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SPLIT CONVEX DOMINATION OF A DIRECTED GRAPH

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Abstract

Directed graphs are an important modelling tool and are being used in establishing relations between many physical situations. Connectedness in directed graphs is used mostly in networking problems. In this paper the existence and non-existence of a split convex dominating set in digraphs is investigated and many results related to this in standard directed graphs are analyzed. A convex dominating set D of a graph $G = (V, E)$ is said to be a split convex dominating set if $\langle V - D \rangle$ is disconnected.

1. 1ntroduction

The concept of split domination in graphs was first introduced by Kulli and Janakiram in ([1]). Convexity in graphs has been investigated and studied in ([2], [3]) and convex domination in a graph has been studied in ([4], [5]). The topic split convex domination number of a graph is studied in ([10]). Throughout this paper $D = (V, A)$ is a finite directed graph with neither loops nor multiple arcs (but pairs of arcs are allowed) and $G = (V, E)$ is an undirected graph with neither loops nor multiple edges. For basic terminology on graphs and digraphs, we refer to Chartrand and Lesniak ([7]).

Let $G = (V, E)$ be a graph. A subset D of V is called dominating set of G

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if every vertex in $V - D$ is adjacent to at least one vertex in D The minimum cardinality of dominating set of G is called domination number of G and is denoted by $\gamma(D)$. A dominating set D of G is convex dominating set if for every $u, v \in D$ all $u - v$ shortest path of G entirely contained in $\langle D \rangle$. A convex domination number $\gamma_{conv}(G)$ of G is the minimum cardinality of a convex dominating set. A convex dominating set D of G is said to be split convex dominating set if the induced sub graph $\langle V - D \rangle$ is disconnected.

Let $G = (V, E)$ be a directed graph. A subset S of V is called dominating set of D if every vertex in $V - S$ is adjacent to at least one vertex in S. The minimum cardinality of dominating set of D is called domination number of D and is denoted by $\gamma(D)$. A dominating set S of D is convex dominating set if for every $u, v \in S$ all $u - v$ shortest path of D entirely contained in $\langle S \rangle$. A convex domination number $\gamma_{conv}(D)$ of D is the minimum cardinality of a convex dominating set. A convex dominating set S of D is said to be split convex dominating set if the induced sub graph $\langle V - D \rangle$ is disconnected. The minimum in degree, the minimum out degree, the maximum in degree and maximum out degree of D are denoted by δ^-, δ^+, Δ^- and Δ^+ respectively ([1]). An out- domination set of di-graph D is a set S^+ of vertices of D such that every vertex of $V - S^+$ is adjacent from some vertex of S. The minimum cardinality of out-domination set for D is the out-domination number $\gamma+(G)$. The in-domination number $\gamma^-(G)$ is defined as expected.

A directed star $K_{1,n}$ is the digraph where $K_{1,n} = (V, A)$ where $V = v_0, v_1, \ldots, v_n$ and $A = (v_0, v_1), (v_0, v_2), \ldots, (v_0, v_n)$. The vertex v_0 is called the central vertex of the directed star.

A directed wounded spider $S_{m,n}$ of order $m + n - 1$ is a digraph having subdivision of $n (1 \leq n \leq m)$ arcs of the directed star $K_{1,m}$. If $m = n$ then it is called directed spider $S_{n,n}$.

The directed Comet $C_{m,n}$ where m, n are positive integers denotes the out tree obtained by identifying the central vertex of the directed star directed star $K_{1, n}$ with the vertex of the directed path P_m of maximum out degree 0.

A digraph D is quasi-transitive if, for every triple of distinct vertices of D such that xy and yz are the arcs of, there is at least one arc between x and z.

Although domination and other related concepts have been extensively studied for undirected graphs, the respective analogue on directed graphs have not received much attention.

γ- set is the set of all vertices in dominating set with # $\gamma(D)$

γ^{+}- set is the set of all vertices in out dominating set with # $\gamma^{+}(D)$

γ^{-}- set is the set of all vertices in in-dominating set with # $\gamma^{-}(D)$

γ_s- set is set of all vertices in split dominating set with # $\gamma_s(D)$

γ_s^{+}- set is set of all vertices in split out dominating set with # $\gamma_s^{+}(D)$

γ_s^{-}- set is set of all vertices in split in dominating set with # $\gamma_s^{-}(D)$

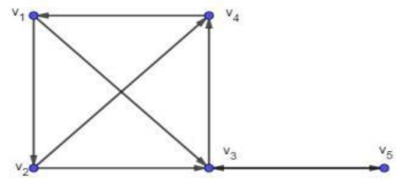
γ_{scon}^{+}- set is set of all vertices in split convex out dominating set with # $\gamma_{scon}^{+}(D)$

γ_{scon}^{-}- set is set of all vertices in split convex in dominating set with $\gamma_{scon}^{-}(D)$

2. Results and Observation

Definition 2.1. A convex dominating set S of a graph $= (V, A)$ is said to be a split convex dominating set if $\langle V - D \rangle$ is disconnected. A split convex domination number of D is the minimum cardinality of a split convex dominating set and it is denoted by $\gamma_{scon}(D)$.

Example.



Consider the above graph $D = (V, A)$. Let $S^{+} = \{v_2, v_3, v_4\}$ be the dominating set of D. Then $V - S^{+}$ will have $\{v_1, v_5\}$ Therefore, $\langle V - S^{+} \rangle$ is disconnected.

Hence S^{+} is the split convex dominating set of D and $\gamma^{+}_{scon}(D) = 3$.

Observation 2.1. For directed path P_n, $n \geq 2$, $\gamma^{+}_{scon}(D) = 0$

Observation 2.2. For directed cycle C_n, $n \geq 3$, $\gamma^{+}_{scon}(D) = 0$

Observation 2.3. For complete digraph $\gamma^{+}_{scon}(D) = 0$

Observation 2.4. For wheel digraph, $4 \leq n \leq 6$, $\gamma^{+}_{scon}(D) = 0$

Observation 2.5. For directed spider, $\gamma^{+}_{scon}(D) = 0$

Theorem 2.1. *Let D be a directed star with $\Delta^{+} = n$ and $\Delta^{-} = 1$. Then,* $\gamma^{+}_{scon}(D) = 1$.

Proof of theorem 2.1. Let D be a directed star with $\Delta^{+} = n$ and $\Delta^{-} = 1$. Let S^{+} be the dominating set of D. Then, S^{+} will have only one vertex with maximum out degree which is the central vertex that dominates all the other n vertices in $V - S^{+}$ such that $\langle V - S^{+} \rangle$ is disconnected. Hence S^{+} is the split convex dominating set with at most one vertex which implies $\gamma^{+}_{scon}(D) = 1$.

Theorem 2.2. *Let D be a directed star with $\Delta^{+} = n$ and $\Delta^{-} = 1$. Then* $\gamma^{+}_{scon}(D) = 0$.

Proof of theorem 2.2. Let D be a directed star with $\Delta^{+} = 1$ and $\Delta^{-} = n$

Let S^+ be the dominating set of D. Then, S^+ will have all the n vertices which dominates exactly one vertex in $V - S^+$ which is the central vertex. Since $\langle V - S^+ \rangle$ will have only one vertex, therefore it is connected. Hence S^+ is not split convex dominating set which implies $\gamma^+_{scon}(D) = 0$.

Theorem 2.3. *Let $S_{m,\, n}$ be a directed wounded spider of order $m + n + 1$ having subdivision of $n(1 \leq n \leq m)$ arcs of the directed star $K_{1,\, m}$. Then, the split convex dominating set exist only for $S_{m,\, 1}$ and $\gamma^+_{scon}(D) = n + 1 = 2$.*

Proof of theorem 2.3. Let $S_{m,\, n}$ be a directed wounded spider of order $m + n + 1$ having subdivision of $n = 1(1 \leq n \leq m)$ arcs of the directed star $K_{1,\, m}$. Let S^+ be the dominating set. Then S^+ will have only have the central vertex, one of the m vertices of star along with its branching vertex. Then, S^+ will have at most 3 vertices which dominates exactly $m - 1$ vertices in $V - S^+$. Since $\langle V - S^+ \rangle$ will have $m - 1$ vertices of star, therefore it is disconnected.

Hence S^+ is split convex dominating set which implies $\gamma^+_{scon}(D) = n + 1 = 2$. On contrary, for $n > 1$ the $\langle V - S^+ \rangle$ will be connected and split convex dominating set will not exists.

Theorem 2.4. *Let $C_{m,\, n}$ be a directed comet with directed star $K_{1,\, n}$ and directed path P_m. Then, $\gamma^+_{scon}(D) = m$.*

Proof of theorem 2.4. Let $C_{m,\, n}$ be a directed comet with a directed star $K_{1,\, n}$ and directed path P_m. Let S^+ be the dominating set. Then S^+ will have all the m vertices of the directed path which will include the central vertex of the directed star. Then, $V - S^+$ will have n vertices of the directed star dominated by S^+ and $\langle V - S^+ \rangle$ will be disconnected. Hence S+ is split convex dominating set and $\gamma^+_{scon}(D) = m$.

Theorem 2.5. *Let A and B be two distinct strong components of quasi transitive digraph D with at least two arcs from A to B.*

Then $\gamma^{+}_{scon}(D) = n - 2,$ *where n is the number of vertices in D.*

Proof of theorem 2.5. Let A and B be two distinct strong components of quasi transitive digraph D with at least two arcs from A to B. Then by choice of $x \in A$ and $y \in B$ there exists a path from x to y in D. Since x does not dominate y, either $y \to x$ or there exists vertices $u, v \in V(D) - \{x, y\}$ such that $x \to u \to v \to y$. Since the path of $x \to y$ passes through the cut vertices, S^{+} contains all the cut vertices together with all the other dominating vertices. Hence there will only be two vertices left in $V - S^{+}$ one from A and one from B. Then $\langle V - S^{+} \rangle$ will be disconnected. Hence S^{+} is the split convex dominating set of D and $\gamma^{+}_{scon}(D) = n - 2,$ where n is the number of vertices in D.

Theorem 2.6. *Let A and B be two distinct strong components of a strongly connected digraph D with at least two arcs from A to B. Then,* $\gamma^{+}_{scon}(D) = n - 2,$ *where n is the number of vertices in D.*

Proof of theorem 2.6. Let A and B be two distinct strong components of a strongly connected digraph D with at least two arcs from A to B. Then by choice of $x \in A$ and $y \in B$ there exists a path from x to y in D. Since x does not dominate y, either $y \to x$ or there exists vertices $u, v \in V(D) - \{x, y\}$ such that $x \to u \to v \to y$. Since the path of $x \to y$ passes through the cut vertices, S^{+} contains all the cut vertices together with all the other dominating vertices. Hence there will only be two vertices left in $V - S^{+}$ one from A and one from B. The $\langle V - S^{+} \rangle$ will be disconnected. Hence S^{+} is the split convex dominating set of D and $\gamma^{+}_{scon}(D) = n - 2,$ where n is the number of vertices in D.

Theorem 2.7. *Let D be a strongly connected digraph with cut vertices and an end vertex of degree 2. Then* $\gamma^{+}_{scon}(D) = n - 2,$ *where n is the number of vertices in D.*

Proof of theorem 2.7. Let D be a strongly connected digraph with cut vertices with an end vertex of degree 2. Let S^{+} be the split convex dominating set of D. Then $V - S^{+}$ will contain one vertex from the component and the end vertex with degree 2 such that vertices in $V - S^{+}$ will belong to different components. Thus, $\langle V - S^{+} \rangle$ will be disconnected. Hence, $\gamma^{+}_{scon}(D) = n - 2,$ where n is the number of vertices in D.

3. Conclusion

The concept of connectedness play an important role in many networks. Directed graphs are an excellent modeling tool and are widely used to establish many types of relations amongst any physical situations. In this paper we have introduced the concept of split convex domination in directed graphs and some of the interested results related to the above are proved. There has been interesting increase in study in the field of neuroscience using the digraph theory and networks. Neural networks are considered to be directed graphs that allows broad range application of analytical tools from digraph theory.

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