Advances and Applications in Mathematical Sciences
Volume 22, Issue 6, April 2023, Pages 1353-1360
ISSN 0974-6803

NARAYANA PRIME CORDIAL GRAPH LABELING

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Abstract

The labeling of graph is an assignment between the numbers and vertices/edges. In this paper, the results on the labeling of graph are studied using Narayana numbers and prime numbers for graph namely; Wheel, Star, Complete-bipartite graph, Bistar graph and join of graphs.

1. Introduction and Preliminaries

Graph labeling have enormous applications within mathematics, computer science and communication networks refer [5, 10]. The readers are well known about the following notions.

Binary vertex labeling (BVL)**.** Let $G(V, E)$ be finite and undirected, then g from $V(G)$ to $\{0, 1\}$ is called BVL of G. Let $g(v)$ is the labeling of

2020 Mathematics Subject Classification: 05C78.
Keywords: graph labeling, Narayana numbers, prime numbers.
Received: July 7, 2022; Accepted December 22, 2022

vertex v. For an edge $e = uv$, $g^* : E(G)$ to $\{0, 1\}$ is given by

$g^*(e) = |g(u) - g(v)|$, is called the induced edge labeling.

(1) The notions of $G, v_g(0)$ and $v_g(1)$; vertices number with labels 0 and 1.

(2) The notions of G, $e_{g^*}(0)$ and $e_{g^*}(1)$; edges number with labels 0 and 1.

Cordial labeling (CL) [3]**.** A BVL of G is CL, provided $|v_g(0) - v_g(1)| \le 1$ and $|e_{g^*}(0) - e_{g^*}(1)| \le 1$. Every cordial graph admits CL.

Prime Cordial labeling (PCL) [1, 2, 11, 12, 13]**.** A PCL of G is an onto mapping $e : V(G)$ to $\{1, 2, 3, \ldots, V(G)\}$, here $V(G)$ is vertex set and $g^* : E(G) \to \{0, 1\}$ is defined by, $g^*(uv) = 1$; if $\gcd(g(u); g(v)) = 1$, otherwise 0, also $|e_{g^*}(0) - e_{g^*}(1)| \le 1$. Every prime cordial graph admits PCL.

Narayana number (NN) [4]**.** For all set of non-negative integer N_0, the numbers, $a \in N_0$. The NN can be expressed as, $N(a, k) = \frac{1}{a} a_{c_k} a_{c_{k+1}}$ where $0 \le k < a$. Relevant work on labeling found in ([5]-[10], [14]).

2. Main Results

The Narayana-prime-cordial-labeling (NPCL) for graphs namely; Wheel, Star, Complete bipartite graph, Bistar graph, Comb Graph are discussed using the following definition.

Definition. For all set of non negative integer, the numbers, $a \in N_0$. The NN can be expressed as,

$$N(a, k) = \frac{1}{a} a_{c_k} a_{c_{k+1}} \text{ where } 0 \le k < a.$$

Theorem 2. 1. *A Wheel graph W_s is a Narayana prime cordial graph.*

Proof. Let W_s be the wheel with s vertices $V = \{v_{\{j\}} : 1 \le j \le s\}$ and the set of edges $E = \{v_j\ v_{j+1} : 1 \le j \le n-1\} \cup \{v_s\ v_1\} \cup \{v_j\ v_s : 1 \le j \le s-1\}$.

Describe one to one function $g : V \to N_0$ in such a way that,

Type (a) when $s \equiv 0 \bmod (2)$

$g(v_j) = 2^{j+1} - 1; 1 \leq j \leq s-1;$ and $g(v_s) = 2^{s+1}.$

In this labeling, $e_{g^*}(0) = s-1$ and $e_{g^*}(1) = s-1,$ which satisfies the constraint $| e_{g^*}(0) - e_{g^*}(1) | \leq 1.$

Type (b) when $s \equiv 1 \bmod (2)$

$g(v_j) = 2^{j+1} - 1; 1 \leq j \leq s-1;$ and $g(v_s) = 2^{s+1}.$

In this labeling, $e_{g^*}(0) = s-1$ and $e_{g^*}(1) = s-1,$ which satisfies the constraint $| e_{g^*}(0) - e_{g^*}(1) | \leq 1.$

Hence, both types of W_s admits a NPCL. Therefore a wheel graph W_s is a NPC graph.

Theorem 2.2. *A Star graph* St_s *is a Narayana prime cordial graph.*

Proof. Consider the star graph St_s of s vertices of vertex set $V = \{v_0\} \cup \{V_j / 1 \leq 1 \leq j \leq s\}.$ Let v_0 be the central vertex with $g(v_0) = 1.$

Describe one to one function $g : V \to N_0$ in such a way that,

Type (a) when $s \equiv 1 (\bmod 2)$

$g(v_0) = 1$

$g(v_j) = 2^{j+1} - 1; 1 \leq j \leq s;$ and $j \equiv 1 (\bmod 2)$

$g(v_j) = 2^{j+1}; 1 \leq j \leq s;$ and $j \equiv 1 (\bmod 2).$

In this labeling, $e_{g^*}(0) = \frac{s-1}{2}$ and $e_{g^*}(1) = \frac{s-1}{2},$ which satisfies the constraint $| e_{g^*}(0) - e_{g^*}(1) | \leq 1.$

Type (b) when $s \equiv 0 (\bmod 2)$

$g(v_0) = 1$

$g(v_j) = 2^{j+1} - 1;\ 1 \leq j \leq s;$ and $j \equiv 1(\mathrm{mod}\ 2)$

$g(v_j) = 2^{j+1};\ 1 \leq j \leq s;$ and $j \equiv 0(\mathrm{mod}\ 2)$

In this labeling, $e_{g^*}(0) = \frac{s}{2}$ and $e_{g^*}(1) = \frac{s}{2} - 1$, which satisfies the constraint $| e_{g^*}(0) - e_{g^*}(1) | \leq 1$.

Hence, both types of Star St_s admit a NPCL. Therefore a Star graph St_s is a NPC graph.

Theorem 2.3. *A complete bipartite graph $K_{m,k}$ admits Narayana prime cordial labeling.*

Proof. Let $K_{m,k}$ be the complete bipartite graph with $m + k$ vertices. Vertex set of graph can be written as; $V = X \cup Y$ where $X = \{v_1, v_2, \ldots, v_m\}$ and $Y = \{v_{m+1}, v_{m+2}, \ldots, v_{m+k}\}$.

Describe one to one function $g : V \to N_0$ in such a way that,

Type (a) when m is even and k is even,

$g(v_j) = 2^{j+1} - 1;\ 0 \leq j \leq m + k;\ j \equiv 0(\mathrm{mod}\ 2)$

$g(v_j) = 2^{j+1}; 1 \leq j \leq m + k;\ j \equiv 0(\mathrm{mod}\ 2)$.

In this labeling, $e_{g^*}(0) = \frac{mk}{2}$ and $e_{g^*}(1) = \frac{mk}{2}$, then $| e_{g^*}(0) - e_{g^*}(1) | = \left| \frac{mk}{2} - \frac{mk}{2} \right| = 0 \leq 1$.

Type (b) when m is odd and k is odd.

Consider any one of the vertices of Y as a starting vertex v_o with $g(v_0) = 1$,

$g(v_j) = 2^{j+1} - 1;\ 0 \leq j \leq m + k;\ j \equiv 0(\mathrm{mod}\ 2)$

$g(v_j) = 2^{j+1};\ 1 \leq j \leq m + k;\ j \equiv 1(\text{mod } 2).$

In this labeling, $e_{g^*}(0) = \frac{m(k-1)}{2} + \frac{m+1}{2}$ and $e_{g^*}(1) = \frac{m(k-1)}{2}$ $+ \frac{m+1}{2}$.

Then, $|\, e_{g^*}(0) - e_{g^*}(1) \,| \leq 1$.

Type (c) when m is even and k is odd

Consider any one of the vertices of Y as a starting vertex v_o with $g(v_0) = 1$,

$g(v_j) = 2^{j+1} - 1;\ 0 \leq j \leq m + k;\ j \equiv 0(\text{mod } 2)$

$g(v_j) = 2^{j+1};\ 1 \leq j \leq m + k;\ j \equiv 1(\text{mod } 2).$

In this labeling, $e_{g^*}(1) = \frac{m(k-1)}{2} + \frac{m}{2}$ and $e_{g^*}(0) = \frac{m(k-1)}{2} + \frac{m}{2}$. Then $|\, e_{g^*}(0) - e_{g^*}(1) \,| = \left| \frac{m(k-1)}{2} - \frac{m}{2} - \frac{m(k-1)}{2} - \frac{m}{2} \right| = 0 \leq 1.$

Type (d) when m is even and k is odd.

Consider any one of the vertices of Y as a starting vertex v_o with $g(v_o) = 1$,

$g(v_j) = 2^{j+1} - 1;\ 0 \leq j \leq m + k;\ j \equiv 0(\text{mod } 2)$

$g(v_j) = 2^{j+1};\ 1 \leq j \leq m + k;\ j \equiv 1(\text{mod } 2).$

In this labeling, $e_{g^*}(1) = \frac{m(k-1)}{2} + \frac{m}{2}$ and $e_{g^*}(0) = \frac{m(k-1)}{2} + \frac{m}{2}$

Then $|\, e_{g^*}(0) - e_{g^*}(1) \,| = \left| \frac{m(k-1)}{2} + \frac{m}{2} - \frac{m(k-1)}{2} - \frac{m}{2} \right| = 0 \leq 1.$

Hence, the four types of complete bipartite graph $K_{m,k}$ admit a NPCL.

Theorem 2.4. *The bistar graph* $BS_{m,k}$ *admits Narayana prime cordial labeling.*

Proof. Consider a bistar graph $BS_{m,k}$ of $m+k$ vertices. Vertex set of the graph can be written as, $V = X \cup Y$ where $X = \{v_0, v_1, v_2, \ldots, v_{m-1}\}$ and $Y = \{v_m, v_{m+1}, \ldots, v_{m+1-1}\}$. Where v_o and v_m are the centres. Describe one to one function $g : V \to N_0$ in such a way that,

Type (a) when m is even and k is odd

$g(v_j) = 2^{j+1} - 1;\ 0 \le j \le m+k;\ j \equiv 0 (\bmod 2)$ and

$g(v_j) = 2^{j+1};\ 1 \le j \le m+k;\ j \equiv 1 (\bmod 2)$

In this labeling, $e_{g^*}(1) = \frac{m-2}{2} + \frac{k-1}{2}$ and $e_{g^*}(0) = \frac{m-2}{2} + \frac{k-1}{2} + 1$.

Then $|e_{g^*}(0) - e_{g^*}(1)| = \left| \frac{m-2}{2} + \frac{k-1}{2} + 1 - \frac{m-2}{2} - \frac{k-1}{2} \right| = 1$

Hence the constraint $|e_{g^*}(0) - e_{g^*}(1)| \le 1$ is proved.

Type (b) when m is even and k is even.

$g(v_j) = 2^{j+1} - 1;\ 0 \le j \le m+k;\ j \equiv 0 (\bmod 2)$ and

$g(v_j) = 2^{j+1};\ 1 \le j \le m+k;\ j \equiv 1 (\bmod 2)$

In this labeling, $e_{g^*}(1) = \frac{m}{2} + \frac{k}{2}$ and $e_{g^*}(0) = \frac{m-2}{2} + \frac{k-2}{2} + 1$. Then

$|e_{g^*}(0) - e_{g^*}(1)| \le 1 = \left| \frac{m-2}{2} + \frac{k-2}{2} + 1 - \frac{m}{2} + \frac{k}{2} \right| = 1$

Type (c) when m is odd and k is odd.

$g(v_j) = 2^{j+1} - 1;\ 0 \le j \le m+k;\ j \equiv 0 (\bmod 2)$ and

$g(v_j) = 2^{j+1};\ 1 \le j \le m+k;\ j \equiv 1 (\bmod 2)$

In this labeling, $e_{g^*}(1) = \frac{m-1}{2} + \frac{m-1}{2} + 1$ and $e_{g^*}(0) = \frac{m-1}{2} + \frac{k-1}{2}$

Then $|e_{g^*}(0) - e_{g^*}(1)| = \left| \frac{m-1}{2} + \frac{k-1}{2} - \frac{k-1}{2} - \frac{m-1}{2} - 1 \right| = 1$

Type (d) when m is odd and k is even.

$g(v_j) = 2^{j+1} - 1;\ 0 \le j \le m + k;\ j \equiv 0 (\mathrm{mod}\ 2)$ and

$g(v_j) = 2^{j+1};\ 1 \le j \le m + k;\ j \equiv 1 (\mathrm{mod}\ 2)$

In this labeling, $e_{g^*}(1) = \frac{m-1}{2} + \frac{k-2}{2} + 1$ and $e_{g^*}(0) = \frac{k}{2} + \frac{m-1}{2}$. Then $| e_{g^*}(0) - e_{g^*}(1) | = \left| \frac{k}{2} + \frac{m-1}{2} - \frac{m-1}{2} - \frac{k-2}{2} - 1 \right| = 0$

Therefore in all the types, $BS_{m,k}$ admits a NPCL.

Theorem 2.5. *Every graph G with vertices with s admits a Narayana-prime-cordial-labeling.*

Proof. Let the graph G with s vertices, $V = \left\{ v_j : 1 \le j \le \frac{s}{2} \right\} \cup \left\{ u_j : \frac{s}{2} < j \le s \right\}$ and edge set $E = \left\{ v_j v_{j+1} : 1 \le j \le \frac{s}{2} \right\} \cup \left\{ v_j u_j : 1 \le j \le \frac{s}{2} \right\}$.

Describe one to one function $g : V \to N_0$ in such a way that,

$g(v_j) = 2^{j+1} - 1;\ 1 \le j \le s;\ j \equiv 1 (\mathrm{mod}\ 2);$

$g(v_j) = 2^{j+1};\ 1 \le j \le s;\ j \equiv 0 (\mathrm{mod}\ 2);$

In this labeling, $e_{g^*}(0) = \frac{s}{2} - 1$ and $e_{g^*}(1) = \frac{s}{2}$. Then $| e_{g^*}(0) - e_{g^*}(1) | \le 1$.

Therefore, the graph G admits NPCL.

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