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FIXED POINT THEOREMS IN FUZZY 2-BANACH SPACE USING E.A PROPERTY

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Abstract

The purpose of this paper is to obtain common fixed point theorems for weakly compatible mappings satisfying the property (E.A.) using implicit relation in fuzzy 2-banach space allows replacing the completeness requirement of the space with a more natural condition of closeness of the range.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [25], which plays a major role in almost all branches of Science and Engineering. In 2002, M. Aamri and D. El Moutawakil [1] defined a property (E.A.) for self maps. For the reader convenience, we recall some terminology from the theory of fuzzy 2-Banach space.

2. Preliminaries

Definition 1 [22]**.** Let D be a vector space over a field K (where K is R or C) and * be a continuous t-norm. A fuzzy set N in $D^2 \times [0, \infty]$ is called a fuzzy 2-norm on D if it satisfies the following conditions:

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(i) $N(p, q, 0) = 0 \; \forall \; p, q \in D$

(ii) $N(p, q, t) = 1 \; \forall \, t > 0$ and at least two among the three points are equal

(iii) $N(p, q, t) = N(q, p, t)$

(iv) $N(p + q + r, t_1 + t_2 + t_3) \geq N(p, q, t_1) * N(p, r, t_2) * N(p, q, t_3) \; \forall \; p, q, r \in S$ and $t_1, t_2, t_3 \geq 0$

(v) For every $p, q \in D, N(p, q, \cdot)$ is left continuous and $\lim_{t \to \infty} N(p, q, t) = 1$

The triple $(D, N, *)$ will be called fuzzy 2-normed linear space $(F2 - NLS)$.

Definition 2 [22]**.** A sequence $\{P_n\}$ in a $F2 - NLS(D, N, *)$ is converge to $p \in D$ if and only if $\lim_{n \to \infty} N(p_n, p, t) = 1 \; \forall \, t > 0$.

Definition 3 [22]**.** Let $(D, N, *)$ be a $F2 - NLS$. A sequence $\{P_n\}$ in D is called a fuzzy Cauchy sequence if and only if $\lim_{m, n \to \infty} N(p_m, p_n, t) = 1 \; \forall \, t > 0$.

Definition 4 [22]**.** A linear fuzzy 2-normed space which is complete is called a fuzzy 2-Benach space.

Definition 5 [22]**.** Self mappings A and S of a fuzzy 2-Banach space $(D, N, *)$ are said to be weakly commuting if $N(ASp, SAp, t) \geq N(Ap, Sp, t)$ $\forall \; p \in D$ and $t > 0$.

Definition 6 [22]**.** Self mapping A and S of a fuzzy 2-Banach space $(D, N, *)$ are said to be compatible if and only if $\lim_{n \to \infty} N(ASp_n, SAp_n, t) = 1 \; \forall \, t > 0$.

Whenever $\{P_n\}$ is a sequence in D such that $Ap_n, Sp_n \to p$ for some $p \in D$ as $n \to \infty$.

Definition 7 [22]**.** Two self maps A and S are said to be commuting if $ASp = SAp$ for all $p \in D$.

Definition 8 [22]**.** Let A and S be two self maps on a set D, if $Ap = Sp$ for some $p \in D$ then p is called a coincidence point of A and S.

Definition 9 [22]**.** Two self maps A and S of a fuzzy 2-Banach space $(D, N, *)$ are said to be weakly compatible if they commute at their coincidence points. That is if $Ap = Sp$ for some $p \in D$ then $ASp = SAp$.

Definition 10 [22]**.** Suppose A and S be two self mappings of fuzzy 2-Banach space $(D, N, *)$. A point $p \in D$ is called a coincidence point of A and S if and only if $Ap = Sp$, then $w = Ap = Sp$ is called a point of coincidence of A and S.

Definition 11 [1]**.** A pair (A, S) of self mapping of a fuzzy 2-Banach space $(D, N, *)$ is said to satisfy property (E.A) if there exists a sequence $\{P_n\}$ in D such that $\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = z$ for some $z \in D$.

Definition 12 [1]**.** Two pairs (A, S) and (B, T) of a self mapping of a fuzzy 2-Banach space $(D, N, *)$ are said to satisfy the common property (E.A) if there exist two sequence $\{p_n\}, \{q_n\}$ in D. Such that $\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = \lim_{n\to\infty} Bq_n = \lim_{n\to\infty} Tqn = z$ for some $z \in D$.

3. Implicit Relations [3]

Let $\{\emptyset\}$ be the set of all real continuous function $\emptyset : (R^+)^6 \to R^+$ satisfying the following condition:

(i) $\emptyset(u, v, u, v, v, u) \geq 0$ imply $u \geq v$ for all $u, v \in [0, 1]$

(ii) $\emptyset(u, v, v, u, u, v) \geq 0$ imply $u \geq v$ for all $u, v \in [0, 1]$

(iii) $\emptyset(u, u, v, v, u, u) \geq 0$ imply $u \geq v$ for all $u, v \in [0, 1]$.

Example 1. Let $(D, N, *)$ be a fuzzy 2-Banach space.

Define the two self maps A and S as follows:

$$AD = \begin{cases} 1 - p & \text{if } p \in \left[0, \frac{1}{2}\right] \\ 0 & \text{if } p \in \left(\frac{1}{2}, 1\right) \end{cases}; \quad SD = \begin{cases} \frac{1}{2} & \text{if } p \in \left[0, \frac{1}{2}\right] \\ \frac{3}{4} & \text{if } p \in \left(\frac{1}{2}, 1\right) \end{cases}$$

Consider the sequence $\{p_n\} = \left\{\frac{1}{2} - \frac{1}{n}\right\}, n \geq 2$

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} A\left(\frac{1}{2} - \frac{1}{n}\right)$$

$$= A\left(\frac{1}{2}\right)$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

$$\lim_{n\to\infty} Sp_n = \lim_{n\to\infty} S\left(\frac{1}{2} - \frac{1}{n}\right)$$

$$= S\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}$$

$$\lim_{n\to\infty} A\left(\frac{1}{2} - \frac{1}{n}\right) = \lim_{n\to\infty} S\left(\frac{1}{2} - \frac{1}{n}\right) = \frac{1}{2}$$

Therefore, E.A property satisfied

$$A\left(\frac{1}{2}\right) = \frac{1-1}{2} = \frac{1}{2}, \, S\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\Rightarrow AS\left(\frac{1}{2}\right) = A\left(\frac{1}{2}\right) = \frac{1}{2} \text{ and } SA\left(\frac{1}{2}\right) = S\left(\frac{1}{2}\right) = \frac{1}{2}$$

we obtain $AS\left(\frac{1}{2}\right) = SA\left(\frac{1}{2}\right) = \frac{1}{2}$.

That is A and S are weakly compatible

$\therefore\ p = \frac{1}{2}$ is the unique coincidence point

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} A\left(\frac{1}{2} - \frac{1}{n}\right)$$

$$= A\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}$$

$$\lim_{n\to\infty} Sp_n = \frac{1}{2}$$

$$\lim_{n\to\infty} ASp_n = \lim_{n\to\infty} AS\left(\frac{1}{2} - \frac{1}{n}\right)$$

$$= AS\left(\frac{1}{2}\right)$$

$$= A\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}$$

$$\neq 1.$$

$$\therefore \lim_{n\to\infty} ASp_n \neq 1$$

$\therefore$ A and S are not compatible.

Thus we can conclude that the property E.A. does not imply compatibility.

Example 2. Let $(D, N, *)$ be a fuzzy 2-Banach space where $D = [0, 3]$.

Define the self maps A and S as follows:

$$AD = \begin{cases} 1 - \frac{p}{3} & \text{if } p \in \left[0, \frac{1}{3}\right] \\ 0 & \text{if } p \in \left(\frac{1}{3}, 3\right) \end{cases}; \quad SD = \begin{cases} \frac{1}{3} & \text{if } p \in \left[0, \frac{1}{3}\right] \\ 0 & \text{if } p \in \left(\frac{1}{3}, 3\right) \end{cases}$$

Consider the sequence $\{p_n\} = \left\{3 - \frac{1}{n}\right\}, n \geq 1$

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} A\left(3 - \frac{1}{n}\right)$$

$$= A(3) = 0$$

$$\lim_{n\to\infty} Sp_n = \lim_{n\to\infty} S\left(3 - \frac{1}{n}\right)$$

$$= S(3) = 0$$

Therefore, E.A property satisfied

$$A(3) = 0,\ S(3) = 0$$

But $AS(3) = 1$ and $SA(3) = \frac{1}{3}$.

A and S are not weakly compatible

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} A\left(3 - \frac{1}{n}\right)$$

$$= A(3)$$

$$= 0$$

$$\lim_{n\to\infty} Sp_n = 0$$

we have

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} AS\left(3 - \frac{1}{n}\right)$$

$$= AS(3)$$

$$= A(0)$$

$$= 1 - \frac{0}{3} = 1$$

$$\therefore \lim_{n\to\infty} ASp_n = 1.$$

we obtain A and S are compatible.

Thus we can conclude that the property E.A. does not imply weak compatibility.

Example 3. Let $(D, N, *)$ be a fuzzy 2-Banach space where $D = [0, 2]$.

Define the self maps A and S as follows:

$$AD = \begin{cases} 1 & \text{if } p \in [0, 1] \\ 0 & \text{if } p \in (1, 2] \end{cases}; \quad SD = \begin{cases} p & \text{if } p \in [0, 1] \\ 0 & \text{if } p \in (1, 2] \end{cases}$$

Consider the sequence $\{p_n\} = \left\{1 - \frac{2}{n}\right\}, n \geq 1$

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} A\left(1 - \frac{2}{n}\right)$$

$$= A(1) = 1$$

$$\lim_{n\to\infty} Sp_n = \lim_{n\to\infty} S\left(1 - \frac{2}{n}\right)$$

$$= S(1)$$

$$= \frac{2 \times 1}{2} = 1$$

$$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = 1$$

We obtain A and S satisfies the property E.A.

$$A(1) = 1, S(1) = 1$$

$$AS(1) = A(1) = 1$$

$$SA(1) = S(1) = 1$$

$$AS(1) = SA(1) = 1$$

We get A and S are weakly compatible.

$p = 1$ is the unique common coincidence point.

$$\lim_{n\to\infty} ASp_n = \lim_{n\to\infty} AS\left(1 - \frac{2}{n}\right)$$

$$= AS(1)$$

$$= A(1)$$

$$= 1$$

We get that A and S are compatible. Thus we can conclude that the self maps A and S satisfies the property E.A., and the self maps A and S are compatible and weekly compatible.

Lemma 4[4]**.** *Let* $(D, N, *)$ *be a fuzzy* 2-*Banach space. If there exists* $k \in (0, 1)$ *such that* $N(p, q, kt) \geq N(p, q, t)$ *for all* $p, q \in D$ *and* $t > 0$ *then* $p = q$.

Lemma 5[6]**.** *Let* $(D, N, *)$ *be a fuzzy* 2-*Banach space. Then for all* $p \in D$, $N(p, q, \cdot)$ *is a non-decreasing function.*

Lemma 6. *Let A and S be compatible two self maps of a fuzzy* 2-*Banach space* $(D, N, *)$ *and* $Ap = Sp$ *for some* $p \in D$ *then* $ASp = SAp = AAp = SSp$.

Lemma 7. *Two self mapping A and S of a fuzzy* 2-*Banach space* $(D, N, *)$ *are compatible then* (A, S) *is weakly compatible.*

Proof. Suppose $Ap = Sp$ for some p in D. Consider a sequence $\{p_n\} \to p$.

Now, $\{Ap_n\} \to Ap$ and $\{Ap_n\} \to Sp(Ap)$

As A and S are compatible, we have $N(ASp_n, SAp_n, t) = 1$ for all $t > 0$ as $n \to \infty$.

Thus, $ASp_n = SAp_n$

$\Rightarrow ASp = SAp$ for some $p \in D$,

i.e. $Ap = Sp$ implies $ASp = SAp$ for some $p \in D$.

Therefore, we get that (A, S) is weakly compatible.

Lemma 8. *Two self mappings A and S of a fuzzy* 2-*Banach space*

$(D, N, *)$. (A, S) *is weakly compatible. w is a point of coincidence of A and S then the pair* (A, S) *satisfies the property E.A.*

Proof. Let $Ap = Sp$ for some $p \in D$ then $ASp = SAp$ for some $p \in D$ (A and S are weakly compatible).

Let $Ap = Sp$ for some $p \in D$ is a coincidence point of A and S then $w = Ap = Sp$ is a point of coincidence of A and S.

Claim that (A, S) satisfies the property E.A.

Consider a sequence $\{p_n\} \to p$.

We have $\{Ap_n\} \to Ap$ and $\{Ap_n\} \to Sp(Ap)$

We obtain $\{Ap_n\} \to Ap(= w)$ and $\{Sp_n\} \to Ap(= w)$

Therefore,

$$\lim_{n\to\infty} Ap_n = w = \lim_{n\to\infty} Sp_n$$

Hence (A, S) satisfies the property E.A.

Theorem 9. *Two self mappings A and S of a fuzzy* 2*-Banach space* $(D, N, *)$.

(i) *Pair* (A, S) *is weakly compatible.*

(ii) *Pair* (A, S) *is compatible.*

(iii) *w is a point of coincidence of A and S.*

(iv) *Pair* (A, S) *satisfies the property E.A then A and S have a fixed point.*

Proof. Let $Ap = Sp$ for some c then $ASp = SAp$ for some $p \in D$ (A and S are weakly compatible).

Consider a sequence $\{P_n\} \to p$.

We have

$$\lim_{n\to\infty} AP_n = Ap$$

$$\lim_{n\to\infty} AP_n = w\,(Ap = Sp = w)$$

now

$$\lim_{n\to\infty} SP_n = Sp$$

$$= Ap$$

$$= w\ (\because\ Ap = Sp = w)$$

We obtain

$$\lim_{n\to\infty} Ap_n = w = \lim_{n\to\infty} Sp_n$$

$$\Rightarrow Ap = w$$

$$= Sp\ (\text{since} \lim_{n\to\infty} P_n = p)$$

We have $Ap = Sp$

Then $ASp = SAp$

$$\Rightarrow Aw = Sw\ (\text{Since } Ap = w = Sp)$$

Since A and S are compatible, $Ap_n, Sp_n \to p$ for some $p \in D$ as $n \to \infty$.

$$\lim_{n\to\infty} ASP_n = AP = w \tag{1}$$

$$\lim_{n\to\infty} ASP_n = ASp$$

$$= Aw\ (\because\ Ap = Sp) \tag{2}$$

$$= w$$

Comparing equations (1) and (2), we get $Aw = w$

$$\lim_{n\to\infty} ASP_n = ASP = Aw = Sw \tag{3}$$

$$(\because\ Aw = w)$$

Comparing equations (1) and (3), we obtain $Sw = w$

Hence $Aw = w$ and $Sw = w$

Hence w is a fixed point of A and S.

Theorem 10. *Let $(D, N, *)$ be a fuzzy 2-Banach space with * continuous t-norm. Let A, B, S, T be self mappings of D satisfying*:

(i) $A(D) \subseteq T(D)$ *and* $B(D) \subseteq S(D)$

(ii) *Pairs (A, S) or (B, T) satisfies the property $(E.A)$*

(iii) *For some $\phi \in (\phi)$ and for all $p, q \in D$ and every $t > 0$,*

$$\phi\{N(Ap, Bq, t), N(Sp, Tq, t), N(Sp, Ap, t), N(Tp, Bq, t),$$

$$N(Sp, Bq, t), N(Tq, Ap, t)\} \geq 0$$

(iv) *Pairs (A, S) and (B, T) are weakly compatible,*

(v) *One of $A(D), B(D), S(D)$ or $T(D)$ is a closed subset of D.*

Then A, B, S, T have a unique common fixed points in D.

Proof. Suppose that (B, T) satisfies the property (E.A). Then there exists a sequence $\{P_n\}$ in D such that

$$\lim_{n\to\infty} Bp_n = \lim_{n\to\infty} Tq_n = z \text{ for some } z \in D$$

Since $B(D) \subseteq S(D)$, there exists a sequence $\{q_n\}$ in D

$$\lim_{n\to\infty} Bp_n = \lim_{n\to\infty} Sq_n = z$$

Now we show that

$$\lim_{n\to\infty} Aq_n = z$$

on putting $p = q_n$ and $q = p_n$ in (iii) we have

$$\phi\{N(Aq_n, Bp_n, t), N(Sq_n, Tp_n, t), N(Sq_n, Tq_n, t), N(Tp_n, Bp_n, t),$$

$$N(Tp_n, Bp_n, t), N(Tp_n, Aq_n, t)\} \geq 0$$

Proceeding limit $n \to \infty$,

$$\phi\{N(Aq_n, z, t), N(z, z, t), N(z, Aq_n, t), N(z, z, t), N(z, z, t), N(z, Aq_n, t)\}$$

$$\geq 0$$

$$[\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$$

$$\phi\{N(Aq_n, z, t), 1, N(z, Aq_n, t), 1, 1, N(z, Aq_n, t)\} \geq 0$$

we have

$$\lim_{n \to \infty} Aq_n = z$$

Since $S(D)$ is a closed subset of D.

$\therefore z = su$ for some $u \in D$, subsequently, we have

$$\lim_{n \to \infty} Bp_n = \lim_{n \to \infty} Tp_n = \lim_{n \to \infty} Sq_n = \lim_{n \to \infty} Aq_n = Su = z$$

From (iii)

putting $p = u, q = p_n$

we have,

$$\phi\{N(Au, Bp_n, t), N(Su, Tp_n, t), N(Su, Au, t), N(Tp_n, Bp_n, t),$$

$$N(Su, Bp_n, t), N(Tp_n, Au, t)\} \geq 0$$

letting $n \to \infty$, in view of ϕ,

we have $Au = Su$.

The weak compatibility of A and S implies that

$ASu = SAu$ and then $Au = ASu = Sz = Su = sz$.

Since $A(D) \subseteq T(D)$ therefore, there exists a point $v \in D$ such that $Au = Tv$ we claim that $Tv = Bv$ putting $p = u$ and $q = v$ in (iii), we have

$$\phi\{N(Au, Bv, t), N(Su, Tv, t), N(Su, Au, t), N(Tv, Bv, t), N(Su, Bv, t),$$

$$N(Tv, Au, t)\} \geq 0$$

i.e. $\phi\{N(Au, Bv, t), 1, 1, N(Au, Bv, t), 1\} \geq 0$

we obtain $Au = Bv = Tv \, [\because \phi(u, v, v, u, u, v) \geq 0 \Rightarrow u \geq v]$

Therefore, $Au = Su = Tv = Bv = z.$

The weak compatibility of B and T implies that

$BTv = TBv$ and $TTv = TBv = BTv = BBv$

i.e. $Tz = Bz$

Now we prove that $Au (= z)$ is a common fixed point of A, B, S and T.

From (iii) it follows that

$$\phi\{N(Az, Bv, t), N(Sz, Tv, t), N(Sz, Az, t), N(Tv, Bv, t),$$

$$N(Sz, Bv, t), N(Tv, Az, t)\} \geq 0$$

$\phi\{N(Az, z, t), N(Az, z, t), N(Az, Az, t), N(z, z, t), N(Az, z, t),$

$N(z, Az, t)\} \geq 0$

i.e. $\phi\{N(Az, z, t), N(Az, z, t), 1, 1, N(Az, z, t), N(z, Az, t)\} \geq 0$

we obtain $Au = z = Su \, [\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$

Hence, $z = Az = Sz$ and z is a common fixed point of A and S.

Similarly, we can prove that $Bz = z$ is also a common fixed point of B and T.

Therefore, we conclude that z is a common fixed point of A, B, S and T.

The proof is similar when $T(D)$ is assumed to be a closed subset of D. The cases in which $A(D)$ or $B(D)$ is a closed subset of D are similar to the case in which $T(D)$ or $S(D)$ respectively is closed.

Uniqueness.

suppose $z \neq w$ be another fixed point, from (iii),

$$\phi\{N(Az, Bw, t), N(Sz, Tw, t), N(Sz, Az, t), N(Tw, Bw, t),$$

$$N(Sz, Bw, t), N(Tw, Az, t)\} \geq 0$$

i.e. $\phi\{N(z, w, t), N(z, w, t), N(z, z, t), N(w, w, t), N(z, w, t),$

$N(w, z, t)\} \geq 0$

i.e. $\phi\{N(z, w, t), N(z, w, t), 1, 1, N(z, w, t), N(w, z, t)\} \geq 0$

we have $z = w[\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$

Hence z is a unique common fixed point of A, B, S and T respectively.

Remark 11. Since two non compatible self mappings of a fuzzy 2-Banach space $(D, N, *)$ satisfy property E.A, we obtain the following Corollary.

Corollary 12. *Let A, B, S and T be self mappings of a fuzzy 2-Banach space $(D, N, *)$ satisfying*:

(i) $A(D) \subseteq T(D)$ *and* $B(D) \subseteq S(D)$

(ii) *Pairs (A, S) and (B, T) satisfies the property* (E.A)

Suppose that (A, S) or (B, T) are noncompatible and the pairs (A, S) and (B, T) are weakly compatible. If the range of one of A, B, S and T is closed then A, B, S and T have a unique common fixed point in D.

Theorem 13. *Let $(D, N, *)$ be a fuzzy 2-Banach space with * continuous t-norm. Let A, B, S, T be self mappings of D satisfying*:

(i) *For some $\phi \in (\phi)$ and for all $p, q \in D$ and every $t > 0$,*

$$\phi\{N(Ap, Bq, t), N(Sp, Tq, t), N(Sp, Ap, t), N(Tq, Bq, t),$$
$$N(Sp, Bq, t), N(Tq, Ap, t)\} \geq 0$$

(ii) *Pairs (A, S) or (B, T) are weakly compatible*

(iii) $T(D)$ *and* $S(D)$ *are closed subsets of D.*

(iv) *Pairs (A, S) and (B, T), satisfy common property* (E.A)

Then A, B, S, T have a unique common fixed points in D.

Proof. Suppose that (A, S) and (B, T) satisfy common property (E.A), then there exists two sequences $\{p_n\}$ and $\{q_n\}$, such that

$$\lim_{n\to\infty} Bp_n = \lim_{n\to\infty} Tp_n = \lim_{n\to\infty} Sq_n = \lim_{n\to\infty} Aq_n = z \text{ in } D.$$

Since $T(D)$ and $S(D)$ are closed subsets of D, therefore $z = Su = Tv$ for some $u, v \in D$,

we claim that $Au = z$,

Replacing p by u and q by p_n in (i) we have

$$\phi\{N(Au, Bp_n, t), N(Su, Tp_n, t), N(Su, Au, t), N(Tp_n, Bp_n, t),$$

$$N(Su, Bp_n, t), N(Tp_n, Au, t)\} \geq 0$$

Letting $n \to \infty$

we have $Au = z = Su[\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$

Now we prove that $Bv = Tv$

Put $p = u$ and $q = v$ in (i) we have

$$\phi\{N(Au, Bv, t), N(Su, Tv, t), N(Su, Au, t), N(Tv, Bv, t),$$

$$N(Su, Bv, t), N(Tv, Au, t)\} \geq 0$$

i.e. $\phi\{N(Tv, Bv, t), N(Tv, Tv, t), N(Tv, Tv, t), N(Tv, Bv, t), N(Tv, Bv, t),$

$N(Tv, Tv, t)\} \geq 0$

i.e. $\phi\{N(Tv, Bv, t), 1, 1, N(Tv, Bv, t), N(Tv, Bv, t), 1\} \geq 0$

we have $Tv = Bv[\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$

Hence $Au = z = Su = Bv = Tv$

The rest of the proof follows from the above theorem.

Example 14. Let $D = [0, 2]$ equipped with the $(D, N, *)$ is a fuzzy 2-Banach space.

Define the self maps A, B, S and $T : D \to D$ by

$$Ad = \begin{cases} 0 & \text{if } d = 0 \\ 0.25 & \text{if } d > 0, \end{cases} \quad Bd = \begin{cases} 0 & \text{if } d = 0 \\ 0.45 & \text{if } d > 0, \end{cases}$$

$$Sd = \begin{cases} 0 & \text{if } d = 0 \\ 0.40 & \text{if } 0 < d \leq 0.6, \\ x - 0.45 & \text{if } d > 0.6 \end{cases} \quad Td = \begin{cases} 0 & \text{if } d = 0 \\ 0.25 & \text{if } 0 < d \leq 0.6 \\ x - 0.25 & \text{if } d > 0.6 \end{cases}$$

$AD = 0.25$, $BD = 0.45$, $SD = 0U(0.15, 1.55)$, $TD = O \cup 0.25U(0.35, 1.75)$.

Let us consider the sequence $\{P_n\} = 0.60 + \frac{1}{n}$, then $Ap_n \to 0.25$, $Bp_n \to 0.45$, $Sp_n \to 0.15$, $Tp_n \to 0.35$, $ASp_n \to 0.25$, $SAp_n \to 0.40$, $BTp_n \to 0.45$, $TBp_n \to 0.25$.

Pairs (A, S) and (B, T) are non compatible.

If we take $t = 1$ then A, B, S and T satisfy all the conditions of the Theorem 3.10 and 0 is the unique common fixed point of A, B, S and T.

4. Conclusion

The aim of this paper is to strengthen the results and to emphasize the role of property E.A. in the existence of common fixed points and prove our main results for the pair of weakly compatible mappings along with property E.A. Our improvements in this paper are given below:

(i) to weaken the completeness requirement of the space.

(ii) to minimize the commutativity requirement of the maps to the point of coincidence.

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