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A NEW PRODUCT ROOT SUM MEAN LABELING ON SOME SPECIAL GRAPHS

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Abstract

In this paper we introduce New Labeling called Product Root Sum Mean Labeling. It is defined as let $G = (p, q)$ be a graph an injective function $f : v \to \{1, 2, 3, \cdots, q+1\}$ is said to be a product root sum mean labeling if the induced function f^* defined by $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$ yield different values. Here we examine the existence of this labeling for some graphs such as Path, Middle graph of a Path, Total graph of a Path, Split graph of a Path, Shadow graph of a Path, Cycle, Middle graph of a Cycle, Triangular Snake, Alternate Triangular Snake, Double Triangular Snake, Alternate Triangular Snake, Kite, Quadrilateral snake graph, Double Quadrilateral snake graph and Alternative Quadrilateral snake graph.

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1. Introduction

Graph theory is the fast-growing area of combinatorics. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for a wide range of applications. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, cordial labeling, prime cordial labeling. Magic labelling, Anti magic labeling etc., have been studied in over 2100 papers [2]. Several researchers refer to Rosa's [4] work. The concept of Root Square Mean Labeling in Graphs has been introduced by Sandhya, Somasundaram and Anusa [5]. Motivated by the above works in this paper we introduce a new type of labeling called Product Root Sum Mean labeling and investigate the existence of this for graphs namely Path, Middle graph of a Path, Total graph of a Path, Split graph of a Path, Shadow graph of a Path, Cycle, Middle graph of a Cycle, Triangular Snake, Alternate Triangular Snake, Double Triangular Snake, Alternate Triangular Snake, Kite, Quadrilateral snake graph, Double Quadrilateral snake graph and Alternative Quadrilateral snake graph.

2. Preliminaries

Definition 2.1. A walk in which the vertices $u_1, u_2, \cdots, u_n$ are distinct is called a path. A path on *n* vertices is denoted by P_n. A closed path is called a cycle. A cycle on *n* vertices is denoted by C_n.

Definition 2.2. The middle graph of a connected graph G denoted by $M(G)$ is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

Definition 2.3. The total graph, $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.

Definition. 2.4. For a graph G the split graph is obtained by adding to each vertex v a new vertex such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as $\mathrm{spl}(G)$.

Definition 2.5. The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G" join each vertex v' in G; to the neighbours of the corresponding vertex v" in G".

Definition 2. 6. Square graph of a graph G denoted by G^2 has the same vertex set as of G and two vertices are adjacent in G^2, if they are at a distance of 1 or 2 apart from G.

Definition 2. 7. A Triangular Snake T_n is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a triangle C_3.

Definition 2.8. A Quadrilateral Snake Q_n is obtained from a path $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining v_i and w_i. That is every edge of a path is replaced by a cycle C_4.

Definition 2.9. The Kite graph is obtained by attaching a path of length 'm' with a cycle of length 'n' and it is denoted as $K_{n,m}$.

3. Main Results

Theorem 3.1. *The Path graph P_n is a Product Root Sum Mean graph.*

Proof. Let the vertex set be $V = \{v_1, v_2, \cdots, v_{n-1}, v_n\}$. The edge set is $E = \{v_i, v_{i+1}\}$ for all values of n. Define a map $f : V \to \{1, 2, 3, \cdots, q+1\}$. Such that $f(v_i) = i, 1 \leq i \leq n$; Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$. The edge labels are obtained as

$$f^*(v_i v_{i+1}) = \frac{[i * (i+1) + \sqrt{i + (i+1)}]}{2} = \frac{(i^2 + i) + \sqrt{2i+1}}{2}, \; 1 \leq i \leq n-1.$$

Thus the edges $\left\{2, 5, 8, \cdots, \frac{(i^2 + i) + \sqrt{2i+2}}{2}\right\}$ are all distinct. Hence the theorem.

Theorem 3.2. *The Middle graph of path* $M(P_n)$ *admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \cdots, u_{n-1}, v_1, v_2, \cdots v_{n-1}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n-2 \cup u_i v_i, 1 \le i \le n-1 \cup u_i v_{i+1}, 1 \le i \le n-1\}$; for all values of n. Define a map $f : V \to \{1, 2, 3, \cdots q+1\}$. Such that $f(u_i) = 2i, 1 \le i \le n-1$; $f(v_i) = 2i-1, 1 \le i \le n$. Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$. The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[2i * (2i+2) + \sqrt{2i + (2i+2)}]}{2} = \frac{(4i^2 + 4i) + \sqrt{4i+2}}{2}, 1 \le i \le n-2.$$

$$f^*(v_i u_i) = \frac{[(2i-1) * 2i + \sqrt{(2i-1) + 2i}]}{2} = \frac{(4i^2 + 4i) + \sqrt{4i+2}}{2}, 1 \le i \le n-1.$$

$$f^*(u_i v_{i+1}) = \frac{[2i * (2i+1) + \sqrt{2i + (2i+1)}]}{2} = \frac{(4i^2 + 2i) + \sqrt{4i+1}}{2}, 1 \le i \le n-1.$$

Thus the edges $\{5, 13, 25, \cdots, \frac{(4i^2 + 4i) + \sqrt{4i+2}}{2}; 1, 7, 16, \cdots, \frac{(4i^2 + 4i) + \sqrt{4i+2}}{2}; 4, 11, 22, \ldots, \frac{(4i^2 + 2i) + \sqrt{4i+1}}{2}$ are all distinct. Hence the theorem.

Theorem 3.3. *The Total graph of path* $T(P_n)$ *is a Product Root Sum Mean graph.*

Proof. Let the vertex set be $V = \{u_1, u_2, \cdots, u_{n-1}, v_1, v_2, \cdots, v_n\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n-2 \cup v_i u_i, 1 \le i \le n-1 \cup u_i v_{i+1}, 1 \le i \le n-1; v_i v_{i+1}, 1 \le i \le n-1\}$ for all values of n. Define a map $f : V \to \{1, 2, 3, \cdots q+1\}$. Such that $f(u_i) = 2i, 1 \le i \le n-1$; $f(v_i) = 2i-1, 1 \le i \le n$. Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$. The edge labels are obtained as

$$f^*(u_iu_{i+1}) = \frac{[2i*(2i+2)+\sqrt{2i+(2i+2)}]}{2} = \frac{(4i^2+4i)+\sqrt{4i+2}}{2}, 1 \le i \le n-2.$$

$$f^*(v_iu_i) = \frac{[(2i-1)*2i+\sqrt{(2i-1)+2i}]}{2} = \frac{(4i^2+4i)+\sqrt{4i+2}}{2}, 1 \le i \le n-1.$$

$$f^*(u_iv_{i+1}) = \frac{[2i*(2i+1)+\sqrt{2i+(2i+1)}]}{2} = \frac{(4i^2+2i)+\sqrt{4i+1}}{2}, 1 \le i \le n-1.$$

$$f^*(v_iv_{i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{(4i^2-1)+\sqrt{4i}}{2}, 1 \le i \le n-1.$$

Thus the edges $\{5, 13, 25, \cdots, \frac{(4i^2+4i)+\sqrt{4i+2}}{2}; 1, 7, 16, \cdots, \frac{(4i^2+4i)+\sqrt{4i+2}}{2}; 4, 11, 22, \ldots, \frac{(4i^2+2i)+\sqrt{4i+1}}{2}; 2, 8, 19, \ldots, \frac{(4i^2-1)+\sqrt{4i}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3. 4. *The Split graph of path $Spl(P_n)$ admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n-1 \cup v_i u_{i+1}, 1 \le i \le n-1 \cup u_i v_{i+1}, 1 \le i \le n-1\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that $f(u_i) = 2i, 1 \le i \le n$; $f(v_i) = 2i-1, 1 \le i \le n$. Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. The edge labels are obtained as

$$f^*(u_iu_{i+1}) = \frac{[2i*(2i+2)+\sqrt{2i+(2i+2)}]}{2} = \frac{(4i^2+4i)+\sqrt{4i+2}}{2}, 1 \le i \le n-1.$$

$$f^*(v_iu_{i+1}) = \frac{[(2i-1)*(2i+2)+\sqrt{(2i-1)+(2i+2)}]}{2} = \frac{(4i^2+2i-2)+\sqrt{4i+1}}{2},$$

$$1 \le i \le n-1.$$

$$f^*(u_i v_{i+1}) = \frac{[2i*(2i+1)+\sqrt{2i+(2i+1)}]}{2} = \frac{(4i^2+2i)+\sqrt{4i+1}}{2}, 1 \le i \le n-1.$$

Thus the edges $\{5, 13, 25, \cdots, \frac{(4i^2+4i)+\sqrt{4i+2}}{2}; 3, 10, 21, \cdots, \frac{(4i^2+2i-2)+\sqrt{4i+1}}{2}; 4, 11, 22, \ldots, \frac{(4i^2+2i)+\sqrt{4i+1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.5. *The Shadow graph of path* $D_2(P_n)$ *is a Product Root Sum Mean graph.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n \cup v_i v_{i+1}, 1 \le i \le n \cup v_i u_{i+1}, 1 \le i \le n-1 \cup u_i v_{i+1}, 1 \le i \le n-1\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that $f(u_i) = 2i, 1 \le i \le n$; $f(v_i) = 2i-1, 1 \le i \le n$. Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$. The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[2i*(2i+2)+\sqrt{2i+(2i+2)}]}{2} = \frac{(4i^2+4i)+\sqrt{4i+2}}{2}, 1 \le i \le n-1.$$

$$f^*(v_i u_{i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{(4i^2-1)+\sqrt{4i}}{2},$$

$$1 \le i \le n-1.$$

$$f^*(u_i v_{i+1}) = \frac{[(2i-1)*(2i+2)+\sqrt{(2i-1)+(2i+2)}]}{2} = \frac{(4i^2+2i-2)+\sqrt{4i+1}}{2},$$

$$1 \le i \le n-1.$$

$$f^*(u_i v_{i+1}) = \frac{[2i*(2i+1)+\sqrt{2i+(2i+1)}]}{2} = \frac{(4i^2+2i)+\sqrt{4i+1}}{2}, 1 \le i \le n-1.$$

Thus the edges $\{5, 13, 25, \ldots, \frac{(4i^2+4i)+\sqrt{4i+2}}{2}; 2, 8, 19, \ldots,$

$\frac{(4i^2-1)+\sqrt{4i}}{2}$; 3, 10, 21, ..., $\frac{(4i^2+2i-2)+\sqrt{4i+1}}{2}$ 4, 11, 22, ...,

$\frac{(4i^2+2i)+\sqrt{4i+1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.6. *The Cycle graph* C_n *is a Product Root Sum Mean graph.*

Proof. Case (i): If n is Odd.

Let the vertex set be $V = \{u_1, u_2, \ldots, u_{(n+1)/2}, v_1, v_2, \ldots, v_{(n-1)/2}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le \frac{n-1}{2}; v_i v_{i+1}, 1 \le i \le \frac{n-3}{2}; u_1 v_1; \frac{u_{n+1}}{2} \frac{v_{n-1}}{2}\}$ for all values of n. Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that

$$f(u_i) = 2i - 1, 1 \le i \le \frac{n+1}{2}.$$

$$f(v_i) = 2i, 1 \le i \le \frac{n-1}{2}.$$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$. The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(2i-1)*(2i+1)+\sqrt{(2i-1)+(2i+1)}]}{2} = \frac{[(4i^2-1)+\sqrt{4i}]}{2}, 1 \le i \le \frac{n-1}{2}.$$

$$f^*(v_i u_{i+1}) = \frac{[(2i)*(2i+2)+\sqrt{(2i)+(2i+2)}]}{2} = \frac{[(4i^2+4i)+\sqrt{4i+2}]}{2},$$

$$1 \le i \le \frac{n-3}{2}.$$

$$f^*(u_1 v_1) = \frac{2+\sqrt{3}}{2} = 2.$$

$$f^*(u_{\frac{n+1}{2}} v_{\frac{n-1}{2}}) = \frac{n*(n-1)+\sqrt{n+n-1}}{2} = \frac{n^2-n+\sqrt{2n-1}}{2}.$$

Thus the edges $\{2, 8, 19, \ldots, \frac{[(4i^2-1)+\sqrt{4i}]}{2}$; 5, 13, 25, ..., $\frac{[(4i^2+4i)+\sqrt{4i+2}]}{2}$; 2; $\frac{n^2-n+\sqrt{2n-1}}{2}\}$ are all distinct.

Case (i). If n is even.

Let the vertex set be $V = \{u_1, u_2, \ldots, u_{(n/2)}, v_1, v_2, \ldots, v_{(n/2)}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le \frac{n-1}{2}; v_i v_{i+1}, 1 \le i \le \frac{n-3}{2}; u_1 v_1; u_{\frac{n-1}{2}} v_{\frac{n-1}{2}}\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that

$$f(u_i) = 2i - 1, 1 \le i \le \frac{n}{2}.$$

$$f(v_i) = 2i, 1 \le i \le \frac{n}{2}.$$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$. The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(2i-1)*(2i+1) + \sqrt{(2i-1)+(2i+1)}]}{2} = \frac{[(4i^2-1)+\sqrt{4i}]}{2}, 1 \le i \le \frac{n-2}{2}.$$

$$f^*(v_i v_{i+1}) = \frac{[(2i)*(2i+2) + \sqrt{(2i)+(2i+2)}]}{2} = \frac{[(4i^2+4i)+\sqrt{4i+2}]}{2},$$

$$1 \le i \le \frac{n-2}{2}.$$

$$f^*(u_1 v_1) = \frac{2+\sqrt{3}}{2} = 2.$$

$$f^*(u_{\frac{n}{2}} v_{\frac{n}{2}}) = \frac{(n-1)*n + \sqrt{n-1+n}}{2} = \frac{n^2 - n + \sqrt{2n-1}}{2}.$$

Thus the edges $\{2, 8, 19, \ldots, \frac{[(4i^2-1)+\sqrt{4i}]}{2}; 5, 13, 25, \ldots, \frac{[(4i^2+4i)+\sqrt{4i+2}]}{2}; 1; \frac{n^2 - n + \sqrt{2n-1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.7. *The triangular snake T_n admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-1}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n-1 \bigcup u_i v_i, 1 \le i \le n-1 \bigcup v_i u_{i+1}, 1 \le i \le n-1\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that

$f(u_i) = 2i - 1, 1 \le i \le n.$

$f(v_i) = 2i, 1 \le i \le n-1.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(2i-1) * (2i+1) + \sqrt{(2i-1) + (2i+1)}]}{2} = \frac{(4i^2 - 1) + \sqrt{4i}}{2},$$

$$1 \le i \le n-1$$

$$f^*(u_i u_{i+1}) = \frac{[(2i-1) * (2i) + \sqrt{(2i-1) + (2i)}]}{2} = \frac{(4i^2 - 2i) + \sqrt{4i-1}}{2},$$

$$1 \le i \le n-1$$

$$f^*(v_i u_{i+1}) = \frac{[(2i) * (2i+1) + \sqrt{(2i) + (2i+1)}]}{2} = \frac{(4i^2 + 2i) + \sqrt{4i+1}}{2},$$

$$1 \le i \le n-1.$$

Thus the edges $\{2, 8, 20, \ldots, \frac{(4i^2 - 1) + \sqrt{4i}}{2}; 1, 7, 16, \ldots, \frac{(4i^2 - 2i) + \sqrt{4i-1}}{2}, 4, 11, 22, \ldots, \frac{(4i^2 + 2i) + \sqrt{4i+1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.8. *The Alternate triangular snake AT_n admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n/2}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n-1 \cup u_{2i-1} v_i, 1 \le i \le n/2 \cup v_i u_{2i}, 1 \le i \le n/2\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \dots, q+1\}$. Such that

$f(u_i) = 2i - 1, 1 \le i \le n.$

$f(v_i) = 2i, 1 \le i \le n/2.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(2i-1)*(2i+1) + \sqrt{(2i-1)+(2i+1)}]}{2} = \frac{(4i^2-1)+\sqrt{4i}}{2},$$

$$1 \le i \le n-1$$

$$f^*(u_{2i-1} v_i) = \frac{[2(2i-1)*(2i) + \sqrt{2(2i-1)+(2i)}]}{2} = \frac{(8i^2-4i)+\sqrt{6i-1}}{2},$$

$$1 \le i \le n/2.$$

$$f^*(v_i u_{2i}) = \frac{[(2i)*(4i-1) + \sqrt{(2i)+(4i-1)}]}{2} = \frac{(8i^2-2i)+\sqrt{6i-1}}{2},$$

$$1 \le i \le n/2.$$

Thus the edges $\{2, 8, 19, \dots, \frac{(4i^2-1)+\sqrt{4i}}{2}; 1, 11, 28, \dots, \frac{(8i^2-4i)+\sqrt{6i-1}}{2}, 4, 15, 35, \dots, \frac{(8i^2-2i)+\sqrt{6i-1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.9. *The Double triangular snake DT_n admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1},$

$w_1, w_2, \ldots, w_{n-1}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \leq i \leq n-1 \cup u_i v_i, 1 \leq i \leq n-1 \cup v_i u_{i+1}, 1 \leq i \leq n-1 \cup u_i w_i, 1 \leq i \leq n-1 \cup w_i u_{i+1}, 1 \leq i \leq n-1\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that

$f(u_i) = 3i - 2, 1 \leq i \leq n.$

$f(v_i) = 3i - 1, 1 \leq i \leq n-1.$

$f(w_i) = 3i, 1 \leq i \leq n-1.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(3i-2)*(3i+1) + \sqrt{(3i-2)+(3i+1)}]}{2} = \frac{(9i^2 - 3i - 2) + \sqrt{6i-1}}{2},$$

$1 \leq i \leq n-1$

$$f^*(u_i v_i) = \frac{[(3i-2)*(3i-1) + \sqrt{(3i-2)+(3i-1)}]}{2} = \frac{(9i^2 - 9i + 2) + \sqrt{6i-3}}{2},$$

$1 \leq i \leq n-1.$

$$f^*(v_i u_{i+1}) = \frac{[(3i-1)*(3i+1) + \sqrt{(3i-1)+(3i+1)}]}{2} = \frac{(9i^2 - i) + \sqrt{6i}}{2},$$

$1 \leq i \leq n-1$

$$f^*(u_i w_i) = \frac{[(3i-2)*(3i) + \sqrt{(3i-2)+(3i)}]}{2} = \frac{(9i^2 - 6i) + \sqrt{6i-2}}{2},$$

$$f^*(w_i u_{i+1}) = \frac{[(3i)*(3i+1) + \sqrt{(3i)+(3i+1)}]}{2} = \frac{(9i^2 + 3i) + \sqrt{6i+1}}{2},$$

$1 \leq i \leq n-1.$

Thus the edges $\{3, 15, 37, \ldots, \frac{(9i^2 - 3i - 2) + \sqrt{6i-1}}{2}; 1, 11, 29, \ldots,$

$\frac{(9i^2-9i+2)+\sqrt{6i-3}}{2}, 5, 19, 42, \ldots, \frac{(9i^2-1)+\sqrt{6i}}{2}, 2, 13, 33, \ldots, \frac{(9i^2-6i)+\sqrt{6i-2}}{2};$

$7, 22, 47, \ldots, \frac{(9i^2+3i)+\sqrt{6i+1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.10. *The Alternate double triangular snake ADT_n admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-3}, w_1, w_2, \ldots, w_{n-3}\}$.

The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n-1 \cup u_{2i-1}v_i, 1 \le i \le n/2 \cup v_i u_{2i}, 1 \le i \le n/2 \cup u_{2i-1}w_i, 1 \le i \le n/2 \cup w_i u_{2i}, 1 \le i \le n/2\}$ for all values of n.

Define a map $f: V \to \{1, 2, 3, \ldots, q+1\}$. Such that

$f(u_i) = 3i-2, 1 \le i \le n.$

$f(v_i) = 3i-1, 1 \le i \le n/2.$

$f(w_i) = 3i, 1 \le i \le n/2.$

Define the induced function $f^*: E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u)+f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(3i-2)*(3i+1)+\sqrt{(3i-2)+(3i+1)}]}{2} = \frac{(9i^2-4i-2)+\sqrt{6i-1}}{2},$$

$1 \le i \le n-1.$

$$f^*(u_{2i-1}v_i) = \frac{[3(2i-1)*(3i-1)+\sqrt{3(2i-1)+(3i-1)}]}{2} = \frac{(18i^2-21i+5)+\sqrt{9i-6}}{2},$$

$1 \le i \le \frac{n}{2}.$

$$f^*(v_i u_{2i}) = \frac{[(3i-1)*(6i-2)+\sqrt{(3i-1)+(6i-2)}]}{2} = \frac{(18i^2-12i+2)+\sqrt{9i-3}}{2},$$

$1 \leq i \leq \frac{n}{2}$.

$$f^*(u_{2i-1}w_i) = \frac{[3(2i-1)*(3i)+\sqrt{3(2i-1)+(3i)}]}{2} = \frac{(18i^2-9i)+\sqrt{9i-1}}{2},$$

$1 \leq i \leq \frac{n}{2}$.

$$f^*(w_i u_{2i}) = \frac{[(3i)*(6i-2)+\sqrt{(3i)+(6i-2)}]}{2} = \frac{(18i^2-6i)+\sqrt{9i-2}}{2},$$

$1 \leq i \leq \frac{n}{2}$.

Thus the edges $\{3, 15, 37, \ldots, \frac{(9i^2-4i-2)+\sqrt{6i-1}}{2}; 1, 19, 54, \ldots, \frac{(18i^2-21i+5)+\sqrt{9i-1}}{2}, 5, 26, 66, \ldots, \frac{(18i^2-12i+2)+\sqrt{9i-3}}{2}, 2, 22, 60, \ldots, \frac{(18i^2-9i)+\sqrt{9i-1}}{2}; 7, 32, 74, \ldots, \frac{(18i^2-6i)+\sqrt{9i-2}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.11. *The Kite graph* $K_{3,n}$ *is a Product Root Sum Mean graph.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_{3+n}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \leq i \leq n+2 \cup u_1u_3$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots, q+1\}$. Such that

$f(u_i) = i, 1 \leq i \leq 3+n.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u)*f(v)+\sqrt{f(u)+f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(i)*(i+1)+\sqrt{(i)+(i+1)}]}{2} = \frac{(i^2+i)+\sqrt{2i+1}}{2},$$

$1 \leq i \leq n+2.$

$$f^*(u_1 u_3) = \frac{[(1) * (3) + \sqrt{(1) + (3)}]}{2} = \frac{(3) + \sqrt{4}}{2} = 3.$$

Thus the edges $\left\{1, 4, 7, \ldots, \frac{(i^2 + 1) + \sqrt{2i + 1}}{2}; 3\right\}$ are all distinct. Hence the theorem.

Theorem 3.12. *The quadrilateral sanke* Q_n *admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-1}, w_1, w_2, \ldots, w_{n-1}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \le i \le n - 1 \cup u_i v_i, 1 \le i \le n - 1 \cup v_i w_i, 1 \le i \le n - 1 \cup w_i u_{i+1}, 1 \le i \le n - 1\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots q + 1\}$. Such that

$f(u_i) = 3i - 2, 1 \le i \le n.$

$f(v_i) = 3i - 1, 1 \le i \le n - 1.$

$f(w_i) = 3i, 1 \le i \le n - 1.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$.

The edge labels are obtained as

$$f(u_i u_{i+1}) = \frac{(3i - 2)(3i + 1) + \sqrt{6i - 1}}{2} = \frac{9i^2 - 3i - 2 + \sqrt{6i - 1}}{2}, 1 \le i \le n - 1.$$

$$f(u_i v_i) = \frac{(3i - 1)(3i - 1) + \sqrt{(3i - 2) + (3i - 1)}}{2} = \frac{9i^2 - 9i + 2 + \sqrt{6i - 3}}{2}, 1 \le i \le n - 1.$$

$$f(v_i w_i) = \frac{(3i - 1)(3i) + \sqrt{(3i - 1) + (3i)}}{2} = \frac{9i^2 - 3i + 2 + \sqrt{6i - 1}}{2}, 1 \le i \le n - 1.$$

$$f(w_i u_{i+1}) = \frac{(3i)(3i + 1) + \sqrt{(3i) + (3i + 1)}}{2} = \frac{9i^2 + 3i + \sqrt{6i + 1}}{2}, 1 \le i \le n - 1.$$

Thus the edges $\{3, 15, 37, 67, \ldots, \frac{9i^2 - 3i - 2 + \sqrt{6i-1}}{2}; 1, 11, 29, \ldots, \frac{9i^2 - 9i + 2 + \sqrt{6i-3}}{2}, 4, 16, 38, \ldots, \frac{9i^2 - 9i + 2 + \sqrt{6i-1}}{2}; 7, 22, 47, \ldots, \frac{9i^2 + 3i + \sqrt{6i+1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.13. *The Double quadrilateral snake DQ_n admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-1}, w_1, w_2, \ldots, w_{n-1}, x_1, x_2, \ldots, x_{n-1}, y_1, y_2, \ldots, y_{n-1}\}$.

The edge set is $E = \{u_i u_{i+1}, 1 \leq i \leq n-1 \cup v_i w_i, , 1 \leq i \leq n-1 \cup x_i y_i, 1 \leq i \leq n-1 \cup u_i v_i, 1 \leq i \leq n-1 \cup w_i u_{i+1}, 1 \leq i \leq n-1 \cup u_i x_i, 1 \leq i \leq n-1 \cup y_i u_{i+1}, 1 \leq i \leq n-1\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \ldots q+1\}$. Such that

$f(u_i) = 5i - 4, 1 \leq i \leq n.$

$f(v_i) = 5i - 3, 1 \leq i \leq n-1.$

$f(w_i) = 5i - 1, 1 \leq i \leq n-1.$

$f(x_i) = 5i - 2, 1 \leq i \leq n-1.$

$f(y_i) = 5i, 1 \leq i \leq n-1.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{[(5i-4)*(5i+1) + \sqrt{(5i-4)+(5i+1)}]}{2} = \frac{(25i^2 - 15i - 4) + \sqrt{10i-3}}{2},$$

$$1 \leq i \leq n-1$$

$$f^*(v_i w_i) = \frac{[(5i-3)*(5i-1) + \sqrt{(5i-3)+(5i-1)}]}{2} = \frac{(25i^2 - 20i + 3) + \sqrt{10i-4}}{2},$$

$1 \leq i \leq n-1$

$$f^*(x_i y_i) = \frac{[(5i-2)*(5i) + \sqrt{(5i-2)+(5i)}]}{2} = \frac{(25i^2 - 10i) + \sqrt{10i-2}}{2},$$

$1 \leq i \leq n-1$

$$f^*(u_i v_i) = \frac{[(5i-4)*(5i-3) + \sqrt{(5i-4)+(5i-3)}]}{2} = \frac{(25i^2 - 35i + 12) + \sqrt{10i-7}}{2},$$

$1 \leq i \leq n-1$

$$f^*(w_i u_{i+1}) = \frac{[(5i-1)*(5i+1) + \sqrt{(5i-1)+(5i+1)}]}{2} = \frac{(25i^2 - 1) + \sqrt{10i}}{2},$$

$1 \leq i \leq n-1$

$$f^*(u_i x_i) = \frac{[(5i-4)*(5i-2) + \sqrt{(5i-4)+(5i-2)}]}{2} = \frac{(25i^2 - 30i + 8) + \sqrt{10i-6}}{2},$$

$1 \leq i \leq n-1$

$$f^*(y_i u_{i+1}) = \frac{[(5i)*(5i+1) + \sqrt{(5i)+(5i+1)}]}{2} = \frac{(25i^2 + 5i) + \sqrt{10i+1}}{2},$$

$1 \leq i \leq n-1$.

Thus the edges $\{5, 36, 91, \ldots, \frac{(25i^2 - 15i - 4) + \sqrt{10i-3}}{2}$; $6\ 34, 87, \ldots, \frac{(25i^2 - 20i + 3) + \sqrt{10i-4}}{2}$; $9, 43, 101, \ldots, \frac{(25i^2 - 10i) + \sqrt{10i-2}}{2}$; $2, 23, 69, \ldots, \frac{(25i^2 - 35i + 12) + \sqrt{10i-7}}{2}$; $14, 52, 115, \ldots, \frac{(25i^2 - 1) + \sqrt{10i}}{2}$; $3, 26, 74, \ldots, \frac{(25i^2 - 30i + 8) + \sqrt{10i-6}}{2}$; $17, 58, 123, \ldots, \frac{(25i^2 - 5i) + \sqrt{10i-1}}{2}\}$ are all distinct. Hence the theorem.

Theorem 3.14. *The Alternative quadrilateral snake* AQ_n *admits Product Root Sum Mean Labeling.*

Proof. Let the vertex set be $V = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n/2}, w_1, w_2, \ldots, w_{n/2}\}$. The edge set is $E = \{u_i u_{i+1}, 1 \leq i \leq n-1 \cup u_{2i-1} v_i, 1 \leq i \leq n/2$

$\cup v_i w_i, 1 \le i \le n/2 \cup w_i u_{i+1}, 1 \le i \le n/2\}$ for all values of n.

Define a map $f : V \to \{1, 2, 3, \dots q+1\}$. Such that

$f(u_i) = 2i - 1, 1 \le i \le n.$

$f(v_i) = 4i - 2, 1 \le i \le n/2.$

$f(w_i) = 4i, 1 \le i \le n/2.$

Define the induced function $f^* : E \to N$ such that $f^*(uv) = \frac{f(u) * f(v) + \sqrt{f(u) + f(v)}}{2}$.

The edge labels are obtained as

$$f^*(u_i u_{i+1}) = \frac{(2i-1)*(2i+1) + \sqrt{2i - 1 + 2i + 1}}{2} = \frac{4i^2 - 1 + \sqrt{4i}}{2},$$

$$1 \le i \le n - 1$$

$$f^*(u_{2i-1} v_i) = \frac{(2i-1)*(4i-2) + \sqrt{(2i-1)+(4i-2)}}{2} = \frac{8i^2 - 6i + 2 + \sqrt{6i-3}}{2},$$

$$1 \le i \le \frac{n}{2}.$$

$$f(v_i w_i) = \frac{(4i-2)(4i) + \sqrt{(4i-2)+(4i)}}{2} = \frac{16i^2 - 8i + \sqrt{8i-2}}{2},$$

$$1 \le i \le \frac{n}{2}.$$

$$f(w_i u_{2i}) = \frac{(4i)(4i-1) + \sqrt{(4i)+(4i-1)}}{2} = \frac{16i^2 - 4i + \sqrt{8i-1}}{2},$$

$$1 \le i \le \frac{n}{2}.$$

Thus the edges $\{3, 9, 20, \dots, \frac{4i^2 - 1 + \sqrt{4i}}{2}; 2, 17, 48, \dots, \frac{8i^2 - 6i + 2 + \sqrt{6i-3}}{2}, 6, 26, 63, \dots, \frac{16i^2 - 8i + \sqrt{8i-2}}{2}; 8, 30, 69, \dots,$

$\frac{16i^2 - 4i + \sqrt{8i-1}}{2}\}$ are all distinct. Hence the theorem.

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