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"FUZZY ADDITIVE AND FUZZY MULTIPLICATIVE GROUP"

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Abstract

Since inception, for fuzzy group only one algebraic operation on the crisp set $\breve{G}$ is used which gives identical values of membership function. For better results the identical values of membership functions are inefficient. Due to this problem, in this paper two algebraic operations, addition and multiplication on the crisp set $\breve{G}$ has were introduced. For which membership functions are defined differently so that $\mu(x+y) \leq \max\{\mu(x), \mu(y)\}$ and $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ are satisfied. The main purpose of this research is to introduce a fuzzy field structure on the membership values which provide future resources for new researchers in fuzzy mathematics.

1. Introduction

In the 1960's Zadeh [7] proposed a new theory known as Fuzzy Sets, a generalization of classical set theory. This fuzzy set theory deals with a vagueness which the classical set theory was unable to do so. In 1986, Rosenfield, A., [12] has proposed Intuitionistic Fuzzy groups. In continuation, a lot of extensions covering various applications are being developed and some of them are ortho-pair, neutrosophic, etc. The literature review [1-6, 8-11] done by us shows that different operations were defined on fuzzy sets for the fuzzy group and their applications are shown. The use of fuzzy subgroup is used in solving fuzzy equations [13-15] and many other areas of pure and

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applied sciences. The study of the modification of fuzzy group may use to understand the difference between oriented fuzzy Subgroup and Fuzzy Field [16-18].

In this paper, we define the algebraic operations on the membership values from a different angle. Here we define traditionally. However, we define them with a purpose in mind that, at the later stage we can have a field structure on the membership values, so that one can solve the fuzzy algebraic equations in a meaningful way. We think of two operations addition and multiplication on the crisp group $\widetilde{G}$ and then think of membership functions defined in particular ways so that $\mu(x+y) \leq \max\{\mu(x), \mu(y)\}$, and $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ are satisfied. We call them additive and multiplicative fuzzy subgroups. We study them in some detail.

2. Preliminaries

A. Fuzzy subgroup. Let G be a group. Let fuzzy set A of the G is said to be fuzzy subgroup if it is satisfying the following axioms:

(i) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$ for all $x, y \in G$

(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$ for all $x, y \in G$.

B. Level subgroup. Let A be a fuzzy subgroup of a group G. The subgroup A_t of G such that $t \leq \mu_A(e)$ is called a level subgroup of G. were for $t \in [0, 1]$.

C. Fuzzy normal. Let A fuzzy subgroup of a group G. Then A is called fuzzy normal if $\mu_A(xy) = \mu_A(yx)$.

D. Theorem. *Let A be a fuzzy normal subgroup of group G. Then for any* $g \in G$, *we have* $\mu_A(gxg^{-1}) = \mu_A(g^{-1}xg)\ \forall\, x \in G$.

E. Theorem. *A fuzzy subgroup A of a group G is normalized iff* $\mu_A(e) = 1$, *where 'e' is an identity element of group G.*

F. Conjugate fuzzy subgroups. Let A and B be two fuzzy subgroups of a group G. Then A and B are said to be conjugate fuzzy subgroups of G if for some $g \in G$, $\mu_A(x) = \mu_B(g^{-1}xg)$ for every $x \in G$.

G. Fuzzy subring. A fuzzy subset A of a ring R is called a fuzzy subring of R if for all $x, y \in R$ the following requirements are

(i) $\mu_A(x - y) \geq \min(\mu_A(x), \mu_A(y))$ and

(ii) $\mu_A(xy) \geq \min(\mu_A(x), \mu_A(y))$

Now if condition (ii) is replaced by $\mu_A(xy) \geq \min(\mu_A(x), \mu_B(y))$ then A will be called a fuzzy ideal of R.

H. Theorem. *Let A be any fuzzy subring/fuzzy ideal of a ring R. If, for some $x, y \in R$, $\mu_A(x) < \mu_B(y)$, then* $\mu_A(x - y) = \mu_A(x) = \mu_A(y - x)$.

I. Theorem. *The intersection of any family of fuzzy subrings (fuzzy ideals) of a ring R is again a fuzzy subring of R.*

J. Fuzzy prime. A non-constant fuzzy ideal A of a ring R is called fuzzy prime if for any fuzzy ideals A_1 and A_2 of R the condition $A_1 A_2 \in A$ implies that either $A_1 \in A$ or $A_2 \in A$.

K. Fuzzy field. Let X be a field and be a fuzzy set in X with membership function μ_F then F is a fuzzy field in X if and only if the following conditions are satisfied

(i) $\mu_F(x + y) \geq \min(\mu_F(x), \mu_F(y)\}, \forall x, y \in X$

(ii) $\mu_F(x^{-1}) \geq \mu_F(x), \forall x \in X$

(iii) $\mu_F(xy) \geq \min(\mu_F(x), \mu_F(y)\}, \forall x, y \in X$

(iv) $\mu_F(x^{-1}) \geq \mu_F(x), \forall x \in X$ and $x \neq 0$

(v) $\mu_F(0) = 1, \mu_F(1) = 1$.

L. Theorem. *F is a fuzzy field in X if and only if*

(i) $\mu_F(x + y) \geq \min(\mu_F(x), \mu_F(y)\}, \forall x, y \in X$

(ii) $\mu_F(xy^{-1}) \geq \min(\mu_F(x), \mu_F(y)\}, \forall x, y \in X$

M. Theorem. (i) *Let A be a fuzzy subset of F and* $s, t \in \mathrm{Im}(A)$ (*the image of A*), *then* $s \leq t$ *if and only if* $A_s \supseteq A_t$ *and* $s = t$ *if and only if* $A_s = A_t$.

(ii) *Let A be a fuzzy subfield of F then for all* $x \in F \neq 0$,

$$\mu_A(0) \geq \mu_A(1) \geq \mu_A(A) = \mu_A(-x) = \mu_A(x^{-1})$$

N. Theorem. *Let A be a fuzzy subset of F. If At is a subfield of F for all* $t \in \mathrm{Im}(A)$ *then A is a fuzzy subfield of F. Conversely if A is a fuzzy subfield of F then for all t such that* $0 \leq t \leq A(1)$ *and* A_t *is a subfield of F.*

3. Fuzzy Additive Group

A. Fuzzy additive group. Let $(G, +)$ be an additive group and $\mu_1 : G \to [0, 1]$ be a member function then the group G with the operation + and the membership function μ_1 is said to be a fuzzy additive group if

(i) $\mu_1(x + y) \leq \max\{\mu_1(x), \mu_1(y)\}$ for all $x, y \in G$

(ii) $\mu_1(-x) = \mu_1(x)$ for all $x \in G$

It is denoted by $(G, +, \mu_1)$.

Note: Let the identity element in the group $(G, +)$ be denoted by 0.

B. Theorem 1. *For any fuzzy additive group* $(G, +, \mu_1)$, $\mu_1(0) \leq \mu_1(x)$ *for all* $x \in G$.

Proof. $\mu_1(0) = \mu_1(x - x) \leq \max\{\mu_1(x), \mu_1(-x)\} = \mu_1(x)$

Since $\mu_1(-x) = \mu_1(x)$ for all $x \in G$.

C. Theorem 2. *Let* $(G, +)$ *be an abelian additive group and* $(G, +, \mu_1)$ *be a fuzzy additive group*.

Then $\mu_1(x + y) = \max\{\mu_1(x), \mu_1(y)\}$ *if* $\mu_1(x) \neq \mu_1(y)$, *for all* $x, y \in G$.

Proof. Let $\mu_1(x) \neq \mu_1(y)$, for all $x, y \in G$.

Without loss of generality assume that $\mu_1(x) > \mu_1(y)$ (1)

First we try to prove that $\mu_1(y) < \mu_1(x+y)$

Suppose $\mu_1(y) \geq \mu_1(x+y)$

$\mu_1(x) = \mu_1(x+y-y)$

$\leq \max\{\mu_1(x+y), \mu_1(-y)\}$

$= \max\{\mu_1(x+y), \mu_1(y)\}$

$= \mu_1(y)$ (since $\mu_1(y) \geq \mu_1(x+y)$)

Thus $\mu_1(x) \leq \mu_1(y)$

Which is contradict to our assumption $\mu_1(y) > \mu_1(y)$

Our supposition is false

Thus we must have $\mu_1(y) < \mu_1(x+y)$ (2)

Now $\mu_1(y) = \mu_1(x+y-y)$

$\leq \max\{\mu_1(x+y), \mu_1(-y)\}$

$= \max\{\mu_1(x+y), \mu_1(y)\}$

$= \mu_1(x+y)$

$\leq \max\{\mu_1(x), \mu_1(y)\}$

$= \mu_1(x)$

Thus, $\mu_1(y) = \mu_1(x+y) = \max\{\mu_1(x), \mu_1(y)\}$.

D. Theorem 3. *Let* $(G, +, \mu_1)$ *be a fuzzy additive group and* $x \in G$ *then* $\mu_1(x+y) = \mu_1(y)$, *for all* $y \in G$ *if and only if* $\mu_1(x) = \mu_1(0)$.

Proof. First assume that $\mu_1(x+y) = \mu_1(y)$, for all $y \in G$.

Put $y = 0 \in G$

$\mu_1(x+0) = \mu_1(0)$

Therefore $\mu_1(x) = \mu_1(0)$

Thus then $\mu_1(x+y)=\mu_1(y)$, for all $y \in G \Rightarrow \mu_1(x)=\mu_1(0)$

Conversely assume that $\mu_1(x)=\mu_1(0)$

$\mu_1(y)=\mu_1(y+x-x)$

$\leq \max\{\mu_1(x+y), \mu_1(-x)\}$

$=\max\{\mu_1(x+y), \mu_1(x)\}$

$=\max\{\mu_1(x+y), \mu_1(0)\}$

$=\mu_1(x+y)$ (Since $\mu_1(x) \geq \mu_1(0), \forall x \in G$)

Therefore $\mu_1(y) \leq \mu_1(x+y)$ (3)

Also $\mu_1(x+y)=\mu_1(x+y+0)$

$\leq \max\{\mu_1(x+0), \mu_1(y)\}$

$=\max\{\mu_{1\mu_1}(x), \mu_1(y)\}$

$=\max\{\mu_1(0), \mu_1(y)\}$

(since $\mu_1(x)=\mu_1(0)$)

$=\mu_1(y)$ (since $\mu_1(x) \geq \mu_1(0)$)

Therefore $\mu_1(y) \geq \mu_1(x+y)$ (4)

From (3) and (4) $\mu_1(y)=\mu_1(x+y)$.

Example 1. Let $(Z_{12}, +_{12})$ be a group then

$H_1=\{[0]\}$,

$H_2=\{[0],[6]\}$,

$H_3=\{[0],[4],[8]\}$,

$H_4=\{[0],[3],[6],[9]\}$,

$H_5=\{[0],[2],[4],[6],[8],[10]\}$,

$H_6 = \{[0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]\}$

Are subgroups,

Also here $H_1 \subset H_3 \subset H_5 \subset H_6$ and $H_1 \subset H_2 \subset H_4 \subset H_6$

Define $\mu : Z_{12} \to [0, 1]$

$$\mu(x) = \begin{cases} 0 & \text{if } x = [0] \\ 0.2 & \text{if } x \in \{[4], [8]\} \\ 2.4 & \text{if } x \in \{[2], [6], [10]\} \\ 0.5 & \text{if } x \in \{[1], [3], [5], [7], [9], [11]\} \end{cases}$$

Then (i) $\mu(x + y) \leq \max\{\mu(x), \mu(y)\}$ for all $x, y \in Z_{12}$.

(ii) $\mu(-x) = \mu(x)$ for all $x \in Z_{12}$

Therefore $(Z_{12}, +_{12}, \mu)$ is a fuzzy additive group.

4. Fuzzy Multiplicative Group

A. Fuzzy multiplicative group. Let $(G, \cdot)$ be a multiplicative group. Let $\mu_2 : G \to [0, 1]$ be a membership function. Then the group G with the operation and the membership function μ_2 is said to be a fuzzy multiplicative group if

(i) $\mu_2(x \cdot y) \geq \min\{\mu_2(x), \mu_2(y)\}$ for all $x, y \in G$

(ii) $\mu_2(x^{-1}) = \mu_2(x)$ for all $x \in G$

It is denoted by $(G, \cdot, \mu_2)$.

Note: Let the identity element in the group $(G, \cdot)$ be denoted by 1.

B. Theorem 4. *Let* $(G, \cdot, \mu_2)$ *be a fuzzy multiplicative group then* $\mu_2(1) \geq \mu_2(x)$ *for all* $x \in G$.

Proof. $\mu_2(1) = \mu_1(x \cdot x^{-1})$

$\geq \min\{\mu_1(x), \mu_1(x^{-1})\}$

$= \min\{\mu_1(x), \mu_1(x)\}$

$= \mu_1(x)$

Thus $\mu_2(1) \geq \mu_2(x)$.

C. Theorem 5. *Let* $(G, \cdot, \mu_2)$ *be a fuzzy additive group and an abelian then* $\mu_2(x \cdot y) = \min\{\mu_2(x), \mu_2(y)\}$ *with* $\mu_2(x) \neq \mu_2(y)$, *for all* $x, y \in G$.

Proof. Let $\mu_2(x) \neq \mu_2(y)$, for all $x, y \in G$

Without loss of generality assume that $\mu_2(x) < \mu_2(y)$ (5)

First, we try to prove that $\mu_2(x \cdot y) < \mu_2(y)$

Suppose $\mu_2(x \cdot y) \geq \mu_2(y)$

$\mu_2(x) = \mu_2(x \cdot y \cdot y^{-1})$

$\geq \min\{\mu_2(x \cdot y), \mu_2(y^{-1})\}$

$= \min\{\mu_2(x \cdot y), \mu_2(y)\}$

$= \mu_2(y)$ (since $\mu_2(x \cdot y) \geq \mu_2(y)$)

Thus $\mu_2(x) \geq \mu_2(y)$

Which is contradicts our assumption $\mu_2(x) < \mu_2(y)$,

Therefore our supposition is false.

Thus we must have $\mu_2(x \cdot y) < \mu_2(y)$ (6)

Now $\mu_2(x) = \mu_2(x \cdot y \cdot y^{-1})$

$\geq \min\{\mu_2(x \cdot y), \mu_2(y^{-1})\}$

$= \min\{\mu_2(x \cdot y), \mu_2(y)\}$

$= \mu_2(x)$

$= \min\{\mu_2(x), \mu_2(y)\}$

$= \mu_2(x)$

Thus $\mu_2(x) = \mu_2(x \cdot y) = \min\{\mu_2(x), \mu_2(y)\}$.

D. Theorem 6. *Let* $(G, \cdot, \mu_2)$ *be a fuzzy multiplicative group and* $x \in G$ *then* $\mu_2(x \cdot y) = \mu_2(y)$, *for all* $y \in G$ *if and only if* $\mu_2(x) = \mu_2(1)$.

Proof. First assume that $\mu_2(x \cdot y) = \mu_2(y)$, for all $y \in G$.

Put $y = 1 \in G$

$\mu_1(x \cdot 1) = \mu_2(1)$

$\mu_2(x) = \mu_2(1)$

Thus then $\mu_2(x \cdot y) = \mu_2(y)$, for all $y \in G \Rightarrow \mu_2(x) = \mu_2(1)$

Conversely assume that $\mu_2(x) = \mu_2(1)$

$\mu_2(y) = \mu_2(y \cdot x \cdot x^{-1})$

$\geq \min\{\mu_2(x \cdot y), \mu_2(x^{-1})\}$

$= \min\{\mu_2(x \cdot y), \mu_2(x)\}$

$= \min\{\mu_2(x \cdot y), \mu_2(1)\}$

$= \mu_2(x \cdot y)$

Therefore $\mu_1(y) \geq \mu_2(x \cdot y)$ (7)

Also $\mu_2(x \cdot y) \geq \min\{\mu_2(x), \mu_2(y)\}$

$= \min\{\mu_2(1), \mu_2(y)\}$ (since $\mu_2(x) = \mu_2(1)$)

$= \mu_2(y)$ (since $\mu_2(y) \leq \mu_2(1), \forall\, y \in G$)

Therefore $\mu_2(y) \leq \mu_2(x \cdot y)$ (8)

From (7) and (8), we have $\mu_2(y) = \mu_2(x \cdot y)$.

E. Example 2. Let $(Z_{11}^{*}, \times_{11})$ be a group then

$H_1 = \{[1]\}$

$H_2 = \{[1], [10]\}$

$H_3 = \{[1], [3], [4], [5], [9]\}$

$H_4 = \{[1], [2], [3], [4], [5], [6], [7], [8], [9], [10]\}$

are subgroups. Also here

$H_1 \subset H_2 \subset H_4$ and $H_1 \subset H_3 \subset H_4$

And let $\mu_2 : Z_{11}^* \to [0, 1]$

$$\mu_2(x) = \begin{cases} 1 & \text{if } x = [1] \\ 0.8 & \text{if } x \in \{[10]\} \\ 0.6 & \text{if } x \in \{[2], [3], [4], [5], [6], [7], [8], [9]\} \end{cases}$$

Then (i) $\mu_2(x \cdot y) \geq \min\{\mu_2(x), \mu_2(y)\}$ for all $x, y \in Z_{11}^*$

(ii) $\mu_2(x^{-1}) = \mu_2(x)$ for all $x \in Z_{11}^*$

Thus (Z_{11}^*, μ) is Multiplicative fuzzy group.

F. Example 3. Let $U(n)$ be the set of all positive integers less than n and relatively prime to n then $(U(n), \times_n)$ is a group, $n > 1$.

Let k be a divisor of n and $U_k(n) = \{x \in U(n) \setminus x \equiv 1 \bmod k\}$ then $U_k(n)$ is a subgroup of $U(n)$.

Let $U(20) = \{1, 3, 7, 9, 11, 13, 17, 19\}$

Then $U_n(20) = U(20)$

$U_4(20) = \{1, 9, 13, 17\}$

$U_5(20) = \{1, 11\}$

$U_{10}(20) = \{1, 11\}$ are subgroups of $U(20)$

Also here

$U_5(20) \subset U_4(20) \subset U(20)$ and $U_{10}(20) \subset U_4(20) \subset U(20)$

And let $\mu_2 : U(20) \to [0, 1]$

$$\mu_2(x) = \begin{cases} 1 & \text{if } x \in U_5(20) \\ 0.8 & \text{if } x \in U_4(20) - U_5(20) \\ 0.6 & \text{if } x \in U(20) - U_4(20) \end{cases}$$

Then (i) $\mu_2(x \cdot y) \geq \min\{\mu_2(x), \mu_2(y)\}$ for all $x, y \in U(20)$

(ii) $\mu_2(x^{-1}) = \mu_2(x)$ for each $x \in U(20)$

Thus $(U(20), \mu)$ is the Multiplicative fuzzy group.

5. Fuzzy field

A. Fuzzy field. Let F be a field, $(F, +, \mu_1)$ be a fuzzy additive group, and $(F - \{0\}, \cdot, \mu_2)$ be a fuzzy multiplicative group. Then a mapping $\mu : F \to [0, 1]$ is said to be a fuzzy field if

(i) $\mu(x + y) \leq \max\{\mu_1(x), \mu_1(y)\}$

(ii) $\mu(-x) \leq \mu_1(x)$

(iii) $\mu(x \cdot y) \geq \min\{\mu_2(x), \mu_2(y)\}$

(iv) $\mu_2(x^{-1}) \geq \mu_2(x)$.

B. Example 4. Let $(Z_{11}, +_{11}, \mu_1)$ be a fuzzy additive group and $(Z_{11}^*, \cdot_{11}, \mu_2)$ be a fuzzy multiplicative group where $\mu_1 : Z_{11} \to [0, 1]$ be a member function as following

$$\mu_1(x) = \begin{cases} 0 & \text{if } x \in \{[0]\} \\ 0.5 & \text{if } x \in \{[1], [2], [3], [4], [5], [6], [7], [8], [9], [10]\} \end{cases}$$

And $\mu_2 : Z_{11} - \{[0]\} \to [0, 1]$

That is $\mu_2 : Z_{11}^* \to [0, 1]$, define as

$$\mu_2(x) = \begin{cases} 1 & \text{if } x \in \{[1]\} \\ 0.8 & \text{if } x \in \{[10]\} \\ 0.5 & \text{if } x \in \{[2], [3], [4], [5], [6], [7], [8], [9], [10]\} \end{cases}$$

Define: $\mu : Z_{11} \to [0, 1]$ as following

$$\mu(x) = \begin{cases} 0 & \text{if } x \in \{[0]\} \\ 0.5 & \text{if } x \in \{[1], [2], [3], [4], [5], [6], [7], [8], [9], [10]\} \end{cases}$$

Then (i) $\mu(x + y) \leq \max\{\mu_1(x), \mu_1(y)\}$ for all $x, y \in Z_{11}$

(ii) $\mu(-x) \leq \mu_1(x)$ for all $x \in Z_{11}$

(iii) $\mu(x \cdot y) \geq \min\{\mu_2(x), \mu_2(y)\}$ for all $x, y \in Z_{11}^*$

(iv) $\mu(x^{-1}) \geq \mu_2(x)$ for all $x \in Z_{11}^*$

Thus μ fuzzy field.

6. Conclusion

In this paper, we define and study the algebraic operations on the membership values from a different angle. Here we define traditionally. However, we define them with a purpose in mind that we have a field structure on the membership values, so that one can solve the fuzzy algebraic equations. Some result related to maximal distinct membership values of both fuzzy additive and fuzzy multiplicative group. Which provide future resources for new researchers in fuzzy Mathematics.

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