Advances and Applications in Mathematical Sciences
Volume 21, Issue 1, November 2021, Pages 421-434


# STEADY STATE ANALYSIS OF $M/G/1$ RETRIAL QUEUEING MODEL WITH RESTRICTED ADMISSION AND SERVER BREAKDOWN

**E. RATHNAKUMARI**

Assistant Professor
Department of Mathematics
Loganatha Narayanasamy Government College
Ponneri, Tamilnadu, India
E-mail: rrathnakumari11@gmail.com

## Abstract

This paper deals with steady state analysis of $M/G/1$ retrial queuing model with restricted admissibility and random server breakdown. The arriving units are allowed to join the system with state dependent admission control policy. Let $\alpha_1, \alpha_2,$ and $\alpha_3$ be the assigned probabilities for an arrival to join the system during the period of idle, busy, or on breakdown respectively. While the server working, it may breakdown at any instant and the service channel will fail for a short interval of time and it is repaired immediately. The steady state distribution of the server state and the number of customer in the orbit/system are obtained. Numerical example is presented to illustrate the influence of the parameters on several performance characteristics.

## 1. Introduction

Nowadays, retrial queues have become increasingly important in the analysis of computer and communication networks. Retrial queuing system are characterized by the fact that an arriving customer who finds server busy upon arrival will leave the service area and repeats his requests for service after some time. In many queuing situations, the customer's arrival rate depends on the state of the server like idle, busy or on breakdown. In recent years queues with server breakdown have emerged as an important area of

2020 Mathematics Subject Classification: 60K25, 68M20.

Keywords: Restricted admissibility, random server breakdown, retrial queue, supplementary variable technique.

Received May 17, 2021; Accepted June 7, 2021

research due to their various application in production systems, computer networks etc. These queuing models arise in stochastic modelling of many real-life situations. Queuing system with server breakdown is very common in communication systems and manufacturing systems.

Altman et al. [1] considered the state dependent $M/G/1$ type queuing analysis for congestion control in data networks. Madan and Abu-Dayyeh [3] and Madan and Choudhury [4] have investigated classical queuing system with restricted admissibility of arriving batches and Bernoulli server vacation. J. Ebnesar Anna Bagyam and K. Udayachandrika [2] investigated Batch arrival $M/G/1$ retrial queuing system with state dependent admission and Bernoulli vacation. P. Rajadurai and V. M. Chandrasekar [6] studied $M/G/1$ feedback retrial queue with subject to server breakdown and repair under multiple vacation policy. Shan Gao and Deran Zhang [7] analyzed $M/G/1$ retrial queue due to server vacation. Madhu Jain and Sandeep Kumar [5] analyzed $M^x/G/1$ retrial queue with Bernoulli vacation feedback customer reneging and unreliable server.

This paper concerns with $M/G/1$ retrial queuing system with state dependent admission and random server breakdowns.

## 2. Model Description

- New customers arrive from outside according to Poisson process with rate $\lambda$.

- The customers from the orbit try to request his service later and inter retrial times have an arbitrary distribution $A(x)$ with corresponding Laplace Stieljes transform $\widetilde{A}(\theta)$.

- There is a single server who provides service to all arriving customers. The service time follows a general distribution $S(x)$ with corresponding Laplace Stieljes transform $\widetilde{S}(\theta)$ and $n^{\text{th}}$ factorial moment is $s_n$.

- The arriving units are allowed to join the system with state dependent admission control policy. Let $\alpha_1$, $\alpha_2$, $\alpha_3$, be the assigned probabilities for an arriving unit to join the system during the period of idle, busy, or on breakdown respectively.

• It is also assumed that when the server is busy it fails at an exponential rate $\beta$. When the server fails it repaired immediately and the customer first being served before the server breakdown wait for the server until repair completion in order to accomplish its service. The repair time is a random variable with probability distribution function $R(x)$, Laplace Stieljes transform $\widetilde{R}(\theta)$ and the nth factorial moment is $r_n$.

• Various stochastic processes involved in the system are independent of each other.

## 3. System Analysis

In this section, the steady-state difference-differential equations for the retrial system by treating the elapsed retrial time and the elapsed repair time as supplementary variables are developed. After that, the probability generating functions for server state and number of customers in the system/orbit are derived.

In the steady-state, it is assumed that $A(0) = 0$, $A(\infty) = 1$, $S(0) = 0$, $S(\infty) = 1$, and are continuous at $x = 0$, so that $\alpha(x)dx = (dA(x))/(1 - A(x))$, $\mu(x) = (dS(x))/(1 - S(x))$, $\gamma(x) = (dR(x))/(1 - R(x))\alpha(x)dx$ can be interpreted as the conditional probability density of completion of the retrial time, given that the elapsed time is $x$. Both $\mu(x)\,dx$ and $\gamma(x)\,dx$ can refer to the corresponding service and repair respectively. The state of the system at time $t$ is given by

$N(t) =$ number of customers in the orbit

$A(t) =$ the elapsed retrial time

$S(t) =$ the elapsed service time

$R(t) =$ the elapsed repair time.

Let us define the random variable $C(t)$ as follows:

$C(t) = \{0,$ if the server is free at time $t$

1, if the server is busy at time $t$

2, if the server is on repair at time $t$.

Thus the supplementary variables $A-(t)$, $S-(t)$, $V-(t)$ and $R-(t)$ are introduced to obtain a bivariate Markov process $\{C(t), \delta(t)\}$, where $\delta(t) = A-(t)$ if $C(t) = 0$ and $N(t) > 0$, $\delta(t) = S-(t)$ if $C(t) = 1$ and $N(t) \geq 0$, and $\delta(t) = R-(t)$ if $C(t) = 2$ and $N(t) \geq 0$.

Let $\{t_n : n = 1, 2, \ldots\}$ be the sequence of epochs at which either a service period completion occurs or vacation time ends or repair time ends. The sequence of random vectors $Z_n = \{N(t_{n^+}), \delta(t_{n^+})\}$ form a Markov chain which is embedded in the retrial queuing system.

**Theorem 1.** *The embedded Markov chain* $\{z_n;\ n \in N\}$ *is ergodic if and only if* $s_1(\lambda\alpha_2 + \beta b_1\lambda\alpha_3\} < \widetilde{A}(\lambda\alpha_1)$.

Let us define the following probabilities:

$P_0(t) = \Pr\{N(t) = 0,\ C(t) = 0\}$

$P_n(x, t)\,dx = \Pr\{N(t) = n,\ \delta(t) = A(t);\ x < A(t) \leq x + dx\},\ x > 0,\ n \geq 1$

$\Pi_n(x, t)\,dx = \Pr\{N(t) = n,\ \delta(t) = S(t);\ x < S(t) \leq x + dx\},\ x > 0,\ n \geq 1$

$B_n(x, t)\,dx = \Pr\{N(t) = n,\ \delta(t) = R(t);\ x < R(t) \leq x + dx\},\ x > 0,\ n \geq 0.$

In steady-state, it can be set as

$$P_0 = \lim_{t \to \infty} P_0(t)$$

and limiting densities

$$P_n(x) = \lim_{n \to \infty} P_n(x, t), \text{ for } x > 0 \text{ and } n \geq 1$$

$$\tau_n(x) = \lim_{n \to \infty} \pi(x, t), \text{ for } x > 0 \text{ and } n \geq 0$$

$$B_n(x) = \lim_{n \to \infty} B(x, t), \text{ for } x > 0 \text{ and } n \geq 0.$$

The steady state difference-differential equations are

$$\lambda P_0 = \int_0^\infty \pi_0(x)\mu(x)\,dx \tag{1}$$

$$\frac{dp_n(x)}{dx} + (\lambda + \alpha(x))P_n(x) = \lambda(1-\alpha_1)P_n(x) \tag{2}$$

$$\frac{d\pi_0(x)}{dx} + (\lambda + \beta + \mu(x))\pi_0(x) = \lambda(1-\alpha_2)\pi_0(x) \tag{3}$$

$$\frac{d\pi_n(x)}{dx} + (\lambda + \beta + \mu(x))\pi_n(x) = \lambda(1-\alpha_2)\pi_n(x)$$

$$+ \lambda\alpha_2\pi_{n-1}(x) + \int_0^\infty B_n(x, y)\gamma(y)dy \tag{4}$$

$$\frac{dB_0(x, y)}{dx} + (\lambda + \gamma(y))B_0(x, y) = \lambda(1-\alpha_3)B_0(x, y) \tag{5}$$

$$\frac{dB_n(x, y)}{dy} + (\lambda + \gamma(y))B_n(x, y) = \lambda(1-\alpha_3)B_n(x, y) + \lambda\alpha_3 B_n(x, y)$$

$$+\lambda\alpha_3 B_{n-1}(x, y).$$

Boundary conditions at $x = 0$

$$P_n(0) = \int_0^\infty \pi_n(x)\mu(x)dx \tag{7}$$

$$\pi_0(0) = \lambda P_0 + \int_0^\infty P_1(x)\alpha(x)dx \tag{8}$$

$$R_n(x, 0) = \beta\pi_n(x). \tag{9}$$

The normalizing condition is

$$P(1) + \Pi(1) + R(1) = 1. \tag{10}$$

**Theorem 2.** *Under the stability condition,* $s_1(\lambda\alpha_2 + \beta b_1\lambda\alpha_3) < \widetilde{A}(\lambda\alpha_1)$ *the stationary distributions of the number of customers in the system when the server is free, busy, and on breakdown are given by*

$$P(z)=\frac{P_0(z\widetilde{S}(\lambda\alpha_2(1-z)+\beta-\beta\widetilde{R}(\lambda\alpha_3(1-z))(1-z)-z(1-\widetilde{A}(\lambda\alpha_1))}{\alpha_1(z-\widetilde{A}(\lambda\alpha_1))+z(1-\widetilde{A}(\lambda\alpha_1))\widetilde{S}(\lambda\alpha_2(1-z)+\beta-\beta\widetilde{R}(\lambda\alpha_3(1-z)))}$$

$$\pi(z)=\frac{\begin{array}{c} z\lambda P_0-\lambda P_0(\widetilde{A}(\lambda\alpha_1)+z(1-\widetilde{A}(\lambda\alpha_1)) \\ (1-\widetilde{S}(\lambda\alpha_2(1-z)+\beta-\beta\widetilde{R}(\lambda\alpha_3(1-z)))) \end{array}}{\begin{array}{l} z-\widetilde{S}(\lambda\alpha_2(1-z)+\beta-\beta\widetilde{R}(\lambda\alpha_3(1-z)))(\widetilde{A}(\lambda\alpha_1)+z(1-\widetilde{A}(\lambda\alpha_1)) \\ \widetilde{S}(\lambda\alpha_2(1-z)+\beta-\beta\widetilde{R}(\lambda\alpha_3(1-z)))) \end{array}}$$

$$B(z)=\frac{\beta\pi(x,z)(1-\widetilde{R}(\lambda\alpha_3(1-z)))}{\lambda\alpha_3(1-z)}.$$

**Proof.** Generating functions are

$$P(x,z)=\sum_{n=1}^{\infty}P_n(x)z^n \tag{11}$$

$$\pi(x,z)=\sum_{n=1}^{\infty}\pi_n(x)z^n \tag{12}$$

$$B(x,y,z)=\sum_{n=1}^{\infty}B_n(x,y,z)z^n. \tag{13}$$

Multiplying equation (2) by $z, z^2, \ldots, z^n$ and taking summation over $n$

$$\frac{dP(x,z)}{dx}+(\alpha(x)+\lambda\alpha_1)P(x,z)=0. \tag{14}$$

Multiplying equation (4) by $z, z^2, z^3, \ldots, z^n$ and taking summation over $n$

$$\frac{d\pi(x,z)}{dx}+(\beta+\mu(x)+\lambda\alpha_2(1-z))\pi(x,z)=\int_0^{\infty}B(x,y,z)\gamma(y)dy. \tag{15}$$

Multiplying equation (6) by $z, z^2, z^3, \ldots, z^n$ and taking summation over $n$

$$\frac{dB(x,y,z)}{dy}+(\lambda\alpha_3(1-z)+\gamma(y))B(x,y,z)=0. \tag{16}$$

Multiplying equation (7) by $z, z^2, z^3, \ldots, z^n$ and taking summation over $n$

$$P(0,\, z) = -\lambda P_0 + \int_0^\infty \pi(x,\, z)\mu(x)dx. \tag{17}$$

$$\pi(0,\, z) = \int_0^\infty \frac{P(x,\, z)\alpha(x)}{z}\, dx + \lambda\alpha_1 \int_0^\infty P(x,\, z)dx + \lambda P_0. \tag{18}$$

Multiplying equation (9) by $z,\, z^2,\, z^3,\, \ldots,\, z^n$ and taking summation over $n$

$$B(x,\, 0,\, z) = \beta\pi(x,\, z). \tag{19}$$

Solving the partial differential equation (14) to get

$$P(x,\, z) = e^{-\lambda\alpha_1 x}(1 - A(x))P(0,\, z). \tag{20}$$

Solving the partial differential equation (15) to get

$$B(x,\, y,\, z) = \beta\pi(x,\, z)(1 - R(y))e^{-\lambda\alpha_3(1-z)y}. \tag{21}$$

Substituting (19) in (16)

$$\pi(x,\, z) = \pi(0,\, z)(1 - S(x))e^{-(\lambda\alpha_2(1-z)+\beta-\beta\widetilde{R}(\lambda\alpha_3(1-z)))x}. \tag{22}$$

From equations (18) and (21)

$$\pi(0,\, z) = P(0,\, z)\frac{(\widetilde{A}(\lambda\alpha_1) + z(1 - \widetilde{A}(\lambda\alpha_1))}{z} + \lambda P_0. \tag{23}$$

From equations (21) and (22)

$$P(0,\, z) = \pi(0,\, z)\widetilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\widetilde{R}(\lambda\alpha_3(1 - z))) - \lambda P_0. \tag{24}$$

From equation (24) we get

$$P(0,\, z) = \frac{z\lambda P_0\widetilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\widetilde{R}(\lambda\alpha_3(1 - z)))}{z - (\widetilde{A}(\lambda\alpha_1)) + z(1 - \widetilde{A}(\lambda\alpha_1))\widetilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\widetilde{R}(\lambda\alpha_3(1 - z))} \tag{25}$$

$$P(z) = \frac{Nr}{Dr}$$

$$Nr = P_0(z\widetilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\widetilde{R}(\lambda\alpha_3(1 - z))(1 - z) - z(1 - \widetilde{A}(\lambda\alpha_1)))$$

$$Dr = \alpha_1(z - \widetilde{A}(\lambda\alpha_1) + z(1 - \widetilde{A}(\lambda\alpha_1))\widetilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\widetilde{R}(\lambda\alpha_3(1 - z)))$$

$$\pi(z) = \frac{Nr}{Dr}$$

$$Nr = z\lambda P_0(\tilde{A}(\lambda\alpha_1) + z(1 - \tilde{A}(\lambda\alpha_1))(1 - \tilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\tilde{R}(\lambda\alpha_3(1 - z))))$$

$$Dr = z - \tilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\tilde{R}(\lambda\alpha_3(1 - z)))(\tilde{A}(\lambda\alpha_1) + z(1 - \tilde{A}(\lambda\alpha_1))$$

$$\tilde{S}(\lambda\alpha_2(1 - z) + \beta - \beta\tilde{R}(\lambda\alpha_3(1 - z))))$$

$$B(z) = \frac{\beta\pi(x, z)(1 - \tilde{R}(\lambda\alpha_3(1 - z))}{\lambda\alpha_3(1 - z)}.$$

The unknown constant $P_0$ can be determined by using the normalization condition (10)

$$P_0 + P(1) + \pi(1) + B(1) = 1.$$

Thus

$$P_0 = \frac{2\alpha_1\alpha_3(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)(\tilde{A}(\lambda\alpha_1) - \theta\lambda - s_1(\lambda\alpha_2 + \beta r_1\lambda\alpha_3))}{T}. \tag{26}$$

Where

$$T = 3\lambda^3\alpha_1\alpha_3^2\beta\,(\tilde{A}(\lambda\alpha_1) - s_1(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)) + (\lambda\alpha_2 + \beta r_1\lambda\alpha_3)(3\alpha_1\alpha_3)$$

$$(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)(2s_1 + 2s_1\tilde{A}(\lambda\alpha_1) - s_2(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)) - s_1\beta r_2\lambda^2\alpha_3^2.$$

## 4. System Performance Measures

Now, the system performance measures of the $M/G/1$ retrial queue with feedback, modified vacation and random server breakdown is derived. Note that (26) gives the steady state probability that the server is idle but available in the system. From (26) we have, $s_1(\lambda\alpha_2 + \beta b_1\lambda\alpha_3) < \tilde{A}(\lambda\alpha_1)$ which is the stability condition.

The probability generating function of the number of customer in the orbit is

$$\phi_q(z) = P_0 + P(z) + \pi(z) + B(z). \tag{27}$$

The mean number of customers in the orbit $(L_q)$ under steady state condition is obtained by differentiating (27) with respect to $z$ and evaluating at $z = 1$

$$L_q = \lim_{z+1} \frac{d\phi_q(z)}{dz}$$

$$L_q = \phi_q'(1) = P_0\left[\frac{Nr'''(1)Dr''(1) - Dr'''(1)Nr''(1)}{3(Dr''(1))^2}\right]$$

where,

$$Dr'' = 2\alpha_1\alpha_3\lambda(\alpha_2 + \beta b_1\alpha_3)(\widetilde{A}(\lambda\alpha_1) - \lambda s_1(\alpha_2 + \beta b_1\alpha_3))$$

$$Dr''' = 3\lambda^2\alpha_1\alpha_3^2\beta(\widetilde{A}(\lambda\alpha_1) - \lambda s_1(\alpha_2 + \beta b_1\alpha_3))$$

$$+ 3\alpha_1\alpha_3\lambda(\alpha_2 + \beta b_1\alpha_3)((\lambda(\alpha_2 + \beta b_1\alpha_3)(1 - \widetilde{A}(\lambda))$$

$$- \lambda s_2(\alpha_2 + \beta b_1\alpha_3)) - \lambda^2\alpha_3^2 s_1\beta b_2)$$

$$Nr'' = 2\widetilde{A}(\lambda\alpha_1)\lambda\alpha_1\alpha_3(\alpha_2 + \beta r_1) + (\lambda s_1(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)$$

$$(1 - \widetilde{A}(\lambda\alpha_1))(2\alpha_2\alpha_3 + \alpha_3^2\beta r_1 - \alpha_1\alpha_3\beta r_1) + \alpha_1\alpha_3 + \alpha_1\alpha_4)$$

$$Nr''' = s_1(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)(3\lambda\alpha_3(1 - \widetilde{A}(\lambda\alpha_1))(2\alpha_1\beta r_1 - \lambda\beta\alpha_3^2 r_2 + 2\alpha_1\alpha_2 + 2\alpha_2)$$

$$+ (1 - \alpha_3)3\lambda\alpha_1\alpha_3(\lambda\alpha_3\beta r_2 + 2\beta r_1) + 6\lambda\alpha_1\alpha_3(1 + \beta r_1\alpha_3))s_2(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)^2$$

$$(3\lambda(1 - \widetilde{A}(\lambda\alpha_1))(\alpha_1\alpha_3\beta r_1 - \alpha_2\alpha_3 - \lambda^2\alpha_3^4 s_1\beta^2 r_1 r_2(1 - \alpha_3)$$

$$+ \lambda\alpha_1\alpha_3^2\beta r_2 + \lambda^2\alpha_1\alpha_2\alpha_3^2\beta r_1)).$$

The probability generating function of the number of customer in the system is

$$\phi_s(z) = P_0 + P(z) + z(\pi(z) + B(z)).$$

The mean number of customers in the orbit under steady state condition is obtained by differentiating $\phi_s(z)$ with respect to $z$ and evaluating at $z = 1$.

The mean number of customers in the system is given by

$$L_s = \phi_s'(1) = P_0\left[\frac{Nr'''(1)Dr''(1) - Dr'''(1)Nr''(1)}{3(Dr''(1))^2}\right].$$

Where

$$Dr'' = 2\alpha_1\alpha_3\lambda(\alpha_2 + \beta b_1\alpha_3)(\widetilde{A}(\lambda\alpha_1) - \lambda s_1(\alpha_2 + \beta b_1\alpha_3))$$

$$Dr''' = 3\lambda^2\alpha_1\alpha_3^2\beta(\widetilde{A}(\lambda\alpha_1) - \lambda s_1(\alpha_2 + \beta b_1\alpha_3))$$

$$+ 3\alpha_1\alpha_3\lambda(\alpha_2 + \beta b_1\alpha_3)((\lambda(\alpha_2 + \beta b_1\alpha_3)(1 - \widetilde{A}(\lambda))$$

$$- \lambda s_2(\alpha_2 + \beta b_1\alpha_3)) - \lambda^2\alpha_3^2 s_1\beta b_2)$$

$$Nr'' = 2\widetilde{A}(\lambda\alpha_1)\lambda\alpha_1\alpha_3(\alpha_2 + \beta r_1) + (\lambda s_1(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)$$

$$(1 - \widetilde{A}(\lambda\alpha_1))(2\alpha_2\alpha_3 + \alpha_3^2\beta r_1 - \alpha_1\alpha_3\beta r_1) + \alpha_1\alpha_3 + \alpha_1\alpha_4)$$

$$Nr''' = s_1(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)(1 - \widetilde{A}(\lambda\alpha_1))3\lambda\alpha_3(\alpha_3\beta r_2\lambda + 2\alpha_3\beta r_1 + 2\alpha_2 - 3\alpha_1\alpha_3$$

$$\beta r_1 - 2\alpha_1\alpha_2 + \lambda\alpha_1\alpha_3\beta r_2)(s_2(\lambda\alpha_2 + \beta r_1\lambda\alpha_3)^2(1 - \widetilde{A}(\lambda\alpha_1))$$

$$(3\lambda\alpha_3(\lambda^2 s_1\beta^2 b_1 b_2 - \alpha_2) + (1 - \alpha_3)(\alpha_3\beta r_1 + \lambda^2\alpha_3^2\beta^2 s_1 r_1 r_2))$$

$$+ 3\lambda\alpha_1\alpha_3^2\widetilde{A}(\lambda\alpha_1)(\alpha_3 + \lambda^2\alpha_2\beta r_1) - 3\lambda\alpha_1\alpha_2\alpha_3.$$

## 5. Numerical Example

In this section a numerical example is presented in order to illustrate the effect of various parameters in the system performance measures of our system, where all retrial times and service time follows Erlang 2 stage distribution vacation time follows Bernoulli distribution and repair time are exponentially distributed. Arbitrary values are assumed to the parameters such as steady state condition is satisfied.

The following table presented here gives the values of $P_0$, $L_s$, $L_q$ for the various retrial rates breakdown rates and arrival rates.

**Table 1.** The effect of $P_0$, $L_s$, $L_q$ on λ and β.

| Λ | Β | $P_0$ | $L_q$ | $L_s$ |
|---|---|---|---|---|
| 0.2 | 0.3 | 0.587 | 0.1198 | 0.3089 |
| 0.3 | 0.3 | 0.4415 | 0.2921 | 0.5276 |
| 0.4 | 0.3 | 0.3213 | 0.5934 | 0.8396 |
| 0.5 | 0.3 | 0.2208 | 1.1367 | 1.3395 |
| 0.6 | 0.3 | 0.1343 | 2.3031 | 2.3379 |
| 0.2 | 0.4 | 0.5821 | 0.1224 | 0.3179 |
| 0.3 | 0.4 | 0.4359 | 0.2987 | 0.5423 |
| 0.4 | 0.4 | 0.3157 | 0.6082 | 0.8634 |
| 0.5 | 0.4 | 0.2154 | 1.1702 | 1.3818 |
| 0.6 | 0.4 | 0.1294 | 2.3977 | 2.4362 |
| 0.2 | 0.5 | 0.5772 | 0.1250 | 0.3269 |
| 0.3 | 0.5 | 0.4305 | 0.3055 | 0.5571 |
| 0.4 | 0.5 | 0.3102 | 0.6233 | 0.8874 |
| 0.5 | 0.5 | 0.2102 | 1.2052 | 1.4253 |
| 0.6 | 0.5 | 0.1246 | 2.4971 | 2.5386 |
| 0.4 | 0.2 | 0.3329 | 0.5419 | 0.7833 |
| 0.4 | 0.3 | 0.3272 | 0.5559 | 0.8065 |
| 0.4 | 0.4 | 0.3217 | 0.5704 | 0.8300 |
| 0.4 | 0.5 | 0.3163 | 0.5851 | 0.8535 |
| 0.4 | 0.6 | 0.311 | 0.6001 | 0.8772 |
| 0.4 | 0.2 | 0.3309 | 0.5540 | 0.7940 |
| 0.4 | 0.3 | 0.3253 | 0.5684 | 0.8176 |

| | | | | |
|---|---|---|---|---|
| 0.4 | 0.4 | 0.3197 | 0.5828 | 0.8410 |
| 0.4 | 0.5 | 0.3143 | 0.5977 | 0.8648 |
| 0.4 | 0.6 | 0.3089 | 0.6127 | 0.8884 |
| 0.4 | 0.2 | 0.3289 | 0.5662 | 0.8047 |
| 0.4 | 0.3 | 0.3233 | 0.5808 | 0.8285 |
| 0.4 | 0.4 | 0.3177 | 0.5954 | 0.8522 |
| 0.4 | 0.5 | 0.3122 | 0.6104 | 0.8759 |
| 0.4 | 0.6 | 0.3069 | 0.6257 | 0.9000 |
| 0.4 | 0.3 | 0.4471 | 0.2691 | 0.5055 |
| 0.4 | 0.3 | 0.4457 | 0.2748 | 0.5110 |
| 0.4 | 0.3 | 0.4443 | 0.2805 | 0.5165 |
| 0.4 | 0.3 | 0.4429 | 0.2863 | 0.5220 |
| 0.4 | 0.3 | 0.4415 | 0.2921 | 0.5276 |
| 0.3 | 0.4 | 0.3292 | 0.5438 | 0.7957 |
| 0.3 | 0.4 | 0.3272 | 0.5559 | 0.8065 |
| 0.3 | 0.4 | 0.3253 | 0.5684 | 0.8176 |
| 0.3 | 0.4 | 0.3233 | 0.5808 | 0.8285 |
| 0.3 | 0.4 | 0.3213 | 0.5934 | 0.8396 |
| 0.3 | 0.5 | 0.2309 | 1.0278 | 1.2501 |
| 0.3 | 0.5 | 0.2284 | 1.0542 | 1.2720 |
| 0.3 | 0.5 | 0.2259 | 1.0812 | 1.2943 |
| 0.3 | 0.5 | 0.2233 | 1.1084 | 1.3164 |
| 0.3 | 0.5 | 0.2208 | 1.1367 | 1.3395 |

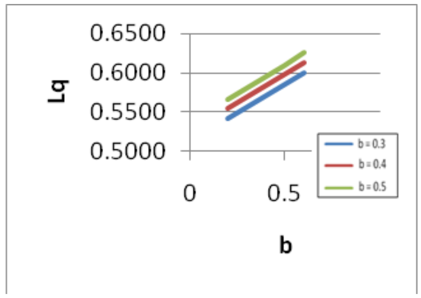


**Figure 1.** The effect of $\beta$ on $L_q$.

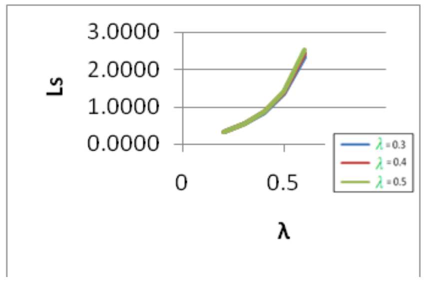


**Figure 2.** The effect of $\lambda$ on $L_s$.

For the effect of parameters $\beta$, and $\lambda$ on the system performance, two dimensional graphs are drawn in Figures 1, 2 such that the stability condition is satisfied. Figure 1 shows that $L_q$ increases for increasing value of $\beta$. Figure 2 shows that $L_s$ increases for increasing value of $\lambda$.

## 6. Conclusion

Retrial queue with single arrival, restricted admissibility and random server breakdown have been investigated in this paper. The necessary and sufficient condition for the system to be stable is obtained. The inputs of the parameters on the performance measures are illustrated. The results obtained in this paper are more realistic and useful for system engineers.

## References

[1] E. Altman, K. Avrachenkov, C. Barakat and R. Nunez-Queija, State dependent $M/G/1$ queuing analysis for congestion in control in data networks, Computer Networks 39(6) (2002), 789-808.

[2] J. Ebnesar Annabagyam and K. Udayachandrika, Batch arrival queuing system with state dependent admission and Bernoulli vacation, International Journal of Research in Engineering and Technology (2013), 374-379.

[3] K. C. Madan and W. Abu-Dayyeh, Restricted admissibility of batches into an $M[x]/G/1$ type bulk queue with modified Bernoulli schedule server vacations, ESSAIM: Probability and Statistics 6 (2002), 113-125.

[4] K. C. Madan and G. Choudhury, Steady state analysis of $M/(G1, G2)/1$ queue with restricted admissibility of arriving batches and modified Bernoulli server vacations under a single vacation policy, Journal of Probability and Statistical Sciences 2(2) (2004), 167-185.

[5] Madhu Jain, Sandeep Kumar, Bernoulli vacation model for $M^x/G/1$ unreliable server retrial queue with Bernoulli feedback, balking and optional reservice, RAIRO Operations Research 55 (2021), S2027-S2053 , Available online at https://doi.org/101051/ro/2020074

[6] P. Rajadurai and V. M. Chandrasekar, A study on $M/G/1$ feedback retrial queue with subject to server breakdown and repair under multiple vacation policy, Alexandria Engineering Journal 57(2) (2018), 947-962.

[7] Shan Gao and Deran Zhang, Performance and sensitivity analysis of $M/G/1$ queue with retrial customers due to server vacations, Ain Shams Engineering Journal 11(3) (2020), 795-803.