**Advances and Applications in Mathematical Sciences**
Volume 19, Issue 11, September 2020, Pages 1125-1131
© 2020 Mili Publications

# FUZZY CHROMATIC NUMBER OF FUZZY SOFT GRAPHS

**R. JAHIR HUSSAIN and M. S. AFYA FARHANA**

PG and Research Department of Mathematics
Jamal Mohamed College (Autonomous)
Tiruchirappalli-620020, India

## Abstract

In this paper, a new concept of fuzzy coloring of fuzzy soft graph is introduced. Fuzzy chromatic number of fuzzy soft graph, fuzzy soft sub graph, fuzzy soft bipartite graph and fuzzy soft tree has been discussed.

## Introduction

Fuzzy Graph Theory was introduced by Azriel Rosenfied in 1975. In 1999, D. Molodtsov [7] introduced the concept of soft set theory. Molodstov's soft sets give us new technique for dealing with uncertainty from the view point of parameters. Rajesh K. Thumbakara and Bobin George [12] introduced the notions of soft graph and investigate some of their properties.

The concept of fuzzy chromatic number was introduced by Munoz et al. [8]. In 2014, R. Jahir Hussain and K. S. Kanzul Fathima [4] introduced a new concept of fuzzy coloring and find fuzzy chromatic number of different fuzzy graphs.

This paper addresses the study of fuzzy chromatic number of fuzzy soft graphs. By using the concept of adjacency of vertices called strong adjacent and weak adjacent, the fuzzy chromatic number of fuzzy soft graphs, fuzzy soft sub graphs, fuzzy soft Bipartite graphs, fuzzy soft trees are studied.

## 1. Preliminaries

**Definition 1.1.** Let $S$ be an initial universe and $R$ be the set of parameters. Let $P(S)$ denotes the power set of $S$. A pair $(F, R)$ is called a soft set over $S$ where $F$ is a mapping given by $F : R \to P(S)$.

2010 Mathematics Subject Classification: 11D09, 11D99.
Keywords: fuzzy chromatic number, fuzzy soft graph, fuzzy soft bipartite graph, fuzzy soft tree.
Received June 4, 2020; Accepted August 10, 2020

**Definition 1.2.** Let $S$ be an initial universe and $R$ be the set of parameters. Let $T \subset R$. A pair $(F, T)$ is called fuzzy soft set over $S$ where $F$ is mapping given by $F : T \to I^S$ where $I^S$ denotes the collection of fuzzy subsets of $S$.

**Definition 1.3.** Let $V$ be a non-empty finite set and $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $(x, y) \in V \times V$. Then the pair $G = (\sigma, \mu)$ is called fuzzy graph over the set $V$.

**Definition 1.4.** Let $V = \{x_1, x_2, x_3, \ldots, x_n\}$ is a non empty set, $R$ is a parameter set and $T \subseteq R$. Also let

(i) $\alpha : T \to F(V)$ (collection of all fuzzy subsets in $V$)

$e \vdash\succ \alpha(e) = \alpha_e$ (say)

$\alpha_e : V \to [0, 1]$

$(T, \alpha)$ : fuzzy soft vertex

(ii) $\beta : T \to F(V \times V)$ (collection of all fuzzy subsets in $V \times V$)

$e \vdash\succ \beta(e) = \beta_e$ (say)

$\beta_e : V \times V \to [0, 1]$

$(T, \beta)$ : fuzzy soft edge

Then $((T, \alpha), (T, \beta))$ is called fuzzy soft graph if and only if $\beta_e(x, y) \leq \alpha_e(x) \wedge \alpha_e(y)$ for all $e \in T$ and this fuzzy soft graphs are denoted by $G^S(T, V)$.

**Definition 1.5.** The fuzzy soft graph $H^S(T, V) = ((T, \zeta), (T, \gamma))$ is called a fuzzy soft sub graph of $G^S(T, V) = ((T, \alpha), (T, \beta))$ if $\zeta_e(x) \leq \alpha_e(x)$, for all $x \in V$, $e \in T$ and $\gamma_e(x, y) \leq \beta_e(x, y)$ for all $x, y \in V$, $e \in T$.

**Definition 1.6.** A Path of length '$n$' in a fuzzy soft graph is a sequence of distinct vertices $x_1, x_2, x_3, \ldots, x_n$ such that $\forall e \in T$ and $\beta_e(x_{i-1}, x_i) > 0 \, \forall i = 1, 2, 3, \ldots, n$.

**Definition 1.7.** The strength of connectedness between 2 nodes $x, y$ in a fuzzy soft graph $G^S(T, V)$ is $\beta_e^\infty(x_i, x_j) = \sup\{\beta_e^k(x_i, x_j) : k = 1, 2, 3, \ldots\}$ where $\beta_e^k(x_i, x_j) = \sup\{\beta_e(x_1, x_{j+1}) \wedge \beta_e(x_2, x_{j+2}) \wedge \ldots \beta_e^k(x_{k-1}, x_j)\}$.

**Definition 1.8.** An arc $(x, y)$ in fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is said to be a strong arc if and only if $\beta_e(x, y) = \beta_e^\infty(x, y)$ and $x, y$ is said to be strong neighbours.

**Definition 1.9.** A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is fuzzy soft forest if it has a spanning fuzzy soft sub graph $H^S(T, V)$, which is a forest, where all arcs $(x, y)$ not in $H^S(T, V)$, $\beta_e(x, y) < \beta_e^\infty(x, y)$.

A connected fuzzy soft forest is called a fuzzy soft tree.

**Definition 1.10.** A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is fuzzy soft bipartite if it has a spanning fuzzy soft sub graph $H^S(T, V)$ which is bipartite where for all edges $(x, y)$ not in $H^S(T, V)$. The weight of $(x, y)$ in $G^S(T, V)$ is strictly less than the strength of the pair $(u, v)$ in $H^S(T, V)$.

(i) $\beta_e(u, v) < \gamma_e^\infty(u, v)$.

## 2. Fuzzy Chromatic Number of Fuzzy Soft Graphs

**Definition 2.1.** Two vertices $x$ and $y$ in a fuzzy soft graph $G^S(T, V)$ are strong adjacent if $(x, y)$ is a strong arc. Otherwise they are said to be weak adjacent.

**Definition 2.2.** The strong adjacent degree of a vertex '$x$' is the number of vertices that are strong adjacent to $x$. It is denoted by $d_e^s(x)$, $e \in T$.

The minimum strong adjacent degree of $G^S(T, V)$ is $\delta_e^s(G^S(T, V)) = \min\{d_e^s(x)/x \,\varepsilon\, V, e \in T\}$.

The maximum strong adjacent degree of $G^S(T, V)$ is $\Delta_e^s(G^S(T, V)) = \max\{d_e^s(x)/x \,\varepsilon\, V, e \in T\}$.

**Definition 2.3.** A $k$-fuzzy soft vertex coloring of a fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is an assignment of $k$-colors, usually denoted by $1, 2, \ldots, k$ to the vertices of $G^S(T, V)$. Thus, a $k$-fuzzy vertex coloring of $G^S(T, V)$ is a mapping $\Omega_e : V(G^S(T, V)) \to \{1, 2, \ldots, k\}$, $e \in T$.

**Definition 2.4.** A fuzzy soft vertex coloring is proper if no two distinct strong adjacent vertices receive the same color. Thus a proper $k$-fuzzy soft vertex coloring $\Omega_e$ of $G^S(T, V) = ((T, \alpha), (T, \beta))$ is a mapping $\Omega_e : V(G^S(T, V)) \to \{1, 2, \ldots, k\}$ such that $\Omega_e(v_1) \neq \Omega_e(v_2)$, $e \in T$ whenever $v_1$ and $v_2$ are strong adjacent in $G^S(T, V)$.

**Definition 2.5.** A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ has a proper $k$-fuzzy soft vertex coloring then the vertex set has the partition $(v_1^e, v_2^e, \ldots, v_k^e)$ where vie denotes all the vertices of $V$ which are colored with the color $i$, each vie is a fuzzy independent set.

**Definition 2.6.** A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is $k$-fuzzy soft vertex colorable if $G^S(T, V)$ has a proper $k$-fuzzy soft vertex coloring.

**Definition 2.7.** The fuzzy chromatic number $\chi_f^e(G^S(T, V))$, $e \in T$ of a fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is the minimum $k$ for which $G^S(T, V)$ is $k$-fuzzy soft colorable. $G^S(T, V)$ is said to be $k$-fuzzy chromatic if $\chi_f^e(G^S(T, V)) = k$.

**Remark.** It is customary to abbreviate a proper fuzzy soft vertex coloring as fuzzy soft coloring, a proper $k$-fuzzy soft vertex coloring as $k$-fuzzy soft coloring and $k$-fuzzy soft vertex colorable as $k$-fuzzy soft colorable.

## 3. Fuzzy Chromatic Number of Fuzzy Soft Sub Graphs

In a crisp graph, for any sub graph $H$ of $G$, $\chi(H) \leq \chi(G)$. But in fuzzy soft graph, it is not satisfied in general. Let $G^S(T, V)$ be fuzzy soft graph and $H^S(T, V)$ be a fuzzy soft sub graph of $G^S(T, V)$.

**Case (i).** If an edge $(x, y)$ is strong (weak) in $G^S(T, V)$ and also the same in $H^S(T, V)$ then the condition $\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))$ is satisfied, since there is no change in minimum number of fuzzy independent sets.

**Case (ii).** If a strong arc in $G^S(T, V)$ becomes weak arc in $H^S(T, V)$ then the condition $\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))$ is satisfied. Since the minimum number of fuzzy independent sets of $H^S(T, V)$ is less than or equal to that of $G^S(T, V)$.

**Case (iii).** If a weak arc in $G^S(T, V)$ becomes strong arc in $H^S(T, V)$. then the condition $\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))$ may not be satisfied, since the minimum number of fuzzy independent sets of $H^S(T, V)$ may not be less than or equal to that of $G^S(T, V)$.

Thus we conclude that, the condition $\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))$ is satisfied if a strong arc in $G^S(T, V)$ becomes strong (weak) in $H^S(T, V)$. But a weak arc in $G^S(T, V)$ becomes strong arc in $H^S(T, V)$, then the condition $\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))$ may not be satisfied.

## 4. Fuzzy Chromatic Number of Fuzzy Soft Bipartite Graphs

**Theorem 4.1.** *A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is 2-fuzzy soft colorable iff $G^S(T, V)$ is fuzzy soft bipartite graph.*

**Proof.** Suppose $G^S(T, V)$ is 2-fuzzy soft colorable.

We have to show that $G^S(T, V)$ is fuzzy soft bipartite.

Since $G^S(T, V)$ is 2-fuzzy soft colorable,

Let $V_1^e$ and $V_2^e$ be the 2 partitions such that $\forall e \in T$, $V_1^e$ is the set of all vertices of $G^S(T, V)$ which receive the color 1 and $V_2^e$ is the set of all vertices of $G^S(T, V)$ which receive the color 2.

Then every strong arc in $G^S(T, V)$ has one end in $V_1^e$ and other end in $V_2^e$.

Thus $V$ can be partitioned into 2 fuzzy independent sets $V_1^e$ and $V_2^e$.

Thus $G^S(T, V)$ is fuzzy soft bipartite graph.

Conversely, Suppose $G^S(T, V)$ is fuzzy soft bipartite.

Then $G^S(T, V)$ can be partitioned into 2 fuzzy independent sets $V_1^e$ and $V_2^e$, since every strong arc in $G^S(T, V)$ has one end in $V_1^e$ and other end in $V_2^e$.

So, we can assign the color 1 to all the vertices in $V_1^e$ and color 2 to all vertices in $V_2^e$.

Hence $G^S(T, V)$ is 2-fuzzy soft colorable.

The generalization of theorem can be stated as follows:

"A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is fuzzy soft $k$-partite iff $G^S(T, V)$ is $k$-fuzzy soft colorable."

**Corollary 4.2.** *Let $G^S(T, V)$ be a fuzzy soft graph such that $G^* = (\mu^*, \rho^*)$ is $P_p^e$, $p \geq 2$, where $P_p^e$ path of length $p$. Then $\chi_e^f(G^S(T, V)) = 2$.*

**Proof.** Clearly every arc in $G^S(T, V)$ is strong.

So vertex set can be partitioned into 2 sets $V_1^e$ and $V_2^e$, $e \in T$ such that every strong arc has one end in $V_1^e$ and other end in $V_2^e$.

Thus $\chi_e^f(G^S(T, V)) = 2$.

**Corollary 4.3.** *For a non-trivial fuzzy soft tree* (*forest*), $\chi_e^f(G^S(T, V)) = 2$.

**Proof.** Since fuzzy soft tree (forest) is fuzzy soft bipartite. Therefore $\chi_e^f(G^S(T, V)) = 2$.

## Conclusion

In this paper, the fuzzy chromatic number of fuzzy soft graphs, fuzzy soft sub graphs, fuzzy soft bipartite graphs and fuzzy trees are studied. We further extend this study on some more types of fuzzy soft graphs.

## References

[1] Anas Al-Masarwah and Majdoleen Abu Qamar, Some new concepts of fuzzy soft graphs, Fuzzy Information and Engineering, (2016).

[2] N. Cagman, S. Enginoglu and F. Citak, Fuzzy Soft Set Theory and its Applications, International Journal of Fuzzy Systems, (2011).

[3] Eslahchi and B. N. Onagh, Vertex strength of fuzzy graphs, International Journal of Mathematics and Mathematical Science, (2006).

[4] R. Jahir Hussain and K. S. Kanzul Fathima, Fuzzy chromatic number of different types of fuzzy graph, International Conference on Mathematical Sciences, (2014).

[5] R. Jahir Hussain and S. Satham Hussain, Domination in fuzzy soft graphs, International Journal of Fuzzy Mathematical Archieve, (2017).

[6] S. Lavanya and R. Sattanathan, Fuzzy vertex coloring in fuzzy graphs, International Review of Fuzzy Mathematics, (2011).

[7] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37(4-5) (1999), 19-31.

[8] S. Munoz, T. Ortuno, J. Ramirez and J. Yanez, Coloring Fuzzy Graphs, Omega 33(3) (2005), 211-221.

[9] Muhammed Akram and Saira Nawaz, On fuzzy soft graphs, Italian Journal of Pure and Applied Mathematics 34 (2015), (497-514).

[10] Muhammed Akram and Fariha Zafar, Fuzzy soft trees, Southeast Asian Bulletin of Mathematics 40(2) (2016), 151-170.

[11] A. Nagoor Gani and V. T. Chandrasekren, First Look at Fuzzy Graph Theory, Allied Publishers, (2010).

[12] Rajesh K. Thumbakara and Bobin George, Soft graphs, Gen. Math. Notes, (2014).

[13] Sumit Mohinta and T. K. Samanta, An introduction to fuzzy soft graph, Mathematica Moravika, (2015).

[14] M. S. Sunitha and Sunil Mathew, Fuzzy Graph Theory: A Survey, Annals of Pure and Applied Mathematics, (2013).