A STUDY AND MATHEMATICAL MODELING OF SYNTHETIC APERTURE RADAR (SAR) MISSILE

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Abstract

In the modern era, to increase the accuracy for any missile the guidance system is mostly automatic. The target should be identified accurately so that the missile can be guided towards it to provide better defense. Synthetic Aperture Radar imaging are having numerous advantages like remote imaging, improved guidance accuracy and better functioning in all day and all weathers. A study on formation for SAR image, SAR missile and its mathematical modeling is discussed in this paper. Analysis of SAR sub-aperture algorithm is done in this study. The technique can be used provide data for targeting before launch or during flight.

1. Introduction

SAR means Synthetic Aperture Radar. SAR is the advanced technique of radar imaging. Two dimensional and three dimensional images are created using this form of radar. High resolution is required for the images at a broader level in the mapping of the earth resources, in military systems and in the monitoring of environment [1-3]. The imaging required in this type of imaging acquires in unpleasant weather or at the night and such capability is provided by Synthetic Aperture Radar (SAR). [4] The radar signals have the propagation of very long range and advantage of this characteristic of radar signals is taken by Synthetic Aperture Radar (SAR). The imagery of high

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resolution is provided by modern digital electronics which has the capability of processing complex information [5-6]. The capabilities of optical imaging and photographic imaging are complemented by SAR because the atmospheric conditions cannot limit it [7-9].

![Figure 1. Transmission of radar pulse.](image)

Transmission of microwave pulses takes place towards the surface of the earth in SAR imaging. The spacecraft measures the microwave energy which is scattered back [10-12]. Now the simple radar principle is used by SAR system in which the image is formed by capturing the time delay caused by the microwave energy signals which were scattered back [13].

![Figure 2. Backscatter of radar pulse to antenna.](image)

The radar signal which is reflected back is driven by the backscatter radar which contains information regarding the surface of the earth. There are many factors which drive this reflection [14-16]. These factors include radar parameters and surface parameters. Radar parameters are polarization, frequency, and wavelength and incidence angle. Surface parameters are dielectric constant, roughness of the surface and the orientation and surface of the objects [17].
2. Algorithm for SAR Image Formation

The algorithm used for the formation of SAR image usually consists of two algorithms. These are RD algorithm and azimuthal preprocess. The workflow is as given in the figure below.

![Figure 3. System Flow.](image)

Firstly, the data which is sampled from A/D converter or the input data is sent for the caching of the data in FGPA and further it is transferred to azimuthal preprocess. In this process the rate of the data transfer is lowered so that the data is suitable for the DSP processing. Then the range processing of the data is done in the next module followed by the transport storage. The azimuth processing is carried out when the data from the cache is read. The computing modulus provides the final output. This algorithm has many advantages such as easy implementation and less calculations. The disadvantage of this algorithm is that the input signal spectrum is stationary because the signal is passed through band pass filter with constant coefficient [18-20].

The basic principle on which the SAR system is explained as follows. High azimuth resolution is obtained by signal processing when the target and the radar have relative motion between them and hence a long aperture is formed. When the filter velocity matched with azimuth equals the platform speed then the best focus is obtained when the target is stationary [21-22]. However when the target is moving, then the relative speed between the target and the platform is considered to obtain the best azimuth focus.
3. Analysis of SAR Missile Sub-Aperture Algorithm

The Doppler bandwidth is very large in azimuth due to the high speed of the missile and due to this there is very high scope of spectrum alias and so the PRF (Pulse Repetition Frequency) must be very high [23]. In order to lower the data transfer rate so that it is suitable for real time processing, an algorithm is required based on sub-aperture and is in time domain [24]. It is better that the sub aperture time is shorter for the amount of data to decrease. The azimuth resolution is given as:

$$\delta_a = \frac{V_a}{B_a} = \frac{V_a}{f_{dr} \cdot \Delta T_s}.$$  (1)

Here, delta represents the azimuth resolution, $V_a$ represents the velocity of missile, $B_a$ represents the Doppler bandwidth and $T_s$ is the time of sub-aperture. The resolution is higher when the sub-aperture time is higher. There are basic steps which are to be followed for this algorithm. Firstly, the pulse compression is carried out with echo signal at range direction. The expression for range reference function is given as:

$$H(f) = \exp \left[ j\pi \left( \frac{f^2}{K_r} \right) \right].$$  (2)

Here, $f$ denotes the frequency and $K_r$ is LFM signal slope.

The expression for the output signal is given as:

$$S_r(t, \tau) = \sin c \left\{ \pi B \left[ t - \frac{2r(t)}{c} \right] \right\} \cdot \exp \left[ -j \frac{4\pi r(t)}{\lambda} \right].$$  (3)

Here, $\Gamma$ is the range time, $t$ is the azimuth time, $r(t)$ is the transient distance and $b$ is the LFM signal bandwidth. The expression for $r(t)$ is given as:

$$r(t) = r_0 - \frac{\lambda f_{dc}}{2} t - \frac{\lambda f_{dr}}{4} t^2.$$  (4)

The distance between point target and radar is given by $r_0$ and $f_{dc}$ represents the Doppler frequency. The expression of $f_{dc}$ is given by:

$$f_{dc} = \frac{2v \cdot \sin \theta_0}{\lambda}$$  (5)
\( \theta_0 \) represents the deviated angle from the central line of the beam along azimuth. Inserting the expression of \( r(t) \) in equation (3) we get:

\[
S_r(t, \tau) = \sin c \left[ \pi B \left( \tau - \frac{2r_0}{c} + \frac{f_{dc}}{f_c} t + \frac{f_{dr}}{2f_c} t^2 \right) \right] \\
\cdot \exp \left[ -j \frac{4\pi r_0}{\lambda} \right] \cdot \exp \left( j2\pi f_{dc} t + j\pi f_{dr} t^2 \right),
\]

where \( f_c \) is frequency of the carrier wave.

The range walk migration is given by the first order function of \( t \) in \( \sin c \). The \( \sin c \) function is given as:

\[
\sin c \left[ \pi B \left( \tau - \frac{2r_0}{c} \right) \right].
\]

The Fourier transform \( S_{r1}(f) \) is given as:

\[
F\left\{ \sin c \left[ \pi B \left( \tau - \frac{2r_0}{c} \right) \right] \right\} = S_{r1}(f).
\]

Then,

\[
F\left\{ \sin c \left[ \pi B \left( \tau - \frac{2r_0}{c} + \frac{f_{dc}}{f_c} t + \frac{f_{dr}}{2f_c} t^2 \right) \right] \right\} = S_{r1}(f) \cdot \exp \left( j2\pi f_{dc} f t \right) \cdot \exp \left( j\pi f_{dr} f t^2 \right).
\]

A compensation factor \( \phi \) is multiplied in frequency domain to carry out the RCMC (Range Cell Migration Correction) after the multiplication with reference function.

\[
\phi_1 = \exp \left( -j2\pi \frac{f_{dc}}{f_c} f t \right)
\]

\( f_{dc} \) represents the centroid of Doppler. A rough estimation of \( f_{dc} \) is made to satisfy the condition of real time processing. This is done by collecting the data from INS (Indian navigational Services) and then clutter clock is used for making the precise estimation [25]. The curvature range is represented in the \( \sin c \) function by the term \( t^2 \). RCMC neglects the range curvature influence using the compensation factor given as:

\[
\phi_2 = \exp \left( -j2\pi f_{dc} t - j\pi f_{dr} t^2 \right).
\]
The flowchart of the sub-aperture algorithm in the time domain is shown in figure 3

![Flowchart of the sub-aperture algorithm in the time domain](image)

**Figure 3.** Flowchart of the algorithm.

### 4. Conclusion

SAR is imaging technique using RADAR that produces high resolution images of a scene by producing coherent phase history of reflected signal over multiple pulses being transmitted. With the increase in the techniques of homing guidance, SAR provides better accuracy even in bad weather conditions. Missile borne SAR can improve inertial guidance system for long range missiles. In this paper, forming of SAR image, sub aperture algorithm has been discussed with mathematical modeling.

### References


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