Advances and Applications in Mathematical Sciences
Volume 21, Issue 4, February 2022, Pages 1687-1701

FUZZY SHORTEST PATH PROBLEM WITH TRIANGULAR INTUITIONISTIC FUZZY NUMBERS AS ARC LENGTHS

L. SUJATHA and N. ANBARASI

Assistant Professor
Department of Mathematics
Auxilium College (Autonomous), Vellore-6
E-mail: sujathajayasankar@yahoo.co.in

M.Sc. Student, Department of Mathematics
Auxilium College (Autonomous), Vellore-6
E-mail: anbarasi2230@gmail.com

Abstract

The shortest path problem is a classical and important network optimization problem in a non-fuzzy network which plays a vital role in many social networks. In this paper, we discuss the shortest path problem from a specified vertex to every other vertex on a network with imprecise arc lengths. A theorem is proposed for the fuzzy shortest path problem where each arc lengths is in the from of take triangular intuitionistic fuzzy numbers. Four different cases of fuzzy shortest path are verified, based on the parameters chosen by the decision makers to point out the conclusion. Suitable numerical example is demonstrated for the proposed approach. Simulation result using C program is included for the general algorithm. Comparison is also made with the existing earlier result.

1. Introduction

Over the past several years, a great deal of attention has been paid to mathematical programs and mathematical models that can be solved through the use of networks. Posing problems on networks not only yields computational advantages, it also serves as a means for visualizing a problem and for developing a better understanding of the problem. It is much easier for a decision maker to draw a picture of what he wants than it is to write

2020 Mathematics Subject Classification: 90B99.

Keywords: Acyclic network, Shortest path problem, Triangular intuitionistic fuzzy numbers, α-cut ranking technique, Decision maker.

Received October 5, 2020; Accepted November 10, 2020

down the constraints. A wide variety of network type problems include models such as location, transportation, flow, reliability and shortest path to name a few. It is the purpose of this paper to concentrate on the most basic network problem, the shortest path problem under fuzzy environment. In traditional shortest path problem the arc lengths (edge weights) of a network takes crisp value. But in real life situations the arc length may represent cost, time etc. There are so many cases that a unique number cannot be assigned to edge weights, because of cost, time variation. Hence we focus on fuzzy shortest path problem (FSPP). Fuzzy set theory, proposed by Zadeh [14], is frequently utilized to deal with uncertainty problems. The fuzzy shortest path problem was first analyzed by Dubois and Prade [3]. However, the major drawback to this classical fuzzy shortest path problem is, a fuzzy shortest path length is found, but it may not correspond to an actual path in the network. Numerous papers have been published under fuzzy shortest path problems [1, 9, 10, 12]. The papers of Yao and Lin [13], Elizabeth and Sujatha [4, 6] motivated us to present this current work. The paper is organized as follows: In section 2, preliminary definitions are reviewed. Section 3, focus on formulation of shortest path problem under fuzzy environment, where a theorem is proposed for fuzzy shortest path problem with the suitable numerical examples verifying three different cases (case (i), case (ii) and case (iii)) of FSPP based on the parameters chosen by the decision makers. In section 4, we have proposed a general algorithm, where case (iv) of FSPP is verified. Simulation result using C language is included for the same. Comparison is also made with the existing earlier results to point out the conclusion. Section 5, concludes the paper.

2. Prerequisites

The concepts of an intuitionistic fuzzy set was introduced by Atanassov [2] to deal with vagueness which can be defined as follows given in definition 2.1.

Definition 2.1. (Intuitionistic Fuzzy Set) Let X be an universe of discourse, then an intuitionistic fuzzy set (IFS) A in X is given by $A = \{(x, \mu_A(x), \gamma_A(x))/x \in X\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\gamma_A(x) : X \to [0, 1]$, determine the degree of membership and non-membership of the element $x \in X$ respectively and for every $x \epsilon X$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.2 (Triangular Intuitionistic Fuzzy Number [11]). The triangular intuitionistic fuzzy number $A = \{(\mu_A(x), \gamma_A(x))/x \epsilon R\} = (\langle a_1, a_2, a_3 \rangle, \langle a'_1, a_2, a'_3 \rangle) = (A_M, A_{NM})$, $a'_1 < a_1 < a_2 < a_3 < a'_3$; $a'_1, a_1, a_2, a_3, a'_3 \epsilon R$. Here M denote the membership function and NM denote the non- membership function.

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a'_1} & \text{for } a'_1 \leq x \leq a_2 \\ 0 & \text{for } x = a_2 \\ \dfrac{x - a_2}{a'_3 - a_2} & \text{for } a_2 \leq x \leq a'_3 \\ 1 & \text{otherwise} \end{cases}$$

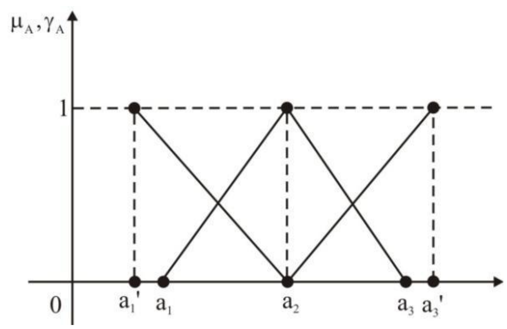

Figure 1. Triangular Intuitionistic Fuzzy Number.

Definition 2.3. (α-cut ranking technique for triangular intuitionistic fuzzy number [5]) Let $A = (\langle a_1, a_2, a_3 \rangle, \langle a'_1, a_2, a'_3 \rangle)$ be a triangular intuitionistic fuzzy number then the α-cut ranking technique of A is denoted by

$$R(A) = (\langle \int_0^1 [a_1 + \alpha(a_2 - a_1)]\alpha d\alpha + \int_0^1 [a_3 - \alpha(a_3 - \alpha_2)]\alpha d\alpha \rangle,$$

$$\langle - \int_0^1 [a_2 - \alpha(a_2 - a_1')]\alpha d\alpha - \int_0^1 [a_2 + \alpha(a_3' - a_2)]\alpha d\alpha \rangle)$$

$$= (\langle \frac{a_1 + 4a_2 + a_3}{6} \rangle, - \langle \frac{a_1' + a_2 + a_3'}{3} \rangle) = (R_M(A), R_{NM}(A))$$

= (α-cut ranking of membership function, α-cut ranking of non-membership function)

Ranking function of α-cut ranking technique: If A and B are triangular intuitionistic fuzzy numbers, then $A < B$ if and only if α-cut ranking of membership function of A is less than or equal to the α-cut ranking of membership function of B and α-cut ranking of non-membership function of A is greater than or equal to the α-cut ranking of non-membership function of B.

Definition 2.4. (Addition operation and Binary Operation on triangular intuitionistic fuzzy numbers) Let $A = (\langle a_1, a_2, a_3 \rangle, \langle a_1', a_2, a_3' \rangle)$ and $B = (\langle b_1, b_2, b_3 \rangle, \langle b_1', b_2, b_3' \rangle)$ be two triangular intuitionistic fuzzy numbers then

$$A \oplus B = (\langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle, \langle a_1' + b_1', a_2 + b_2, a_3' + b_3' \rangle) \quad (2.1)$$

$$R_M(A + B) = R_M(A) + R_M(B), R_{NM}(A + B) = R_{NM}(A) + R_{NM}(B) \quad (2.2)$$

3. Formulation of Fuzzy Shortest path problem

Let $G = (V, E)$ be an acyclic directed graph (network), where V is the set of vertices from source vertex 1 to the destination vertex n and E is the Set of edges. Consider the triangular intuitionistic edge weights of the network

$$\tilde{d}_{ij} = (\langle \tilde{d}_{Mij}, \tilde{d}_{NMij} \rangle)$$

$$= (\langle d_{ij} - \gamma_{ij1}, d_{ij}, d_{ij} + \gamma_{ij2} \rangle, \langle d_{ij} - \gamma'_{ji1}, d_{ij}, d_{ij} + \gamma'_{ij2} \rangle), \quad (3.1)$$

where $\gamma_{ij1}, \gamma_{ij2}, \gamma'_{ij1}$ and γ'_{ij2} are the parameters chosen by the decision makers, which satisfies the following conditions:

$0 < \gamma_{ij1} < d_{ij}, 0 < \gamma_{ij2}, 0 < \gamma'_{ij1} < d_{ij}, 0 < \gamma'_{ij2}, \gamma_{ij1} < \gamma'_{ij1}, \gamma_{ij2} < \gamma'_{ij2},$

$$d_{ij} - \gamma'_{ij1} < d_{ij} - \gamma_{ij1} < d_{ij} < d_{ij} + \gamma_{ij2} < d_{ij} + \gamma'_{ij2} \tag{3.2}$$

From definition (2.3), we obtain

$$d^*_{ij} = R(\tilde{d}_{ij})$$

$$= ((\frac{d_{ij} - \gamma_{ij1} + 4d_{ij} + d_{ij} + \gamma_{ij2}}{6})), -((\frac{d_{ij} - \gamma'_{ij1} + d_{ij} + d_{ij} + \gamma'_{ij2}}{3}))$$

$$= \left(\left(\frac{6d_{ij} + (\gamma_{ij2} - \gamma_{ij1})}{6}\right), \left(\frac{-3d_{ij} + (\gamma'_{ij1} - \gamma'_{ij2})}{3}\right)\right)$$

$$= \left(d_{ij} + \left(\frac{\gamma_{ij2} - \gamma_{ij1}}{6}\right), -d_{ij} + \left(\frac{\gamma'_{ij1} - \gamma'_{ij2}}{3}\right)\right)$$

$$d^*_{ij} = (d^*_{Mij}, d^*_{NMij}) = (d_{ij} + \frac{\Delta_{ij}}{6}, -d_{ij} + \frac{\Delta'_{ij}}{3}), \tag{3.3}$$

Where $\Delta_{ij} = \gamma_{ij2} - \gamma_{ij1}, \Delta'_{ij} = \gamma'_{ij1} - \gamma'_{ij2}$.

In equation (3.3), if $\gamma_{ij2} - \gamma_{ij1}$ and $\gamma'_{ij1} = \gamma'_{ij2}$ then $d^*_{ij} = (d_{ij}, -d_{ij}) = (d^*_{Mij}, d^*_{NMij})$. Thus the fuzzy problem becomes the crisp problem in intuitionistic case.

$d^*_{ij} = (d^*_{Mij}, d^*_{NMij}) = \left(d_{ij} + \frac{\Delta_{ij}}{6}, -d_{ij} + \frac{\Delta'_{ij}}{3}\right)$ is the estimate of edge weights in fuzzy sense. The dynamic programming formulation for the shortest path problem in the crisp intuitionistic case can be given as follows:

$$h_M(1) = h_{NM}(1) = 0$$

$$h_M(j) = \min_{i<j} \{h_M(i) + d_{Mij}, \langle i, j\rangle \epsilon E\}, d_{ij} > 0 \tag{3.4}$$

where d_{Mij} is the edge weights of the directed network and $h_M(j)$ is the length of the shortest path in the crisp sense from vertex 1 to vertex j with respect to membership function.

$$h_{NM}(j) = \max_{i<j} \{h_{NM}(i) + d_{NMij} \langle i, j\rangle \epsilon E\}, d_{ij} < 0 \tag{3.5}$$

Where d_{Mij} is the edge weights of the directed network and $h_M(j)$ is the length of the shortest path in the crisp sense from vertex 1 to vertex j with respect to non-membership function. Because there are finite paths from vertex 1 to vertex n in a network, we conclude that there are also finite number of paths from vertex 1 to vertex j, in the network.

Thus, there exists a path $P = \langle 1, j_1, j_2, \ldots, j_{k(j)}, j\rangle$, (i.e. $\langle 1, j_1\rangle, \langle j_1, j_2\rangle, \ldots, \langle j_{k(j)}, j\rangle)$ for, $h_M(j) = d_{M1j_1} + d_{Mj_1j_2} + \ldots + d_{Mj_{k(j)}j}$. Note that $h_M(j)$ and $h_{MN}(j)$ is the length of the shortest path from vertex 1 to vertex j, in case of membership and non-membership function respectively. Therefore we have $h_M(j) = d_{M1j_1} + d_{Mj_1j_2} + \ldots + d_{Mj_{k(j)}j} \leq d_{M1i_1} + d_{Mi_1i_2} + \ldots + d_{Mi_{m(i)}j}$, where at least one equal sign holds for all possible paths, $P = \langle 1, i_1, i_2, \ldots, i_{m(i)}, j\rangle$, from vertex 1 to vertex j. Thus, $h_M(j) = \min\{d_{M1i_1} + d_{Mi_1i_2} + \ldots + d_{Mi_{m(i)}j}$ for all paths $P = \{\langle 1, i_1, i_2, \ldots, i_{m(i)}, j\rangle\}$. Also $h_{NM}(j) = d_{NM1j_1} + d_{NMj_1j_2} + \ldots + d_{NMj_{k(j)}j} \geq d_{NM1i_1} + d_{NMi_1i_2} + d_{NMi_1i_2} + \ldots + d_{NMi_{m(i)}j}$, where at least one equal sign holds for all possible paths, $P = \langle 1, i_1, i_2, \ldots, i_{m(i)}, j\rangle$, from vertex 1 to vertex j. Thus, $h_{NM}(j) = \max\{d_{M1i_1} + d_{Mi_1i_2} + \ldots + d_{Mi_{m(i)}j}$ for all paths $P = \langle 1, i_1, i_2, \ldots, i_{m(i)}, j\rangle\}$.

We rewrite equation (3.4) as follows: for any fixed j,

$h_M(j) \leq h_M(i) + d_{Mij}$, for all $i < j$, $(i, j) \epsilon E$, where at least one equal sign holds.

$$h_M(j) = d_{M1j_1} + d_{Mj_1j_2} + \ldots + d_{Mj_{k(j)}j}$$

$$h_M(i) = d_{M1i_1} + d_{Mi_1i_2} + \ldots + d_{Mi_{k(i)}i}$$

$$d_{M1j_1} + d_{Mj_1j_2} + \ldots + d_{Mj_{k(j)}j}$$

$$\leq d_{M1i_1} + d_{Mi_1i_2} + \ldots + d_{Mi_{k(i)}i} + d_{Mij} \tag{3.6}$$

Decision maker choose appropriate values for the parameters to satisfy

$$\left.\begin{array}{l}\gamma_{1j_1}+\gamma_{j_1j_2}+\ldots+\gamma_{j_{k(j)}j}\leq\gamma_{1i1}+\gamma_{i1i2}+\ldots+\gamma_{i_{k(i)}i}+\gamma_{ij}\text{ for }\gamma_{ij_1}<\gamma_{ij2}\\ \gamma_{1j_1}+\gamma_{j_1j_2}+\ldots+\gamma_{j_{k(j)}j}\geq\gamma_{1i1}+\gamma_{i1i2}+\ldots+\gamma_{i_{k(i)}i}+\gamma_{ij}\text{ for }\gamma_{ij_1}>\gamma_{ij2}\end{array}\right\}\quad(3.7)$$

From (3.6) and (3.7) we have

$\tilde{d}_{M1j_1}+\tilde{d}_{Mj_1j_2}\ldots+\tilde{d}_{Mj_{k(j)}j}\leq\tilde{d}_{M1i_1}+\tilde{d}_{Mi_1i_2}+\ldots+\tilde{d}_{Mi_{k(i)}i}+\tilde{d}_{Mij}$ for all $i<j$ and $\langle i, j\rangle\epsilon E$

$$R_M(\tilde{d}_{M1j_1}+\tilde{d}_{Mj_1j_2}+\ldots+\tilde{d}_{Mj_{k(j)}j})$$

$$\leq R_M(\tilde{d}_{M1i_1}+\tilde{d}_{Mi_1i_2}+\ldots+\tilde{d}_{Mi_{k(i)}i}+\tilde{d}_{Mij})$$

$$R_M(\tilde{d}_{M1j_1})+R_M(\tilde{d}_{Mj_1i_2})+\ldots+R_M(\tilde{d}_{Mj_{k(j)}j})$$

$$\leq R_M(\tilde{d}_{M1i_1})+R_M(\tilde{d}_{Mi_1i_2})+\ldots+R_M(\tilde{d}_{Mi_{k(i)}i})+R_M(\tilde{d}_{Mij})$$

$$d^*_{M1j_1}+d^*_{Mj_1j_2}+\ldots+d^*_{Mj_{k(j)}}j\leq d^*_{M1i_1}+d^*_{Mi_1i_2}+\ldots+d^*_{Mi_{k(i)}i}+d^*_{Mij}$$

$$h^*_M(j)\leq h^*_M(i)+d^*_{Mij},\text{ for all } i<j \text{ and } \langle i, j\rangle\epsilon E$$

$$h^*_M(j)=\min_{i<j}\{h^*_M(i)+d^*_{Mij},\text{ for all } i<j \text{ and } \langle i, j\rangle\epsilon E$$

We rewrite (3.5) as follows: for any fixed j,

$h_{NM}(j)\geq h_{NM}(i)+d_{NMij}$, for all $i<j$, $(i, j)E$, where at least one equal sign holds.

$$h_{NM}(j)=d_{NM1j_1}+d_{NMj_1j_2}+\ldots+d_{NMj_{k(j)}j}$$

$$h_{NM}(i)=d_{NM1i_1}+d_{NMi_1i_2}+\ldots+d_{NMi_{k(i)}j}$$

$$d_{NM1j_1}+d_{NMj_1j_2}+\ldots+d_{NMj_{k(j)}j}\geq d_{NM1i_1}+d_{NMi_1i_2}+\ldots+d_{NMi_{k(i)}i}+d_{NMij}. \quad (3.8)$$

Decision maker choose appropriate values for the parameters to satisfy

$$\left.\begin{array}{l}\gamma'_{1j1}+\gamma'_{j1j2}+\ldots+\gamma'_{j_{k(j)}j}\geq\gamma'_{1i1}+\gamma'_{i1i2}+\ldots+\gamma'_{ik(i)i}+\gamma'_{ij}\text{ for }\gamma'_{ij_1}<\gamma'_{ij2}\\ \gamma'_{1j1}+\gamma'_{j1j2}+\ldots+\gamma'_{jk(j)j}\leq\gamma'_{1i1}+\gamma'_{i1i2}+\ldots+\gamma'_{ik(i)i}+\gamma'_{ij}\text{ for }\gamma'_{ij_1}>\gamma'_{ij2}\end{array}\right\}\tag{3.9}$$

From (3.8) and (3.9) we have

$$\tilde{d}_{NM1j_1}+\tilde{d}_{NMj_1j_2}+\ldots+\tilde{d}_{NMj_{k(j)}j}\geq\tilde{d}_{NM1i_1}+\tilde{d}_{NMi_1i_2}+\ldots+\tilde{d}_{NMi_{k(i)}i}+\tilde{d}_{NMij}$$

for all $i<j$ and $\langle i,\ j\rangle\in E$

$$R_{NM}(\tilde{d}_{NM1j_1}+\tilde{d}_{NMj_1j_2}+\ldots+\tilde{d}_{NMj_{k(j)}j})$$

$$\geq R_{NM}(\tilde{d}_{NM1i_1}+\tilde{d}_{NMi_1i_2}+\ldots+\tilde{d}_{NMi_{k(i)}i}+\tilde{d}_{NMij})$$

$$R_{NM}(\tilde{d}_{NM1j_1})+R_{NM}(\tilde{d}_{NMj_1j_2})+\ldots+R_{NM}(\tilde{d}_{NMj_{k(j)}j})$$

$$\geq R_{NM}(\tilde{d}_{NM1i_1})+R_{NM}(\tilde{d}_{NMi_1i_2})+\ldots+R_{NM}(\tilde{d}_{NMi_{k(i)}i})+R_{NM}(\tilde{d}_{NMij})$$

$$d^*_{NM1j_1}+d^*_{NMj_1j_2}+\ldots+d^*_{NMj_{k(j)}j}\geq d^*_{NM1i_1}+d^*_{NMi_1i_2}+\ldots+d^*_{NMi_{k(i)}i}+d^*_{NMij}$$

$$h^*_{NM}(j)\geq h^*_{NM}(i)+d^*_{NMij},\text{ for all } i<j \text{ and } \langle i,\ j\rangle\in E$$

$$h^*_{NM}(j)=\max_{i<j}\{h^*_{NM}(i)+d^*_{NMij},\text{ for all } i<j \text{ and } \langle i,\ j\rangle\in E$$

Finally, we summarize the above description in the following theorem.

Theorem for Fuzzy Shortest Path Problem. Consider an acyclic directed network $G(V,\ E)$ where V denote the set of vertices and E denote the set of edges. The edge weights are denoted by $\{(d_{ij},\ -d_{ij})/\langle i,\ j\rangle\in E\}$. Based on the triangular intuitionistic fuzzy number given in equation (3.1) for which the conditions equation (3.2) is satisfied, we define the edges weights

$$d^*_{ij}=(d^*_{Mij},\ d^*_{NMij})=\left(d_{ij}+\left(\frac{\gamma_{ij2}-\gamma_{ij1}}{6}\right),\ -d_{ij}+\left(\frac{\gamma'_{ij1}-\gamma'_{ij2}}{3}\right)\right)$$

$$=\left(d_{ij}+\frac{\Delta_{ij}}{6},\ -d_{ij}+\frac{\Delta'_{ij}}{3}\right),$$

where γ_{ij1}, γ_{ij2}, γ'_{ij1} and γ'_{ij2} are the parameters whose values are chosen by the decision makers to satisfy the conditions in equations (3.7) and (3.9) creating a set of edge weights in the fuzzy sense Then the shortest path problem in the fuzzy sense $\{(d^*_{Mij}, d^*_{NMij})/\langle i, j\rangle \epsilon E\}$. is given by

$$h^*_M(1) = h^*_{NM}(1) = 0 \text{ and}$$

$$h^*_M(j) = \min_{i<j} \{h^*_M(i) + d^*_{Mij}, \text{ for all } i < j \text{ and } \langle i, j\rangle \epsilon E, d^*_{ij} > 0$$

$$h^*_M(j) = \max_{i<j} \{h^*_{NM}(i) + d^*_{NMij}, \text{ for all } i < j \text{ and } \langle i, j\rangle \epsilon E, d^*_{ij} < 0,$$

where $h^*_M(j)$ and $h^*_{NM}(j)$ are the length of the shortest path in the fuzzy sense from vertex 1 to vertex j with respect to membership and non-membership functions respectively.

Numerical Example 1:

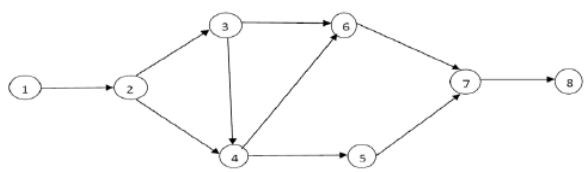

Figure 2. Acyclic network.

Table 1 : Case (i) : $\gamma_{ij1} = \gamma_{ij2}, \gamma'_{ij1} = \gamma'_{ij2}$.

Activity	Duration	α-Cut Ranking
(1, 2)	⟨(21, 25, 29), (18, 25, 32)⟩	(25, –25)
(2, 3)	⟨(15, 20, 25), (14, 20, 26)⟩	(20, –20)
(2, 4)	⟨(60, 63, 66), (54, 63, 72)⟩	(63, –63)
(3, 4)	⟨(37, 41, 45), (35, 41, 47)⟩	(41, –41)
(3, 6)	⟨(54, 57, 60), (50, 57, 64)⟩	(57, –57)
(4, 5)	⟨(12, 15, 18), (10, 15, 20)⟩	(15, –15)

(4, 6)	⟨(8, 9, 10), (7, 9, 11)⟩	(9, –9)
(5, 7)	⟨(70, 75, 80), (65, 75, 85)⟩	(75, –75)
(6, 7)	⟨(65, 75, 85), (60, 75, 90)⟩	(75, –75)
(7, 8)	⟨(20, 25, 30), (15, 25, 35)⟩	(25, –25)

By applying the theorem, we obtain the crisp shortest path (CSP) as 1-2-3-4-6-7-8 and the crisp shortest length is (195, –195).

Table 2. Case (ii): $\gamma_{ij1} < \gamma_{ij2}, \gamma'_{ij1} < \gamma'_{ij2}$.

Activity	Duration	α-Cut Ranking
(1, 2)	⟨(23, 25, 29), (20, 25, 32)⟩	(25.3, –25.7)
(2, 3)	⟨(16, 20, 25), (14, 20, 27)⟩	(20.2, –20.3)
(2,4)	⟨(60, 63, 68), (57, 63, 70)⟩	(63.3,–63.3)
(3, 4)	⟨(38, 41, 46), (35, 41, 48)⟩	(41.3,–41.3)
(3, 6)	⟨(54, 57, 62), (50, 57, 65)⟩	(57.3, –57.3)
(4, 5)	⟨(13, 15, 18), (11, 15, 20)⟩	(15.2, –15.3)
(4, 6)	⟨(6, 9, 13), (5, 9, 15)⟩	(9.2, –9.7)
(5, 7)	⟨(72, 75, 80),(67, 75, 85)⟩	(75.3, –75.7)
(6, 7)	⟨(68, 75, 85), (62, 75, 90)⟩	(75.5, –75.7)
(7, 8)	⟨(23,25,30), (18,25,35)⟩	(25.5, –26)

By applying the theorem, we obtain the fuzzy shortest path (FSP) as 1-2-3-4-6-7-8 and the fuzzy shortest length is (197, -198.7)

Table 3. Case (iii): $\gamma_{ij1} > \gamma_{ij2}, \gamma'_{ij1} > \gamma'_{ij2}$.

Activity	Duration	α-Cut Ranking
(1, 2)	⟨(19, 25, 29), (18, 25, 30)⟩	(24.7,–24.3)
(2, 3)	⟨(14, 20, 25), (12, 20, 26)⟩	(19.8,–19.3)
(2, 4)	⟨(56, 63, 68), (55, 63, 70)⟩	(62.7,–62.7)

(3, 4)	$\langle(36, 41, 44), (32, 41, 48)\rangle$	(40.7, –40.3)
(3, 6)	$\langle(52, 57, 60), (48, 57, 65)\rangle$	(56.7, –56.7)
(4, 5)	$\langle(11, 15, 18), (10, 15, 19)\rangle$	(14.8, –14.6)
(4, 6)	$\langle(7, 9, 10), (6, 9, 11)\rangle$	(8.8, –8.7)
(5, 7)	$\langle(68, 75, 80), (65, 75, 82)\rangle$	(74.7, –74)
(6, 7)	$\langle(63, 75, 85), (66, 75, 88)\rangle$	(74.7, –74.3)
(7, 8)	$\langle(17, 25, 30),(19, 25, 33)\rangle$	(24.5, –24.3)

By applying the theorem, we obtain the fuzzy shortest path as 1-2-3-4-6-7-8 and the fuzzy shortest length is (193.2, –191.2).

Table 4. Results and Discussion 1.

Case		Path length	Path	Result
$\gamma_{ij1} = \gamma_{ij2}$	Member	(195, –195)	1-2-3-4-6-7-8	FSP = CSP
$\gamma'_{ij1} = \gamma'_{ij2}$	Non-member			
$\gamma_{ij1} < \gamma_{ij2}$	Member	(197, –198.7)	1-2-3-4-6-7-8	FSP is extension of CSP
$\gamma'_{ij1} < \gamma'_{ij2}$	Non-member			CSP is an extension of FSP
$\gamma_{ij1} > \gamma_{ij2}$	Member	(193.2, –191.2)	1-2-3-4-6-7-8	CSP is an extension of FSP
$\gamma'_{ij1} > \gamma'_{ij2}$	Non-member			FSP is extension of CSP

4. Proposed General Algorithm for FSPP

Step 1: Let $G(V, E)$ is an acyclic directed network.

Let. $\widetilde{d}_{ij} = (\langle a_{ij}^{(1)}, a_{ij}^{(2)}, a_{ij}^{(3)}\rangle, \langle a'^{(1)}_{ij}, a'^{(2)}_{ij}, a'^{(3)}_{ij}\rangle).$ Each length corresponds to the cost, time etc., in practical problems.

Step 2: Calculate all possible paths P_i, $i = 1$ to n from the source vertex 's' to the destination vertex 'd' and the corresponding path lengths L_i, $i = 1$ to n using addition operation given in equation (2.1). Set $L_i = (\langle a_i, b_i, c_i \rangle, \langle a'_i, b'_i, c'_i \rangle)$.

Step 3: Calculate the α-cut ranking technique for each possible path length, L_i, $i = 1$ to n. The path having the smallest α-cut ranking for membership function and the largest α-cut ranking for non-membership function is identified as the fuzzy shortest path.

Numerical Example 2:

Table 5. Case (iv): $\gamma_{ij1} > \gamma_{ij2}, \gamma'_{ij1} > \gamma'_{ij2}, \gamma_{ij1} < \gamma_{ij2}, \gamma'_{ij1} < \gamma'_{ij2}, \gamma_{ij1} = \gamma_{ij2}, \gamma'_{ij1} = \gamma'_{ij2}$

Activity	Duration
(1, 2)	$\langle(19, 25, 29), (18, 25, 30)\rangle$
(2, 3)	$\langle(15, 20, 25), (14, 20, 27)\rangle$
(2, 4)	$\langle(58, 63, 68), (55, 63, 70)\rangle$
(3, 4)	$\langle(38, 41, 46), (35, 41, 48)\rangle$
(3, 6)	$\langle(54, 57, 62), (50, 57, 65)\rangle$
(4, 5)	$\langle(12, 15, 18), (10, 15, 20)\rangle$
(4, 6)	$\langle(8, 9, 10), (7, 9, 11)\rangle$
(5, 7)	$\langle(70,75, 80), (65, 75, 85)\rangle$
(6, 7)	$\langle(65, 75, 85), (60, 75, 90)\rangle$
(7, 8)	$\langle(20, 25, 30), (15, 25, 35)\rangle$

By applying the algorithm, we obtain Table 6.

Paths (P_i)	Path Lengths L_i	α-Cut ranking for L_i	Ranking
P_1 : 1 –2 –3 –4 –6 –7 –8	$\langle(165, 195, 225),$	(195, –195)	1

	$(149, 195, 241)\rangle$		
$P_2 : 1 -2 -4 -5 -7 -8$	$\langle(179, 203, 225),$ $(163, 203, 240)\rangle$	$(202.6, -202)$	5
$P_3 : 1 -2 -3 -4 -5 -7 -8$	$\langle(174, 201, 228),$ $(157, 201, 245)\rangle$	$(201, -201)$	3
$P_4 : 1 -2 -3 -6 -7 -8$	$\langle(173, 202, 231),$ $(157, 202, 247)\rangle$	$(202, -202)$	4
$P_5 : 1 -2 -4 -6 -7 -8$	$\langle(170, 197, 222),$ $(155, 197, 236)\rangle$	$(196.6, -196)$	2

The fuzzy shortest path is 1-2-3-4-6-7-8 and the fuzzy shortest length is (195, –195).

Simulation Result:

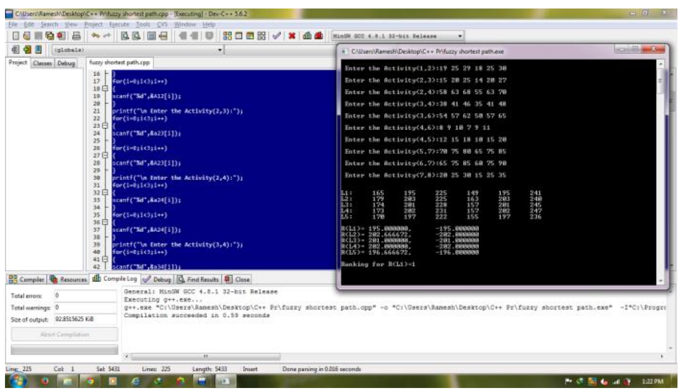

Figure 3. Simulation result using C Language.

Results and Discussion 2: One way to verify the solution obtained is to make an exhaustive comparison. Kung et al. and Okada [7, 8] considered only the membership function for the arc lengths and obtained two fuzzy shortest paths 1-2-4-6-7-8 and 1-2-3-4-6-7-8 for Figure 2. The result obtained in this paper, considering both membership function and non-membership function gives only one fuzzy shortest path (i.e) 1-2-3-4-6-7-8.

5. Conclusion

Many researchers have focused on fuzzy shortest path problem in a network since it is significant to various applications. In this paper, a theorem is presented where the arc length is in the form of triangular intuitionistic fuzzy numbers. Four different cases of fuzzy shortest path is verified, based on the parameters chosen by the decision makers to point out the conclusion. The procedure proposed for triangular intuitionistic fuzzy number will be helpful for the decision maker as they make decision in choosing the best of all the possible path alternatives in case of membership and non-membership functions.

References

[1] Arindam Dey, Rangaballav Pradhan, Anita Pal and Tandra Pal, A genetic algorithm for solving fuzzy shortest path problems with interval type-2 fuzzy arc lengths, Malaysian Journal of Computer Science 31(4) (2018), 255-270.

[2] K. Atanassov, Intuitionistic fuzzy sets, fuzzy sets, and Systems 20(1) (1986), 87-96.

[3] D. Dubois and H. Prade, Fuzzy sets and systems: Theory and Applications, New York Academic press 1980.

[4] S. Elizabeth and L. Sujatha, Fuzzy longest path problem for project scheduling, Advances in Fuzzy Sets and Systems 17(1) (2014) 1-25.

[5] S. Elizabeth and L. Sujatha, Project scheduling method using triangular intuitionistic fuzzy numbers and triangular fuzzy numbers, Applied Mathematical Sciences 9(4) (2015), 185-198.

[6] S. Elizabeth, M. Abirami and L. Sujatha, Finding critical path in a project network under fuzzy environment, Mathematical Sciences International Research Journal 5 (2016), 31-36.

[7] J. Y. Kung, T. N. Chuang and C. T. Lin, Decision making on network problem with fuzzy arc lengths, IMACS Multi Conference on Computational Engineering in Systems Applications (CESA) 578-580, October 4-6, Beijing, China, 2006.

[8] S. Okada, Interactions among paths in fuzzy shortest path problems, IEEE 1 (2001), 41-46.

[9] Ranjan Kumar, S. A. Edalatpanah, Sripati Jha, Sudipta Gayen and Ramayan Singh, Shortest path problems using fuzzy weighted arc length, International Journal of Innovative Technology and Exploring Engineering (IJITEE) 8(6) (2019), 724-731.

[10] Said Broumi, Mohamed Talea, Assia Bakali, Florentin Smarandache, Deivanayagampillai Nagarajan, Malayalan Lathamaheswari and Mani Parimala, Shortest path problem in fuzzy intuitionistic fuzzy and neutrosophic environment, Complex and Intelligent Systems 5 (2019), 371-378.

[11] A. K. Shaw and T. K. Roy, Some arithmetic operations on triangular intuitionistic fuzzy number and its application on reliability evaluation, International Journal of Fuzzy

Mathematics and Systems 2(4) (2012), 363-382.

[12] A. Syarif, K. Muludi, R. Adrian and M. Gen, Solving fuzzy shortest path problem by genetic algorithm, IORA-ICOR 2017, IOP Conference Series: Material Science and Engineering 332 (2018), 1-12.

[13] J. S. Yao and F. T. Lin, Fuzzy shortest-path network problems with uncertain edge weights, Journal of Information Science and Engineering 19 (2003), 329-351.

[14] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.