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GENERALIZED FUZZY *e* SEPARATION AND REGULARITY AXIOMS IN FUZZY TOPOLOGICAL SPACES

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Abstract

In this paper is to discuss generalized fuzzy e separation and regularity axioms in the sense of Šostak's. Also the relationships between these axioms are discussed and investigate some properties of them.

1.Introduction and Preliminaries

Kubiak [14] and Šostak [17] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology [3], in the sense that not only the objects are fuzzified, but also the axiomatics. In [18, 19] Šostak gave some rules and showed how such an extension can be realized. Chatopadhyay et al., [4] have redefined the same concept under the name gradation of openness. A general approach to the study of topological type structures on fuzzy power sets was developed in [8]-[10], [14]-[15]. The notion of fuzzy topology on fuzzy sets was introduced by Chakraborty and Ahsanullah [2] as one of treatments of the problem which may be called the subspace problem in fuzzy topological spaces. One of the

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advantages of defining topology on a fuzzy set lies in the fact that subspace topologies can now be developed on fuzzy subsets of a fuzzy set. Later Chaudhury and Das [5] studied several fundamental properties of such fuzzy topologies. The concept of separation axioms is one of the most important concepts in topology. In fuzzy setting, it had been studied by many authors such as [6, 7, 11, 12, 16]. Later, Kim [13] introduces the separation axioms in Šostak's fuzzy topoogical spaces and investigate some properties of them. The aim of this paper is to introduce the concepts of generalized fuzzy separation and regularity axioms in fuzzy topological spaces.

Throughout this paper, nonempty set will be denoted by X, Y etc., $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I$, $\underline{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t for $t \in I_0$ is an element of I^X such that $x_t(y) = (ty = xoify\ x$. The set of all fuzzy points X of is denoted by $P_t(X)$. The family of all fuzzy subsets of a fuzzy set μ will denoted by A_μ i.e. $A_\mu = (U \in I^X : U \leq \mu\}$. The set $S(\mu) = \{x \in X : \mu(x) > 0\}$ is said to be the support of μ. If $U \in A_\mu$, then the complement of U referred to μ, denoted by $\mu - U$. Let $U, ZV \in A_\mu$ are said to be quasi-coincident referred to μ, denoted by $U_q V[\mu]$, iff there exists $x \in S(\mu)$ such that $U(x) + V(x) > \mu(x)$. If is not quasi-coincident with V referred to μ, we denoted for this by $U_{\bar{q}} V[\mu]$.

Lemma 1.1 [9]. *Let X be a nonempty set and* $\lambda, \mu \in I^X$. *Then*

1. $\lambda q \mu$ *iff there exists* $x_t \in \lambda$ *such that* $x_t q \mu$.

2. $\lambda q \mu$, *then* $\lambda \wedge \mu \neq \overline{0}$.

3. $\lambda \bar{q} \mu$ *iff* $\lambda \leq \overline{1} - \mu$.

4. $\lambda \leq \mu$ *iff* $x_t \in \lambda$ *implies* $x_t \in \mu$ *iff* $x_t q \lambda$ *implies* $x_t q \mu$ *implies* $x_t \bar{q} \lambda$.

5. $x_t \bar{q} \bigvee_{i \in \wedge} \mu_i$ *iff there exists* $i_0 \in \Lambda$ *such that* $x_t \bar{q} \mu_{i_0}$.

Definition 1.1 [1]. Let (X, τ) be a *fts* and $\mu \in I^X$. The mapping $\tau_\mu : A_\mu \to I$ defined by $\tau_\mu(U) = \vee\{\tau(V) : V \in I^X, V \wedge \mu = U\}$ is a fuzzy μ-topology induced over μ by τ. For any $U \in A_\mu$, the number $\tau_\mu(U)$ is called the μ-openness degree of *U*.

Theorem 1.1 [1]. τ_μ *verifies the following properties*:

1. $\tau_\mu(\overline{0}) = \tau_\mu(\overline{1}) = 1$.

2. $\tau_\mu(U \wedge V) \geq \tau_\mu(U) \wedge \tau_\mu$, *for any* $U, V \in A_\mu$.

3. $\tau_\mu(\vee_{i \in \Gamma} U_i) \geq \wedge_{i \in \Gamma} \tau_\mu(U_i)$, *for any* $\{U_i\}_{i \in \Gamma} \subset A_\mu$.

Theorem 1.2 [1]. *Let* (X, τ) *be a fts and* $\mu \in I^X$. *Then for each* $U \in A_\mu$, $r \in I_0$ *we define an operator* $Cl_\mu : A_\mu \times I_0 \to A_\mu$ *as follows*: $Cl_\mu(U, r) = \wedge \{V \in A_\mu : U \leq V, \tau_\mu(\mu - V) \geq r$ (*Equivalently*, *V is* r_μ*- closed*)}. *For* $U, V \in A_\mu$ *and* $r, s \in I_0$ *the operator* Cl_μ *satisfies the following conditions*: [(i)]

1. $Cl_\mu(\overline{0}, r) = \overline{0}$.

2. $U \leq Cl_\mu(U, r)$.

3. $Cl_\mu(U, r) \vee Cl_\mu(V, r) - Cl_\mu(U \vee V, r)$.

4. $Cl_\mu(U, r) \leq Cl_\mu(V, s)$ *if* $r \leq s$.

5. $Cl_\mu(Cl_\mu(U, r), r) = Cl_\mu(U, r)$.

Theorem 1.3 [1]. *Let* (X, τ) *be a fts and* $\mu \in I^X$. *Then for each* $U \in A_\mu$, $r \in I_0$ *we define an operator* $Int_\mu : A_\mu \times I_0 \to A_\mu$ *as follows*: $Int_\mu(U, r) = \vee \{V \in A_\mu : V \leq U, \tau_\mu(V) \geq r$ (*Equivalently*, *V is* r_μ*- open*)}. *For* $U, V \in A_\mu$ *and* $r, s \in I_0$ *the operator* Cl_μ *satisfies the following conditions*: [(i)]

1. $Int_\mu(\mu, r) = \mu$.

2. $Int_\mu(U, r) \leq U$.

3. $Int_{\mu}(U, r) \wedge Int_{\mu}(V, r) = Int_{\mu}(U \wedge V, r)$.

4. $Int_{\mu}(U, r) \leq Int_{\mu}(V, s)$ *if* $s \leq r$.

5. $Int_{\mu}(Int_{\mu}(U, r), r) = Int_{\mu}(U, r)$.

6. $Int_{\mu}(\mu - U, r) = \mu - Cl_{\mu}(U, r)$ *and* $Cl_{\mu}(\mu - U, r) = \mu - Int_{\mu}(U, r)$.

2. Generalized Fuzzy *e*-Separation and Regularity Axioms

Definition 2.1. Let (X, τ) be a *fts*. For $U, V \in I^X$ and $r \in I_0, U$ is called r_{μ}- fuzzy regular open (for short, r_{μ}- fro) (resp. r_{μ}- fuzzy regular closed (for short, r_{μ}- frc)) if $U = Int_{\mu}(Cl_{\mu}(U, r), r)$ (resp. $U = Cl_{\mu}(Int_{\mu}(U, r), r)$; $\delta Cl_{\mu}(U, r) = \wedge\{V \in I^X : V \geq U, V$ is ar μ-frc set} and $\delta Int_{\mu}(U, r) = \vee\{V \in I^X : V \leq U, V$ is a r_{μ}- fro set}; r_{μ}- fuzzy *e*-open (resp. r_{μ}- fuzzy *e*-closed) (briefly, r_{μ}-*feo* (resp. r_{μ}-*fec*)) set if

$$U \leq Cl_{\mu}(\delta Int_{\mu}(U, r), r) \vee Int_{\mu}(\delta CI_{\mu}(U, r), r)$$

(resp. $Cl_{\mu}(\delta Int_{\mu}(U, r), r) \wedge Int_{\mu}(\delta CI_{\mu}(U, r), r) \leq U$); r_{μ}- generalized fuzzy (resp. *e*) closed (briefly, r_{μ}-*gfc* (resp. r_{μ}-*gfec*)) set if $Cl_{\mu}(U, r) \leq V$ (resp. $eCl_{\mu}(U, r) \leq V$) whenever $U \leq V$ and V is r_{μ} open, $r \in I_0$. The complement of a r_{μ}-*gfc* (resp. r_{μ}-*gfec*) set is called r_{μ}- generalized fuzzy (resp. *e*) open (briefly, r_{μ}-*gfo* (resp. r_{μ}-*gfeo*)) set; r_{μ}- generalized fuzzy interior (resp. r_{μ}- generalized fuzzy closure, r_{μ}- generalized fuzzy *e*-interior and r_{μ}- generalized fuzzy *e*-closure) (briefly, $gfInt_{\mu}$ (resp. $gfCl_{\mu}$, $gfeInt_{\mu}$ and $gfeCl_{\mu}$)) of U is $gI\ \mathrm{int}_{\mu}(U, r) = \vee\{V \in I^X : U \geq V, V$ is r_{μ}-*gfo*}

(resp. is $gCl_{\mu}(U, r) = \wedge\{V \in I^X : U \leq V, V$ is r_{μ}-*gfc*},

$$geInt_{\mu}(U, r) = \vee\{V \in I^X : U \leq V, V \text{ is } r_{\mu}\text{-}gfeo\}$$

$$geCl_{\mu}(U, r) = \wedge\{V \in I^X : U \leq V, V \text{ is } r_{\mu}\text{-}gfec\}).$$

Definition 2.2. Let (X, τ) be a fts, $\mu \in I^X$, $x_t \in P_t(\mu)$ and $r \in I_0$. Then any fuzzy set $0_{x_t} \in A_\mu$ with $O_{x_t} \in gfeo_\mu$ contains x_t is called a generalized fuzzy *e* neighbourhood (briefly, *gefnbd*) of x_t. The set of all generalized fuzzy *e* neighbourhoods of x_t will be denoted by $gfeN_\mu(x_t)$. In general for any $O_v \in A_\mu$ with $O_U \in gfeo_\mu$ contains is called *gfenbd* of *U*.

Proposition 2.1. *Let* (X, τ) *be a* $ft\mu$, $\mu \in I^X$, $U \in A_\mu$, $x_t \in P_t(\mu)$ *and* $r \in I_0$. *Then* $x_t qgfeCI_\mu(U, r)[\mu]$ *iff* $O_{x_t} qU[\mu]$, $\forall O_{x_t} \in gfeN_\mu(x_t)$.

Proof. It is easily proved.

Definition 2.3. Let (X, τ) be a fts, $\mu \in I^X$ and $r \in I_0$. μ is said to be:

1. r-$gfe\,\mu R_0$ iff $x_t \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$ implies $gfeCl_\mu(x_t, r)\bar{q}\, y_s[\mu]$ for any distinct fuzzy points $x_t, y_s \in P_t(\mu)$.

2. r-$gfe\,\mu R_1$ iff $x_t \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$ implies that there exist $O_{x_t} \in gfeN_\mu(x_t)$ and $O_{y_t} \in gfeN_\mu(y_s)$ such that $O_{x_t} \bar{q}\, O_{y_x}[\mu]$ for any distinct fuzzy points x_t, y_s.

3. r-$gfe\,\mu R_2$ iff $x_t \bar{q}\, U[\mu]$ with $\tau_\mu(\mu - U) \geq r$ implies that there exist $O_{x_t} \in gfeN_\mu(x_t)$ and $O_U \in gfeN_\mu(U)$ such that $O_{x_t} \bar{q}\, O_U[\mu]$, $U \in A_\mu$ and $x_t \in P_t(\mu)$.

4. r-$gfe\,\mu R_3$ iff $U \bar{q}\, V[\mu]$ with $\tau_\mu(\mu - U) \geq r$ and $\tau_\mu(\mu - V) \geq r$ implies that there exist $O_U \in gfeN_\mu(U)$ and $O_V \in gfeN_\mu(V)$ such that $O_U \bar{q}\, O_v[\mu]$ and $U, V \in A_\mu$.

Definition 2.4. Let (X, τ) be a fts, $\mu \in I^X$ and $r \in I_0$. For any distinct fuzzy points $x_t, y_s \in P_t(\mu)$, μ is said to be:

1. r-$gfe\,\mu T_0$ iff $x_t \bar{q}\, y_s[\mu]$ implies that there exist $O_{x_t} \in gfeN_\mu(x_t)$ such that $O_{x_t} \bar{q}\, y_s[\mu]$ or there exist $O_{y_s} \in gfeN_\mu(y_s)$ such that $x_t \bar{q}\, O_{y_s}[\mu]$.

2. $r\text{-}gfe\,\mu T_1$ iff $x_t \bar{q}\, y_s[\mu]$ implies that there exist $O_{x_t} \in gfeN_\mu(x_t)$ such that $O_{x_t} \bar{q}\, y_s[\mu]$ and there exist $O_{y_s} \in gfeN_\mu(y_s)$ such that $x_t \bar{q}\, O_{y_s}[\mu]$.

3. $r\text{-}gfe\,\mu T_2$ iff $x_t \bar{q}\, y_s[\mu]$ implies that there exist $O_{x_t} \in gfeN_\mu(x_t)$ and $O_{y_s} \in gfeN_\mu(y_s)$ such that $Ox_t \bar{q}\, O_{y_s}[\mu]$.

4. $r\text{-}gfe\,\mu T_3$ iff it is $r\text{-}gfe\,\mu R_2$ and $r\text{-}gfe\,\mu T_1$

5. $r\text{-}gfe\,\mu T_4$ iff it is $r\text{-}gfe\,\mu R_3$ and $r\text{-}gfe\,\mu T_1$.

Theorem 2.1. *Let* (X, τ) *be a* fts, $\mu \in I^X$, $U \in A_\mu$ *and* $x_t, y_s \in P_t(\mu)$. *Then the following statements are equivalent.*

1. μ *is a* $r\text{-}gfe\,\mu R_0$ *space.*

2. $gfeCl\,\mu(x_t, r) \leq O_{x_t}, \forall O_{x_t} \in gfeN_\mu(x_t)$.

3. $gfeCl_\mu(x_t, r) \leq \wedge\{O_{x_t} : O_{x_t} \in gfeN_\mu(x_t)\}$.

4. $x_t \bar{q}\, U[\mu]$ *with* $\tau_\mu(\mu - U) \geq r$ *implies there exists* $O_U \in gfeN_\mu(U)$, $x_t \bar{q}\, O_U[\mu]$.

5. $x_t \bar{q}\, U[\mu]$ *with* $\tau_\mu(\mu - U) \geq r$ *implies* $gfeCl_\mu(x_t, r) \bar{q}\, U[\mu]$.

6. $x_t \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$ *implies* $gfeCl_\mu(x_t, r) \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$.

Proof. (1) $\Rightarrow$ (2) Let $y_s \bar{q}\, gfeCl\,\mu(x_t, r)[\mu]$, from (1) we have $x_t \bar{q}\, gfeCl\,\mu(y_s, r)[\mu]$. From Proposition 2.1, we have $y_s \bar{q}\, O_{x_t}(x_t, r)[\mu] \forall O_{x_t} \in gfeN_\mu(x_t)$, then $gfeCl_\mu(x_t, r) \leq O_{x_t}, \forall O_{x_t} \in gfeN_\mu(x_t)$ (by (5) of Lemma 1.1).

(2) $\Rightarrow$ (3) is obvious.

(3) $\Rightarrow$ (4) Let $x_t \bar{q}\, U[\mu]$ with $\tau_\mu(\mu - U) \geq r$, then $x_t \in (\mu - U)$. By (2), $gfeCl_\mu(x_t, r) \leq (\mu - U)$ implies $U \leq (\mu - gfeCl_\mu(x_\tau, r)) = O_U$, so $x_t \bar{q}(\mu - gfeCl_\mu(x_t, r))[\mu] = O_U$.

(4) ⇒ (5) Let $x_t \bar{q} U[\mu]$ with $\tau_\mu(\mu - U) \geq r$, by (4) there exist O_U such that $x_t \bar{q} O_U[\mu]$ implies $x_t \in (\mu - O_U)$ implies $gfeCl_\mu(x_t, r) \leq (\mu - O_U)$ implies $gfeCl_\mu(x_t, r) \bar{q} O_U[\mu]$, so $gfeCl_\mu(x_t, r) \bar{q} U[\mu]$.

(5) ⇒ (6) and (6) (1) are obvious.

Theorem 2.2. *The following implications hold*: $r - gfe\,\mu R_2 \wedge e - gfe\,\mu R_0 \Rightarrow r - gfe\,\mu R_2 \Rightarrow r - gfe\,\mu R_1 \Rightarrow r - gfe\,\mu R_0$.

Proof. (1) Let μ be a $r - gfe\,\mu R_2 \wedge r - gfe\,\mu R_0$ and let $x_t \bar{q} U[\mu]$ with $\tau_\mu(\mu - U) \geq r$. Then from (5) of the Theorem 2.1, we have $gfeCl_\mu(x_t, r) \bar{q} U[\mu]$. Sinceμ is $r - gfec\,\mu R_2$, then there exists $O_{gfeCl_\mu(x_t, r)}, O_U$ such that $O_{gfeCl_\mu(x_t, r)}, \bar{q} O_U[\mu]$. Now take $O_{x_t} = O_{gfeCl_\mu(x_t, r)}$, then $O_{x_t} \bar{q} O_U[\mu]$, and hence μ is $r - gfe\,\mu R_2$ space.

(2) Let μ be $r - gfe\,\mu R_2$ and $x_t \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$. Then there exist $O_{x_t}, O_{gfeCl_\mu(x_t, r)}$ such that $O_{x_t} \bar{q} O_{gfeCl_\mu(y_s, r)}[\mu]$. Take $O_{y_s} = O_{gfeCl_\mu(y_s, r)}$ then $O_{x_t} \bar{q} O_{y_s, r)}[\mu]$. Hence μ is $r - gfe\,\mu R_1$ space.

(3) Let μ be a $r - gfe\,\mu R_1$ and $y_s \bar{q} O_{x_t}[\mu]$ implies $y_s \leq (\mu - O_{x_t})$ implies $gfeCl_\mu(y_s, r) \leq (\mu - O_{x_t})$ implies $O_{x_t} \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$, so $x_t \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$, there exist $O^*_{x_t}, O_{y_s}$ such that $O^*_{x_t} \bar{q} O_{y_s}[\mu]$ implies $x_t \bar{q} O_{y_s}[\mu]$ implies $y_s \bar{q}\, gfeCl_\mu(x_t, r)[\mu]$, so by (5) of Lemma 1.1, we get $gfeCl_\mu(x_t, r) \leq O_{x_t}, \forall O_{x_t} \in gfeN_\mu(x_t)$. Hence μ is $r - gfe\,\mu R_0$.

Theorem 2.3. μ *is a* $r - gfe\,\mu R_1$ *iff* $\forall x_t, y_s \in P_t(\mu)$ *with* $x_t \bar{q}\, gfeCl_\mu(y_s, r)[\mu]$ *implies that there exists* $U, V \in A_\mu$ *with* $U, V \in gfeo_\mu$, $gfeCl_\mu(x_t, r) \leq U$ *and* $gfeCl_\mu(y_s, r) \leq V$ *such that* $U \bar{q} V[\mu]$.

Proof. Follows from Theorem 2.1(2).

Theorem 2.4. μ *is a* $r - gfe\,\mu R_2$ *iff* $\forall x_t \in P_t(\mu)$ *and* $\forall O_{x_t} \in gfeN_\mu(x_t)$ *there exists* $O^*_{x_t} \in gfeN_\mu(x_t)$ *such that* $gfeCl_\mu(O^*_{x_t}, r) \leq O_{x_t}$.

Proof. Let μ be a $r-gfe\,\mu R_2$, $x_t P_t(\mu)$ and $O_{x_t} \in gfeN_\mu(x_t)$. Then $x_t \overline{q}(\mu - O_{x_t})\mu$ implies there exists $O^*_{x_t} \in gfeN_\mu(x_t)$, $O^{**}_{\mu - O_{x_t}} \in gfeN_\mu(\mu - O_{x_t})$ such that $O^*_{x_t}\overline{q}O^{**}_{(\mu - o_{x_t})}[\mu]$, (since μ is $r-gfe\,\mu R_2$) implies $O^*_{x_t} \leq (\mu - O^{**}_{(x_t - o_{x_t})}$ then $gfeCl_\mu(O^*_{x_t}, r) \leq (\mu - O^{**}_{(x_t - o_{x_t})}) \leq O_{x_t}$.

Conversely, let $x_t \in P_t(\mu)$ and $U \in A_\mu$ with $\tau_\mu(\mu - U) \geq r$ be such that $x_t\overline{q}U[\mu]$. Then $x_t \leq (\mu - U)$ i.e. $(\mu - U) \in gfeN_\mu(x_t)$, so there exists $O^*_{x_t}$ such that $gfeCl_\mu(O^*_{x_t}, r) \leq O_{x_t} = (\mu - U)$ implies $U \leq (\mu - gfeCl_\mu(O^*_{x_t}, r)) = O_U$ and $O_U\overline{q}O^*_{x_t}[\mu]$. Hence μ is $r-gfe\,\mu R_2$.

Theorem 2.5. *μ is a $r-gfe\,\mu R_2$ iff $\forall U \in A_\mu$ with $\tau_\mu(\mu - U) \geq r$ and $\forall O_U \in gfeN_\mu(U)$ there exist $O^*_U \in gfeN_\mu(U)$ such that $gfeCl_\mu(O^*_U, r) \leq O_U$.*

Proof. It is similarly proved as in the above Theorem 2.4.

Theorem 2.6. *μ is a $r-gfe\,\mu T_0$ iff $x_t\overline{q}y_s[\mu]$ implies $x_t\overline{q}\,gfeCl_\mu(y_s, r)[\mu]$ or $gfeCl_\mu(x_t, r)\overline{q}y_s[\mu]$.*

Proof. It is easily proved.

Theorem 2.7. *Let (X, τ) be a fts and $\mu \in I^X$. Then the following statements are equivalent.*

1. *μ is a $r-gfe\,\mu T_1$ space.*

2. *$\forall x_t, y_s \in P_t(\mu)$ with $x_t\overline{q}y_s[\mu] \Rightarrow x_t\overline{q}\,gfeCl_\mu(y_s, r)[\mu]$ and $y_s\overline{q}\,gfeCl_\mu(x_t, r)[\mu]$.*

3. $gfeCl_\mu(x_t, r) = x_t, \forall x_t \in P_t(\mu)$.

Proof. (1) $\Rightarrow$ (2) is clearly from Proposition 2.1.

(1) $\Rightarrow$ (3) Let $x_t\overline{q}y_s[\mu]$ implies there exists O_{y_s}, such that $x_t\overline{q}Oy_s[\mu]$ this implies $O_{y_s} \leq (\mu - x_t)$, thus $\tau_\mu(\mu - x_t) \geq r$ for every $x_t \in P_t(\mu)$ i.e. $gfeCl_\mu(x_t, r) = x_t, \forall x_t \in P_t(\mu)$.

(3) $\Rightarrow$ (1) Let $gfeCl_{\mu}(x_t, r) = x_t, \forall x_t \in P_t(\mu)$ and $x_t \overline{q} y_s[\mu]$. Then $\tau_{\mu}(\mu - x_t) \geq r$ and $\tau_{\mu}(\mu - y_s) \geq r$. Since $y_s \overline{q}(\mu - y_s)[\mu] = O_{\mu_t}$ and $x_t \overline{q}(\mu - x_t)[\mu] = O_{y_s}$, and hence μ is $r - gfe\, \mu T_1$ space.

Theorem 2.8. *The following implications hold*: $r - gfe\, \mu T_4 \Rightarrow r - gfe\, \mu T_2 \Rightarrow r - gfe\, \mu T_2 \Rightarrow r - gfe\, \mu T_1 \Rightarrow r - gfe\, \mu T_0$.

Proof. It is similarly proved as in Theorem 2.2.

From the Theorem 2.8 and Theorem 2.2 we obtain the following result.

Corollary 2.1. *The following implications hold*:

$$\begin{array}{ccccccccc} & & r - gfe\mu T_4 & \Rightarrow & r - gfe\mu T_3 & \Rightarrow & r - gfe\mu T_2 & \Rightarrow & r - gfe\mu T_1 \Rightarrow r - gfe\mu T_0 \\ & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ r - gfe\mu R_3 \wedge r - gfe\mu R_0 & \Rightarrow & r - gfe\mu R_2 & \Rightarrow & r - gfe\mu R_1 & \Rightarrow & r - gfe\mu R_0 & & \end{array}$$

Conclusion

The purpose of this paper is to introduce and investigate generalized fuzzy separation and regularity axioms in Šostak's fuzzy topological spaces. Also, some of their fundamental properties are studied and relation between these generalized separation axioms are discussed.

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