle] \subseteq [AAA] \subseteq P$.

By (3) we get $a \in P$, but $a \notin P$ which is impossible and hence $A \subseteq P$.

Therefore, P is a semiprime hyperideal of H.

Corollary 4.2. *Let H be a commutative ternary semi hyperring and P a hyperideal of H. Then, P is a semi prime hyperideal if and only if* $[aaa] \subseteq P$ *implies* $a \in P$ *for all* $a \in H$.

Proof. Let H be a commutative ternary semihyperring and P a semi prime hyperideal of H and $[aaa] \subseteq P$ for all $a \in H$. Then we have

$[HH[[[HH[aaa]]HH]HH]] \subseteq [HHPHH] \subseteq P$. Since H is a commutative ternary semi hyperring which implies that, $[HHaHHaHHaHH] \subseteq P$. By theorem 4.1, P is semiprime hyperideal of H implies that $a \in P$.

Conversely suppose that $[aaa] \subseteq P$ implies $a \in P$ for all $a \in H$. Let A be a hyperideal of H such that $[AAA] \subseteq P$. Let $A \not\subseteq P$. Let $a \in A$ such that $a \notin P$. Then, for every $a \in A$ we have $[aaa] \subseteq [AAA] \subseteq P$ implies that $a \in P$ which is contradicts to our assumption. Therefore $A \subseteq P$ and hence P is a semiprime hyperideal of H.

Definition 4.3. Let H be a ternary semihyperring. A nonempty subset A of H is called an p-system if for every $a \in A$ there exist the elements $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$ such that $[h_1h_2h_3h_4h_5h_6h_7h_8] \subseteq A$.

Note 4.4. It is clear that in a ternary semi hyperring H every m-system is a p-system and union of p-systems is again a p-system too.

Theorem 4.5. *Let H be a ternary semi hyperring and P a proper hyperideal of H. P is a semi prime hyperideal if and only if its complement $H \setminus P$ is an p-system.*

Proof. Let P be a prime hyperideal of H. Assume that $a \notin P$. Then, $a \in H \setminus P$. Let assume that $H \setminus P$ is not an p-system. Then, for all $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$, $[h_1h_2ahh_3h_4ah_5h_6ah_7h_8] \not\subseteq H \setminus P$. Thus, $[h_1h_2ahh_3h_4ah_5h_6ah_7h_8] \subseteq P$. Since P is semi prime hyperideal of H, so by Theorem 3.4, we get $a \in P$. It is impossible. Hence, $H \setminus P$ is a p-system.

Conversely, suppose that $H \setminus P$ is a p-system. Let $a \in H \setminus P$. Then, there exist $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$ such that $[h_1h_2ahh_3h_4ah_5h_6ah_7h_8] \subseteq H \setminus P$. Thus, $[h_1h_2ahh_3h_4ah_5h_6ah_7h_8] \not\subseteq P$. Hence, if $a \not\subseteq P$, then $[HHaHHaHHaHH] \not\subseteq P$. Thus, by Theorem 4.1, P is a semiprime hyperideal of H.

Definition 4.6. Let H be a ternary semi hyperring and P a hyperideal of H. Then, P is called a completely semi prime hyperideal of H if $[aaa] \in P$ implies $a \in P$ for every element $a \in H$.

Theorem 4.7. *An hyperideal A of a ternary semi hyperring H is completely prime if and only if* $x \in H, n$ *is odd natural number*, $[x_n] \in A \Rightarrow x \in A$.

Proof. Suppose that A is a completely semi prime hyperideal of H.

Let $x \in H$ where n is odd natural number and $[x_n] \in A$.

If $n = 1$ then clearly $x \in A$.

If $n = 3$ then $[xxx] \in A \Rightarrow x$.

If $n = 5$ then $[[xxx]xx] \in A \Rightarrow [xxx] \in A$ or $x \in A \Rightarrow x$.

Therefore by induction of n is an odd natural number, then $[x_n] \in A \Rightarrow x \in A$.

The converse part is trivial.

Theorem 4.8. *Every completely prime hyperideal of a ternary semihyperring H is a completely semi prime hyperideal of H.*

Proof. Let A be a completely prime hyperideal of a ternary semi hyperring H.

Suppose that $x \in H$ and $[xxx] \in A$. Since A is a completely prime hyperideal of H, $x \in A$.

Therefore, H is a completely semiprime hyperideal.

Theorem 4.9. *Let H be a ternary semi hyperring, P be a p-system and Q be a hyperideal of H such that $P \cap Q = \varnothing$. Then, there exists a maximal hyperideal M of H containing Q and $M \cap Q = \varnothing$. Further, M is also a semiprime hyperideal of H.*

Proof. Let $\tau_P = \{B : B$ is a semiprime hyperideal of $H, Q \subseteq B, B \cap P = \varnothing\}$. Since $Q \in \tau_P$, $\tau_P \neq \varnothing$. τ_P is partially ordered set by set inclusion. Let $\{M_i\}$ be an arbitrary chain in τ_P. Since union of hyperideals is a hyperideal, $\bigcup_{i \in I} M_i$ is a hyperideal of H. Since $Q \subseteq M_i$ for all $i \in I$, we have $Q \subseteq \bigcup_{i \in I} M_i$. Assume that $(\bigcup_{i \in I} M_i) \cap P \neq \varnothing$. Then, there exist some $a \in H$ such that $a \in (\bigcup_{i \in I} M_i) \cap P$. This implies $a \in \bigcup_{i \in I} M_i$ and $a \in P$. Thus, $a \in M_i$ for some $i \in I$ and $a \in P$. Thus, $M_i \cap P \neq \emptyset$. It is impossible. Hence, $(\bigcup_{i \in I} M_i) \cap P = \emptyset$. Thus, $\bigcup_{i \in I} M_i$ is an upper bound of $\{M_i\}$. Since $\{M_i\}$ is an arbitrary chain, we have that every chain in τ_P has an upper bound in τ_P. Hence, by Zorn's Lemma the family τ_P contains a maximal element *M*. We will show that *M* is a semiprime hyperideal of *H*. Let *A* is a hyperideals of *H* such that $[AAA] \subseteq M$. Assume that $A \not\subseteq M$. Then, there exist $a \in A$ such that $a \notin M$. Now $\langle a \rangle \cup M$ is hyperideals of *H* properly containing *M*, so $(\langle a \rangle \cup M) \cap P = \emptyset$.

Let $x \in (\langle a \rangle \cup M) \cap P$. Since $x \in P$ and P is p-system, we have $[h_1 h_2 x h_3 h_4 x h_5 h_6 x_{h7} h_8] \subseteq P$ for some $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8 \in H$. Also $[h_1 h_2 x h_3 h_4 x h_5 h_6 x_{h7} h_8] \subseteq [h_1 h_2 (\langle a \rangle \cup M) h_3 h_4 (\langle x \rangle \cup M) h_5 h_6 (\langle x \rangle \cup M) h_7 h_8]$. We have

Case I. If $x \in \langle a \rangle \subseteq A$, then $[h_1 h_2 x h_3 h_4 x h_5 h_6 x_{h7} h_8] \subseteq [[HHA][HHAHH][AHH]] \subseteq [AAA] \subseteq M$.

Case II. If $x \in M$, then $[h_1 h_2 x h_3 h_4 x h_5 h_6 x_{h7} h_8] \subseteq [[HHM][HHMHH][MHH]] \subseteq [MMM] \subseteq M$.

Similarly, $[HMH] \subseteq M$ and $[HHMHH] \subseteq M$. Hence, $P \cap M \neq \phi$, it is impossible. Thus, $A \subseteq M$. Therefore, *M* is a semi prime hyperideal of *H*.

5. Conclusion

The "ternary semihyperring" is a generalization of the concepts of a "semiring", a "semihyperring". Since, the notions of "prime hyperideals", "semiprime hyperideals"; in a "ternary semihyperring" are introduced, and several examples given and characterized those hyperideals.

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