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INFLUENCE OF EPISTEMOLOGICAL BELIEFS ON ACADEMIC PERFORMANCE: A STUDY USING FUZZY RELATIONAL MAP WEIGHTED WITH TRIANGULAR AND TRAPEZOIDAL FUZZY NUMBERS

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Abstract

Fuzzy Relational Map is a framework that illustrates the knowledge stored in a system with a graphical structure. The nodes represent the concepts and the edges represent the causal relationship between the disjoint set of concepts. The strength of concepts and causal relationships is represented by fuzzy numbers. Quantifying the linguistic concepts with advanced fuzzy numbers gives a better view of the problem. This dynamical structure is efficient at predicting the causal relationship between the concepts of two disjoint sets. The causal influence between the concepts is obtained from the inference process and from the fixed-point vector. The causal relationship between epistemological beliefs and the attributes of academic performance is examined in this article using fuzzy relationship maps. With this tool, triangular and trapezoidal fuzzy numbers are employed to quantify the uncertain information of concepts and causal connections. The concepts can be ranked in the order of influence using the average weight of the stable vector and the rankings according to triangular and trapezoidal Fuzzy Relational Maps are compared.

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1. Introduction

Robert Axelrod (1976), a Political Scientist, introduced a formal way of representing social scientific knowledge with the help of a graphical structure called Cognitive Maps [2]. These maps reflect the visualisation of a complex problem as a human being would comprehend. The reasoning process helps to understand the behaviour and dynamics of the problem. The cognitive maps demonstrated whether or not there is influence between the concepts but did not provide any information about the level of influence. A decade later, Bart Kosko (1986) proposed a new idea to overcome this issue by considering fuzzy values for the concepts and fuzzy degrees for the causalities between the concepts [10]. This indeed enhanced the power of cognitive maps, improved their functioning and increased their applications in various fields. Vasantha Kandasamy (2000) introduced Fuzzy Relational Maps (FRM) that can represent the causal association between two disjoint sets of concepts [9].

Fuzzy Relational Map (FRM) methodology is a symbolic representation of a system and it describes the behaviour and dynamics of the system. FRM presents a complex system with a graphical structure where the accumulated knowledge of the system can be represented in terms of concepts and causalities. A concept can represent a state or characteristic of a system, whereas a causality describes the relationship between them. The concepts of FRM belong to two disjoint sets and it is assumed that there is no interaction between the concepts within a set. The FRM methodology integrates both the stored knowledge information in the systems and the expertise of the experts. This results in a conclusion that portrays the future state and behaviour of the system that is studied.

The standard of educational excellence among students is influenced by several factors. Epistemological beliefs are one of the factors that can have a subtle but strong influence on motivating students to perform better in academics. Though there are differing views about the relationship between self-beliefs and academic achievement in the research arena, the general opinion is that young students are strongly influenced by their positive self-beliefs. Research results suggest that among equally achieving students, having positive self-beliefs confers a small but noteworthy advantage on subsequent achievement relative to students who exhibit less favourable self-beliefs [16]. A relation of self-beliefs to academic achievement is evident among students preparing for competitive examinations. The aim of this work is to employ Fuzzy Relational Map to study the causal influence of epistemological beliefs on students' academic performance.

2. Fuzzy Relational Map: A Neuro-Fuzzy Tool

Fuzzy relational map is introduced by Vasantha Kandasamy and Yasmin Sultana as a particularisation of fuzzy cognitive maps. An FRM is a signed directed graph or a map from $D \to R$ with nodes or concepts from domain space (D) and range space (R). This is a dynamical system with feedback representing the elements of a system and the directed connections that establish the causal associations between the concepts [9]. The nodes take values from [0, 1] and edges of take values from [0, 1] or [-1, 1]. The association between the concepts is given by the relational matrix $E = (e_{ij})$ where e_{ij} is the weight of the edge. Every edge $D_i R_j$ (or $R_j D_i$) in the FRM is weighted with a number from [0, 1] or [-1, 1] depending on the application. The weight of the edge gives us an idea about how the nodes in the domain space influence the nodes in the range space.

$$e_{ij} = \begin{cases} -a_{ij}, & \text{increase (or decrease) in } D_i \text{ implies decrease (or increase) in } R_j \\ 0, & \text{increase in } D_i \text{ does not have any effect on } R_j \\ +aij, & \text{increase in } D_i \text{ implies increase in } R_j \end{cases}$$

3. Representation of Concepts and Causal Connections using TrFN and TpFN

Fuzzy sets are classes of objects with grades of membership. The membership functions assign a grade to each object based on a particular characteristic or property. Fuzzy sets are basically characterized by their membership functions. That is, they represent the vague data in terms of membership functions. In recent years, more advanced and sophisticated fuzzy sets have been introduced. The fuzzy sets expressed with single-valued fuzzy membership values provide very limited information about the concepts and causalities. The enhanced membership functions such as triangular and trapezoidal membership functions include more information. A cognitive map with triangular or trapezoidal membership functions enhances the power of the tool to analyse problems [4, 17].

In conventional cognitive map models, the values of the concepts in the system and the weights of causal relations are represented by fuzzy singletons (crisp numbers). The general opinion of the researchers is that the fuzzy singletons are inadequate to capture uncertain or incomplete information in the data. Hence, the cognitive map models are extended with new kinds of fuzzy sets to describe the concepts and causal connections. Anand, M. C. J., and Devadoss, A. V. (2013) introduced the concept of representing the nodes and edges in an FCM with triangular fuzzy numbers [1]. Representing the relations in the cognitive map models with triangular fuzzy numbers (TrFN) increases the capability of the tool to handle uncertain information [17].

A fuzzy number is a convex and normal fuzzy subset in the universe of discourse and they are used to represent imprecise linguistic expressions [11]. Fuzzy numbers are often represented in applications by L-R fuzzy sets and in particular, triangular and trapezoidal fuzzy sets. A triangular fuzzy number A can be defined as $A = (a_1, a_2, a_3)$ where each $a_i \in [0, 1]$. Figure 1 illustrates the membership function of triangular fuzzy number. The triangular number can be represented mathematically as follows [5, 6]:

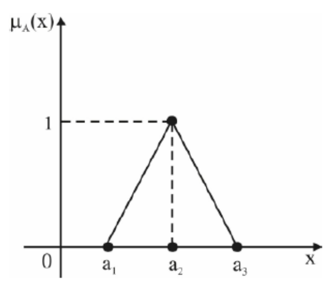

Figure 1. Membership function of Triangular Fuzzy Number.

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 < x < a_3 \\ 0, & x > a_3 \end{cases}$$

A trapezoidal fuzzy number A can be defined as $A = (a_1, a_2, a_3, a_4)$ where each $a_i \in [0, 1]$. The membership function of trapezoidal number is interpreted as given in figure 2 and is represented mathematically as follows [6]:

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ 1, & a_2 < x < a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 < x < a_4 \\ 0, & x > a_4 \end{cases}$$

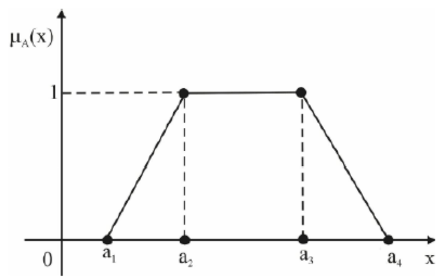

Figure 2. Membership function of Trapezoidal Fuzzy Number.

4. Triangular and Trapezoidal Fuzzy Relational Map

An FRM is called Triangular FRM (TrFRM) if the nodes and causal connections are represented with triangular membership functions. Triangular Fuzzy Relational Maps (TrFRM) are a generalisation of Fuzzy Relational Maps. They are constructed similar to Fuzzy Relational Maps by considering triangular membership values for the concepts and causalities. That is the nodes and edges of the fuzzy relational map are represented with triangular fuzzy numbers. In case of Trapezoidal Fuzzy Relational Maps (TpFRM) the nodes and edges are represented with trapezoidal fuzzy numbers.

A triangular fuzzy relational map (TrFRM) is a signed directed graph between two disjoint sets with concepts (like policies, events) as nodes and the causal influences as edges. The graphical structure of TrFRM represents the causalities between two disjoint sets of concepts. A TrFRM is essentially a dynamical system with feedback as the causal relations flow through a cycle in a revolutionary manner. If the equilibrium state is a unique vector, then it is called the fixed point of the system. Similarly, when a TrFRM reaches equilibrium in which the state vectors repeat in the form $(A_1 \to A_2 \to \ldots \to A_n), (B_1 \to B_2 \to \ldots \to B_m)$, then this equilibrium is known as a limit cycle. The fixed points obtained from simple FRM denote just the ON-OFF position on the nodes, while the fixed points in TrFRM give the precise values and this in turn could be used to calculate the weights of the attributes. The weights of the attributes are considered in determining the order of importance of the concepts. The average weight of the stable vector obtained from the inference process is used to evaluate and prioritize the concepts based on their degree of influence. In this research article, the TrFRM and TpFRM are applied to study the influence of epistemological beliefs on the academic performance of high secondary school students. The epistemological beliefs are ranked using the average sum of the weights of the concepts in the output vector. The ranking order according to TrFRM and TpFRM is compared and the results are analysed.

5. Description of the Problem

Generally, it is believed that academic performance in school determines later success in other areas of life. Numerous cognitive factors influence academic performance, in particular a student's conceptions of knowledge and beliefs about learning. The beliefs related to knowledge and learning are called 'Epistemological beliefs' in the literature. Epistemological beliefs are views about knowledge that relate to what knowledge is, how it is acquired, and the criteria and limitations under which knowledge is defined [8]. Francisco predicted from his research that epistemological beliefs influence student's academic achievement directly and indirectly. Moreover, he claimed that epistemological beliefs change as secondary education progresses, becoming realistic and complex. Also, students' worldview becomes increasingly integrated and sophisticated as they progress through secondary school [3]. Consequently, their dualistic view of knowledge transitions into a relativistic view. The students at this stage progressively moved from holding on to naïve epistemology to sophisticated epistemology [15]. Complex epistemological beliefs are an imperative goal of instruction and a crucial predictor of student achievement as well [7].

Schommer has conceptualized a framework for epistemological beliefs. The framework incorporates the fact that implicit beliefs about learning and knowledge play an instrumental role in how learners think and approach problems [14]. She proposed that a person's epistemological beliefs consist of multidimensional beliefs that are independent of one another to some extent [13]. Her five dimensions of epistemological beliefs about learning and knowledge consist of: 1) Speed of learning - quick to gradual; 2) Stability of knowledge - certain to changing; 3) Source of knowledge - authority to reasoning; 4) Ability to learn - fixed at birth to improvable; and 5) Structure of knowledge - isolated pieces to integrated concepts.

Along these five dimensions, a person holding naïve epistemology generally believes that: knowledge is simple and specific, knowledge resides in authorities and never changes, learning ability is innate and concepts are learned quickly or not at all. In contrast, a person holding sophisticated epistemology believes that knowledge is complex, uncertain and can be acquired gradually through reasoning processes. They develop the mindset that knowledge can be constructed by the learner [12]. This study examines both naive and sophisticated beliefs within a belief cluster. The epistemological beliefs and the factors of academic performance are treated as the disjoint sets of concepts of the triangular or trapezoidal Fuzzy Relational Map. Epistemological beliefs are regarded as elements of the domain space. The factors related to better performance in school are taken to be the elements of range space.

Table 1. shows the factors of FRM.

Domain Space		Range Space
B_1^+ : Gradual Learning	B_1^- : Quick Learning	C_1 : Smartness
B_2^+ : Tentativeness of knowledge	B_2^- : Certainty of knowledge	C_2 : Growth mindset
B_3^+ : Reasoning	B_3^- : Omniscience of Authority	C_3 : Grit
B_4^+ : Acquired learning	B_4^- : Innate ability to	C_4 : Self-esteem

ability	learn	
B_5^+ : Complexity of knowledge	B_5^- : Simplicity of knowledge	C_5 : Motivation

6. Methodology of construction of Triangular and Trapezoidal FRM

The aim of this study is to examine the role of personal beliefs related to knowledge and learning and their influence on academic performance of higher secondary school students. An experimental study is carried out to find out the influence of epistemological beliefs on better performance in school. About 45 students of Class XII (Science Stream) from two different schools, one located in a city and the other in a village, participated in this study. This group of students included both boys and girls having different academic records.

A set of 50 belief statements related to academic performance, both positive and negative, are collected from the Epistemological Beliefs Inventory (EBI) with the help of experts' opinion. These epistemological belief statements are used to assess the academic performance of the secondary school students. This collection included both rational and irrational beliefs one holds on to be true about their learning ability and academic performance. The belief statements were assigned values on a 5-point Likert Scale which ranges from Strongly Agree (5) to Strongly Disagree (1). A hard copy of the epistemological belief statements in Tamil language was issued to the individual students. The student participants were asked to compare the belief statements with their own beliefs and mark each with a value that represents the strength of their beliefs. The filled in forms were collected and the responses were put together for further calculations. The negative responses were reverse-coded and the values are grouped based on the probability of occurrence of a particular value for each belief statement. The Likert scale values are fuzzified using triangular membership values (or trapezoidal membership values) as given in table 2 and summarised to get a final value for each belief statement. The membership value of each belief statement related to factors of academic performance is arranged in the form of a matrix. This is the relational matrix of TrFRM (or TpFRM) which represents the relationship between epistemological beliefs and the factors of academic performance. The relational matrix of TrFRM and TpFRM is presented in table 3 and table 4 respectively.

Table 2. Linguistic terms and their corresponding TrFN and TpFN.

Linguistic Value	**TrFN** (a_1, a_2, a_3)	**Average of TrFN weight**	**TpFN** (a_1, a_2, a_3, a_4)	**Average of TpFN weight**
Strongly Agree	(0.75, 1.00, 1.00)	0.92	(0.70, 0.80, 0.90, 1.00)	0.85
Agree	(0.50, 0.75, 1.00)	0.75	(0.50, 0.60, 0.70, 0.80)	0.65
Neutral	(0.25, 0.50, 0.75)	0.50	(0.30, 0.40, 0.50, 0.60)	0.45
Disagree	(0.00, 0.25, 0.50)	0.25	(0.10, 0.20, 0.30,0.40)	0.25
Strongly Disagree	(0.00, 0.00, 0.25)	0.08	(0.00, 0.00, 0.10, 0.20)	0.08

Table 3. The relational matrix in terms of TrFN.

	TrC_1	TrC_2	TrC_3	TrC_4	TrC_5
TrB_1^+	(0.60, 0.84, 0.96)	(0.48, 0.71, 0.87)	(0.53, 0.77, 0.91)	(0.46, 0.68, 0.82)	(0.42, 0.66, 0.87)
TrB_1^-	(0.19, 0.36, 0.58)	(0.21, 0.39, 0.62)	(0.53, 0.76, 0.87)	(0.38, 0.58, 0.76)	(0.43, 0.67, 0.83)
TrB_2^+	(0.53, 0.77, 0.94)	(0.29, 0.53, 0.75)	(0.36, 0.59, 0.80)	(0.54, 0.79, 0.93)	(0.67, 0.91, 0.96)
TrB_2^-	(0.38, 0.59, 0.77)	(0.33, 0.52, 0.71)	(0.35, 0.57, 0.76)	(0.31, 0.48, 0.67)	(0.47, 0.69, 0.83)
TrB_3^+	(0.65, 0.90, 0.97)	(0.68, 0.92, 0.98)	(0.51, 0.73, 0.88)	(0.64, 0.89, 0.96)	(0.63, 0.87, 0.94)
TrB_3^-	(0.31, 0.49, 0.66)	(0.27, 0.44, 0.64)	(0.35, 0.58, 0.78)	(0.38, 0.58, 0.73)	(0.23, 0.39, 0.61)
TrB_4^+	(0.49, 0.73, 0.86)	(0.66, 0.91, 0.98)	(0.61, 0.85, 0.94)	(0.62, 0.87, 0.96)	(0.49, 0.72, 0.88)
TrB_4^-	(0.38, 0.59, 0.75)	(0.41, 0.62, 0.77)	(0.26, 0.44, 0.62)	(0.33, 0.55, 0.74)	(0.13, 0.27, 0.51)
TrB_5^+	(0.54, 0.78, 0.89)	(0.50, 0.74, 0.89)	(0.55, 0.78, 0.92)	(0.52, 0.76, 0.90)	(0.58, 0.80, 0.89)
TrB_5^-	(0.24, 0.44, 0.68)	(0.41, 0.64, 0.81)	(0.47, 0.69, 0.82)	(0.22, 0.39, 0.61)	(0.31, 0.47, 0.64)

Table 4. The relational matrix in terms of TpFN.

	TrC_1	TrC_2	TrC_3	TrC_4	TrC_5

TrB_1^+	(0.58, 0.68, 0.78, 0.88)	(0.47, 0.56, 0.66, 0.76)	(0.52, 0.61, 0.71, 0.81)	(0.46, 0.54, 0.64, 0.74)	(0.43, 0.53, 0.63, 0.73)
TrB_1^-	(0.22, 0.28, 0.38, 0.48)	(0.24, 0.32, 0.42, 0.52)	(0.51, 0.60, 0.70, 0.80)	(0.38, 0.46, 0.56, 0.66)	(0.44, 0.53, 0.63, 0.73)
TrB_2^+	(0.52, 0.61, 0.71, 0.81)	(0.33, 0.42, 0.52, 0.62)	(0.38, 0.47, 0.57, 0.67)	(0.53, 0.63, 0.73, 0.83)	(0.63, 0.72, 0.82, 0.92)
TrB_2^-	(0.39, 0.47, 0.57, 0.67)	(0.34, 0.41, 0.51, 0.61)	(0.37, 0.46, 0.56, 0.66)	(0.32, 0.39, 0.49, 0.59)	(0.47, 0.56, 0.66, 0.76)
TrB_3^+	(0.62, 0.72, 0.82, 0.92)	(0.64, 0.74, 0.84, 0.94)	(0.50, 0.59, 0.69, 0.79)	(0.61, 0.71, 0.81, 0.91)	(0.60, 0.70, 0.80, 0.90)
TrB_3^-	(0.32, 0.39, 0.49, 0.59)	(0.28, 0.35, 0.45, 0.55)	(0.37, 0.47, 0.57, 0.67)	(0.38, 0.46, 0.56, 0.66)	(0.25, 0.32, 0.42, 0.52)
TrB_4^+	(0.49, 0.59, 0.69, 0.79)	(0.63, 0.73, 0.83, 0.93)	(0.58, 0.68, 0.78, 0.88)	(0.60, 0.70, 0.80, 0.90)	(0.48, 0.58, 0.68, 0.78)
TrB_4^-	(0.39, 0.47, 0.57, 0.67)	(0.41, 0.50, 0.60, 0.70)	(0.28, 0.35, 0.45, 0.55)	(0.35, 0.44, 0.54, 0.64)	(0.16, 0.21, 0.31, 0.41)
TrB_5^+	(0.53, 0.62, 0.72, 0.82)	(0.50, 0.60, 0.70, 0.80)	(0.53, 0.63, 0.73, 0.83)	(0.51, 0.61, 0.71, 0.81)	(0.55, 0.64, 0.74, 0.84)
TrB_5^-	(0.27, 0.35, 0.45, 0.55)	(0.42, 0.51, 0.61, 0.71)	(0.47, 0.56, 0.66, 0.76)	(0.25, 0.32, 0.42, 0.52)	(0.31, 0.38, 0.48, 0.58)

7. Analysis of the Problem Using TrFRM and TpFRM

The association between epistemological beliefs and academic performance is explored using TrFRM and TpFRM. To analyse the level of influence of the factors in the domain space (or range Space) each factor is taken to be the input vector in the inference process one by one. The input vector is passed through the relational matrix of TrFRM (or TpFRM) and the resultant vector is again passed through the relational matrix iteratively until the stable vector is obtained.

7.1. Method of determining the hidden pattern. Following is the Pseudo Code of TrFRM inference process adopted from [1].

Step1. Choose the nodes of Domain and Range spaces of the Triangular FRM.

Step 2. Let A_1 be the input vector. Pass A_1 through the triangular relation matrix Tr(E) by multiplying A_1 with Tr(E). i.e., $A_1 \cdot Tr(E)_{Weight}$.

Step 3. Calculate $A_1 \cdot Tr(E)_{Average}$.

Step 4. Threshold (→) the obtained triangular vector by replacing a_i by 1 if it is the maximum weight of the triangular node or by 0 otherwise. Let

$$B_1 = A_1 \cdot Tr(E)_{Max \cdot weight}(say).$$

Step 5. Steps (1-4) is repeated with B_1 and $Tr(E)^T$. Let $A_2 = B_1 \cdot Tr(E)^T_{Max \cdot weight}$.

Step 6. The dynamical system is ended when $A_1 = A_2$. Otherwise, the procedure is repeated. The above steps are repeated iteratively until a limit cycle or a fixed point is obtained.

Step 7. Calculate the total sum of average weight for each node of the domain.

Step 8. Rank the concepts based on the total sum of average weight.

The above procedure is repeated for Trapezoidal FRM. The input vectors with a particular component in the ON state are passed through the dynamical system and the resultant vector is generated. From the resultant steady state vectors of the triangular FRM and the trapezoidal FRM, the total weightage of attributes is obtained separately. The ranking order of the concepts is determined by the decreasing order of the weightage of attributes. The average weight of the triangular value of the fixed point obtained at each step and the corresponding ranking order based on the total sum of average weight are presented in table 5. The table 6 presents the fixed point of TpFRM and the ranking order of the factors of the domain space.

Suppose that the concept TrB_4 is ON state and other nodes are in OFF states. Let $A^{(4)}$ = (0 0 0 1 0 0 0 0 0 0)

$$A^{(4)}Tr(E)_{weight} = \{(0.4,\ 0.6,\ 0.8),\ (0.3,\ 0.5,\ 0.7),\ (0.4,\ 0.6,\ 0.8),\ (0.3,\ 0.5,\ 0.7)$$

(0.5, 0.7, 0.8)}

$A^{(4)}Tr(E)_{Average} = (0.6\ 0.5\ 0.6\ 0.5\ 0.7)$

$A^{(4)}Tr(E)_{Max\cdot weight} = (0\ 0\ 0\ 0\ 1) = B^{(4)}(say)$

$B^{(4)}(Tr(E))^T = \{(0.4, 0.7, 0.9), (0.4, 0.7, 0.8), (0.7, 0.9, 1), (0.5, 0.7, 0.8),$

$(0.6, 0.9, 0.9), (0.2, 0.4, 0.6), (0.5, 0.7, 0.9), (0.1, 0.3, 0.5), (0.6, 0.8, 0.9),$

$(0.3, 0.5, 0.6)\}$

$B^{(4)}(Tr(E))^T_{Average} = (0.7\ 0.6\ 0.8\ 0.7\ 0.8\ 0.4\ 0.7\ 0.3\ 0.8\ 0.5)$

$B^{(4)}(Tr(E))^T_{Max\cdot weight} = (0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 0) = A_1^{(4)}$

$A_1^{(4)}Tr(E)_{weight} = \{(1.7, 2.4, 2.8), (1.5, 2.2, 2.6), (1.4, 2.1, 2.6), (1.7, 2.4, 2.8)$

$(1.9, 2.6, 2.8)\}$

$A_1^{(4)}Tr(E)_{Average} = (2.3\ 2.1\ 2.0\ 2.3\ 2.4)$

$A_1^{(4)}Tr(E)_{Max\cdot weight} = (0\ 0\ 0\ 0\ 1) = B_1^{(4)}$

$B_1^{(4)}(Tr(E))^T = \{(0.4, 0.7, 0.9), (0.4, 0.7, 0.8), (0.7, 0.9, 1), (0.5, 0.7, 0.8),$

$(0.6, 0.9, 0.9), (0.2, 0.4, 0.6), (0.5, 0.7, 0.9), (0.1, 0.3, 0.5), (0.6, 0.8, 0.9),$

$(0.3, 0.5, 0.6)\}$

$B_1^{(4)}(Tr(E))^T_{Average} = (0.7\ 0.6\ 0.8\ 0.7\ 0.8\ 0.4\ 0.7\ 0.3\ 0.8\ 0.5)$

$B_1^{(4)}{}_1^{(4)}(Tr(E))^T_{Max\cdot weight} = (0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 1\ 0) = A_2^{(4)} = A_1^{(4)}.$

Table 5. Ranking based on Triangular weightage of Attributes.

ON state	TrB_1^+	TrB_1^-	TrB_2^+	TrB_2^-	TrB_3^+	TrB_3^-	TrB_4^+	TrB_4^-	TrB_5^+	TrB_5^-
B_1^+	0.80	0.38	0.75	0.58	0.84	0.49	0.69	0.57	0.74	0.46
B_1^-	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
B_2^+	0.65	0.64	0.84	0.66	0.82	0.41	0.70	0.30	0.76	0.47
B_2^-	0.65	0.64	0.84	0.66	0.82	0.41	0.70	0.30	0.76	0.47
B_3^+	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
B_3^-	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
B_4^+	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
B_4^-	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
B_5^+	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
B_5^-	0.68	0.41	0.52	0.52	0.86	0.45	0.85	0.60	0.71	0.62
Weight	6.883	4.509	6.100	5.526	8.487	4.459	8.037	5.381	7.226	5.730
Average	0.688	0.451	0.610	0.553	0.849	0.446	0.804	0.538	0.723	0.573
Rank	4	9	5	7	1	10	2	8	3	6

Table 6. Ranking based on Trapezoidal weightage of Attributes.

ON state	TrB_1^+	TrB_1^-	TrB_2^+	TrB_2^-	TrB_3^+	TrB_3^-	TrB_4^+	TrB_4^-	TrB_5^+	TrB_5^-
B_1^+	0.64	0.49	0.62	0.51	0.74	0.45	0.70	0.45	0.67	0.48
B_1^-	0.66	0.66	0.52	0.51	0.64	0.52	0.73	0.41	0.68	0.61
B_2^+	0.58	0.58	0.78	0.61	0.75	0.37	0.63	0.28	0.69	0.44
B_2^-	0.58	0.58	0.78	0.61	0.75	0.37	0.63	0.28	0.69	0.44
B_3^+	0.65	0.41	0.61	0.48	0.77	0.46	0.72	0.52	0.66	0.45
B_3^-	0.64	0.49	0.62	0.51	0.74	0.45	0.70	0.45	0.67	0.48
B_4^+	0.61	0.44	0.58	0.46	0.78	0.46	0.76	0.52	0.65	0.47

B_4^-	0.61	0.44	0.58	0.46	0.78	0.46	0.76	0.52	0.65	0.47
B_5^+	0.65	0.41	0.61	0.48	0.77	0.46	0.72	0.52	0.66	0.45
B_5^-	0.61	0.44	0.58	0.46	0.78	0.46	0.76	0.52	0.65	0.47
Weight	6.208	4.969	6.267	5.080	7.491	4.480	7.132	4.473	6.682	4.736
Average	0.621	0.497	0.627	0.508	0.749	0.448	0.713	0.447	0.668	0.474
Rank	5	7	4	6	1	9	2	10	3	8

7.2. Results and Discussion. The ranking from the triangular FRM is as follows: $TrB_3^+ \succ TrB_4^+ \succ TrB_5^+ \succ TrB_1^+ \succ TrB_2^+ \succ TrB_5^- \succ TrB_2^- \succ TrB_4^- \succ TrB_1^- \succ TrB_3^-$. It is inferred from the ranking order that TrB_3^+ (Reasoning) is the most influential sophisticated belief and TrB_3^- (Authority) is the least influential from the naive belief clusters. The ranking from the total weight of the trapezoidal FRM is as follows: $TrB_3^+ \succ TrB_4^+ \succ TrB_5^+ \succ TrB_2^+ \succ TrB_1^+ \succ TrB_2^- \succ TrB_1^- \succ TrB_5^- \succ TrB_3^- \succ TrB_4^-$. It is observed from the ranking order that TrB_3^+ (Reasoning) is the most influential from the cluster of sophisticated beliefs and TrB_4^- (Innate ability) is the least influential from the cluster of simple beliefs.

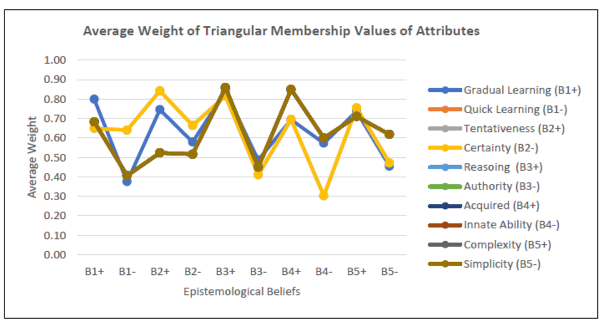

Figure 3. Pictorial Representation of the Average Weight of Attributes in TrFRM.

The pictorial representation of the influence of concepts or attributes based on their average weight of membership triangular and trapezoidal values are presented in figure 3 and 4 respectively. The belief attribute at which the stable vectors of all the inputs coincide is the most influential. From figures 3 and 4 it can be easily seen that TpB_3^+ (Reasoning) is the most influential epistemological belief that enhances the learning capacity and academic performance of the students.

Though the most influential sophisticated belief is same in both triangular and trapezoidal analysis, there is some variation in the ranking order obtained from TrFRM and TpFRM. This difference may be due to the fact that the region of trapezoidal membership values is broader than that of triangular membership function. However, the ranking order obtained from TpFRM is more sensitive and realistic as it includes more information compared to TrFRM. An advanced method can be adopted to confirm the best and accurate ranking order. With regard to absence of epistemological beliefs the ranking order is entirely different from one another. The ranking order of simple beliefs are totally different in case of TrFRM and TpFRM. This may be due to the unawareness, lack of clarity and misunderstanding about the simple beliefs among the students.

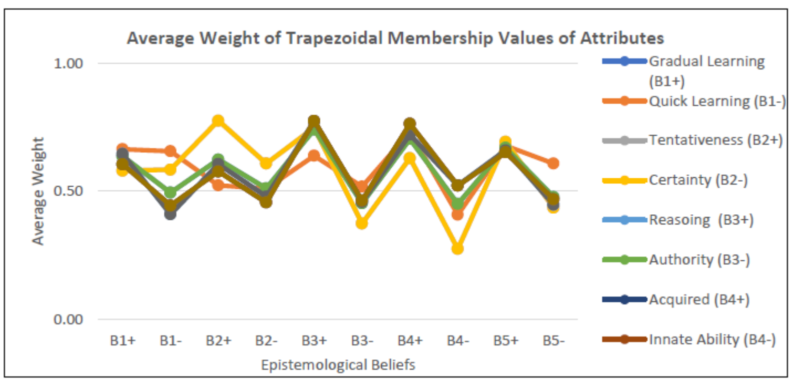

Figure 4. Pictorial Representation of the Average Weight of Attributes in TpFRM.

In FCM approach, correlations between causal associations among concurrently operating units are promoted. FCM is therefore only applicable when causal associations exist among concepts within the same set. Though the FCM approach is a much celebrated one and applied to numerous problems, it is unable to handle the situation where the concepts come from two disjoint sets. FRM helps address this issue as it demonstrates causal associations between disjoint sets. When the concepts in a system cannot be put in a single set due to their distinct nature, FRMs might be helpful. The uncertain information representation of FRMs is increased when they are weighted with triangular (or trapezoidal) fuzzy numbers. As Psychology and Sociology are highly subjective sciences, it is difficult to distinguish between concepts and determine how one influences the other. Therefore, whenever the elements of fuzzy sets are highly indistinguishable, assigning triangular (or trapezoidal) fuzzy numbers to the concepts and causal associations enhances the inference process and problem analysis.

8. Conclusion

In conventional FRM, weights have singleton fuzzy values, which make them unable to represent uncertain data. The ability to represent uncertain information in FRM models is improved with triangular (or trapezoidal) fuzzy numbers as weights. The behaviour of the dynamical system is studied using FRM reasoning process. The impact of the concepts is calculated from the average weight of the concepts that are turned on in the stable vector. The results of both TrFRM and TpFRM are compared and analyzed. Presenting uncertain information in FRM with weights that are in terms of TRFN (or TpFN) has several advantages, including: 1) Experts are able to express their uncertain knowledge more freely. 2) Knowledge of many experts can be aggregated and expressed in one FRM demonstrating the overall structure of the problem 3). The capacity of presentation of uncertain information by the FRM is increased 4). The reasoning process of FRM is enhanced as more uncertain information is included 5). The FRM model is simulated for various initial states that the resultant vectors are more sensitive 6). The predicted behaviour of the system is relatively accurate and more realistic 7). Based on the average weight values of the resulting vector, various concepts of FRM might be ranked.

The present study examines a real-world problem by using FRM weighted with triangular/trapezoidal fuzzy numbers. The uncertainty in the data is represented with triangular/trapezoidal fuzzy numbers, which improved the efficiency of the tool and increased the sensitivity of the results. In the future, FRMs will be extended to include more advanced fuzzy numbers. In addition, more advanced defuzzification methods will be adopted to reduce the loss of information in the process of defuzzification. Also, highly developed aggregation methods will be employed to include vague information and arrive at more realistic results.

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