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FUZZY STOCHASTIC MARKOV CHAIN TRANSITIONS PROBABILITY FOR CLIMATIC MODEL WITH UNCERTAINTIES

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Abstract

In this paper deals Fuzzy Stochastic Markov chain transition probability represented as triangular fuzzy numbers and Hepta value using uncertainty. The model discussed where the fuzzy values are calculated in Heptagonal fuzzy numbers that was taken as linguistic values. All metrological sub-divisional stations in Tamilnadu and Puducherry annual rainfall from 1901-2020 derived as frequency distribution tabulated form. The states means the class interval and transition probability matrix (t_{pm}) is performed variations in annual rainfall such as climatic factors.

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1. Introduction

In Annual seasonal occasions, Rainfall is most important role for environmental situations. The information from rainfall data got into online India Metrological department (IMD) and made to account of long term process for the period 1901 to 2020. This paper to analyses the annual rainfall of Tamilnadu and Puducherry. For this article fuzzy value assumed in the interval $[0, 1]$. The transition probability matrix can be measured from linguistic value, also classified into class interval with uncertainties.

Tamilnadu lies within 11˚.0' N latitude and 78˚05' E longitude and Puducherry has 10 ˚55' N latitude and 79.52' E longitude approximately. The average rainfall annual 810.7 mm and SD 25.3 mm. The average trend from the period (1900 – 2020) 0.18 mm/year.

1.1. A short description of fuzzy sets:

Some basic definitions about fuzzy set theory are given below.

Definition 1.1.1. Fuzzy matrix (F_m). The $m \times n$ order fuzzy matrix is defined as $F_m(A) = [(a_{ij}, a_{ij\tau})]_{m \times n}$, where $a_{ij}, a_{ij\tau}$ denotes a membership measure for an element a_{ij} in $F_m(A)$, then, we write $F_m(A) = a_{ij\tau}$.

Definition 1.1.2. Fuzzy square matrix (F_m^s). Fuzzy square matrix is defined as a fuzzy matrix with all its elements between $[0, 1]$ and its order *nxn* is a fuzzy matrix belonging to elements in $[0, 1]$.

Definition 1.1.3. Fuzzy square transition probability matrix (Fs_{tp}).

Fuzzy square transition probability matrix '$Fs_{tp}(A)$' is a $n \times n$ fuzzy square transition matrix. The domain of α in which the row n for given membership degree $\alpha \in [0, 1]$ is the set

$$Dom_i(\alpha) = \left(\prod_{i=1}^{n} P_{ij}[\alpha]\right) \cap \Delta_n = \{(p_{i1}, \ldots, p_{in}) \in [P_{ij_\alpha}^{L} P_{ij_\alpha}^{U}] \wedge \sum_j p_{ij} = 1\}.$$

The domain of the whole matrix for a given α is $Dom_i(\alpha) = \prod_{i=1}^{n} D_\iota(\alpha)$ the elements of $D_\iota(\alpha)$ are $n \times n$ matrices and its order n so that the number of rows and columns are equal.

Definition 1.1.4. Membership function of fuzzy Heptagonal number. The fuzzy Heptagonal number by its membership function denoted by $\widetilde{H} = (hp_1, hp_2, hp_3, hp_4, hp_5, hp_6, hp_7; k, \omega)$ is defined as follows:

$$\tau_{\widetilde{H}}(x) = \begin{cases} o & \text{for} \quad x < hp_1 \\ k\left(\dfrac{x - hp_1}{hp_2 - hp_1}\right) & \text{for} \quad hp_1 \le x \le hp_2 \\ k & \text{for} \quad hp_2 \le x \le hp_3 \\ k + (\omega - k)\left(\dfrac{x - hp_3}{hp_4 - hp_3}\right) & \text{for} \quad hp_3 \le x \le hp_4 \\ k + (\omega - k)\left(\dfrac{hp_5 - x}{hp_5 - hp_4}\right) & \text{for} \quad hp_4 \le x \le hp_5 \\ k & \text{for} \quad hp_5 \le x \le hp_6 \\ k\left(\dfrac{x - hp_1}{hp_2 - hp_1}\right) & \text{for} \quad hp_6 \le x \le hp_7 \\ 0 & \text{for} \quad x \ge hp_7 \end{cases}$$

If $\omega = 1$, then the fuzzy Heptagonal number is normal. Also, when $k = 1$, the fuzzy Heptagonal number decreases to fuzzy Triangular number and when $k = 1$ it reduces to fuzzy trapezoidal number.

Definition 1.1.5. Hepta value (H_v). The Hepta value of fuzzy square transition probability matrix is defined by

$$H_v = \left[\frac{1}{2}\sum_{i,j}^{n}(r_i c_i)^{1/7}\right],$$

where r_i denotes the row of the matrix and c_i denotes the column of the matrix.

2. Methodological Analysis

The source of data used in this study were obtained from Table 32. Tamilnadu and Puducherry subdivision, area 130068 sq. km 4.52 per, 15 STN Monthly, Seasonal and Annual rainfall Chennai-Regional Meteorological Department (RMD) from the year 1901 to 2020 (in 10th of mm) 2000-2020 (1871-2014 based on 306 stations and 2015-2020 based on India Metrological Department (IMD) Sub divisional rainfall. The frequencies measured from the respective class intervals. We obtained from the fuzzy value by using fuzzy Heptagonal numbers in which the linguistic values and S. Marimuthu, the linguistic terms are assigned to be corresponding range of class intervals and also taken rainfall states. Specifically, the rainfall states are indicates by S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, S_{16} and S_{17}. The table 1 shows that Linguistic terms, Linguistic value, state space and frequency level from the corresponding class interval along with the fuzzy value.

Table 1. Annual rainfall (1901-2020) of its frequency distribution with fuzzy value.

Linguistic terms	class interval	Linguistic value	state s	frequency f	fuzzy value
Lowest (LT)	439.3-529	(0,0,0,0,0,0,0)	S11	4	0
Very Low (VL)	529-618.5	(0.1,0.15,0.2,0.25, 0.3,0.35,0.45)	S12	10	0.12
Low (L)	618.5-708	(0.2, .25,0.3,0.35, 0.4,0.45,0.5)	S13	22	0.30
Moderate (M)	708-797.5	(0.3, 0.4,0.5,0.6, 0.7,0.8,0.9)	S14	33	0.57
High (H)	797.5-887	(0.4,0.5,0.6,0.7, 0.8,0.9,1)	S15	22	0.76
VeryHigh (VH)	887-976.5	(0.6,0.7,0.8,0.9, 1,1,1)	S16	18	0.91
Highest (H)	976.5-066	(1,1,1,1,1,1,1)	S17	11	1

From the fluctuations of Annual rainfall reflected from Fig.-(1). The trending of time series is prepared and it was frequency distribution convert into rainfall states by prepared on Table 1 for the period (1901-2020). The average rainfall was 824 mm/year and the coefficient of variation was observed nearly 14%.

* ... Dotted line indicates the time in the linear trend from the period 1901-2020.

** The solid blue curve indicates the sub-decadal time scale variation smoothed.

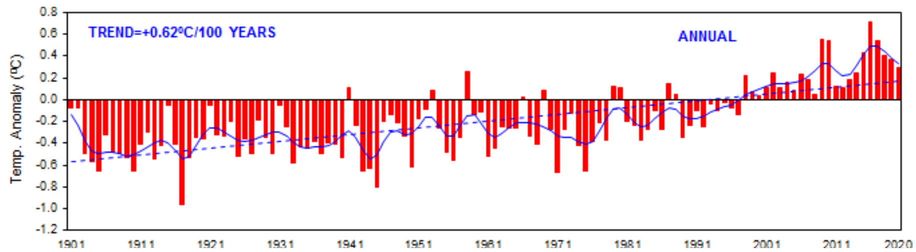

Figure 1. From the period 1901-2020 the Annual mean land surface air temperature anomalies averaged over Tamilnadu and Puducherry.

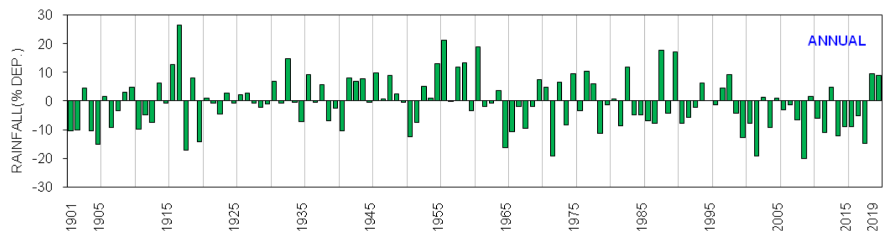

Figure 2. Annual rainfall (1901-2020) percentage departure in time Series of Tamilnadu and Puducherry.

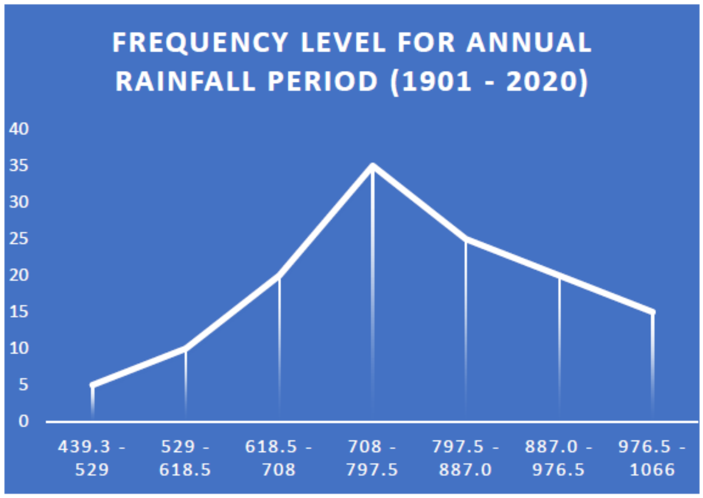

Figure 3. Frequency level of Annual rainfall (1901-2020) percentage of departure in Tamilnadu and Puduherry.

3. Stochastic Markov Model in Time Series

The transition probability of Markov chain $X = \{X_n, n \geq 0\}$ is a sequence or chain with discrete state spaces in which the time from one state i to another such a chain reveals that one-step transition probability. By identifying one variable like fog, frost, wind, cloudiness, precipitation amount and temperature at a time, It is enough to identifying forecast in future one variable like, wind, cloudiness, sunny and temperature at a time, which is represented by a concept called first-order Markov chain. The multi-state stochastic Markov chain for generating daily rainfall depths analyzed here. Stochastic Markov models are not much easy task for exhibiting rainfall such a strong variability in time and space n. The estimation transition probability for the requirement $k(k-1)$ in a 'k state' Markov chain and rest of p_{ij} can be evaluated by using the relation $\sum_{j=1}^{k} p_{ij} = 1.$ The transition probability of stochastic matrix P is derived by calculating k^2 as follows,

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}$$

The determination of the probabilistic behavior of the Markov Chain is the initial state of the chain and the determination of the probabilistic behavior P is known. Let $p_j^{(n)}$ indicates transition probability that Markov chain is in state j as step or time n.

The $1 \times k$ vector $p^{(n)}$ has elements $p_j^{(n)}$.

Thus, $p_j^{(n)} = (p_1^{(n)}, p_2^{(n)}, \ldots, p_k^{(n)})$ and $p^{(1)} = p^{(0)}p$, where $p^{(0)}$ is the initial stochastic vector.

Generally, $p^{(n+k)} = p^{(k)}p^{(n)}$ where, $p^{(n)}$ is the n^{th} power of p.

The two advantages of Markov chain model are

(a) The forecasts are available immediately after the observations are done since the predictors use the local information on the weather.

(b) After the processing of climatological data, they need minimal computation.

3.1. Estimation of parameters. The model to estimate the elements of p_{ij} and transition probability matrix P is formed. The calculating for p_{ij} by $p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{k} n_{ij}}$, where n_{ij} is the observed data number of times moves from i state to another state j. Basically, the changes of rainfall region is divided into many states for the calculation of transition probability of Markov chain according to frequency distribution.

3.2. Transition probability matrix. The annual rainfall indicates seven states and the transition probability matrix is obtained as below:

$$P_{ij} = \begin{bmatrix} \frac{0}{4} & \frac{1}{4} & \frac{0}{4} & \frac{1}{4} & \frac{2}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{1}{10} & \frac{3}{10} & \frac{2}{10} & \frac{1}{10} & \frac{0}{10} & \frac{2}{10} & \frac{1}{10} \\ \frac{2}{22} & \frac{5}{22} & \frac{7}{22} & \frac{5}{22} & \frac{3}{22} & \frac{0}{22} & \frac{0}{22} \\ \frac{1}{33} & \frac{3}{33} & \frac{5}{33} & \frac{7}{33} & \frac{8}{33} & \frac{4}{33} & \frac{5}{33} \\ \frac{1}{22} & \frac{2}{22} & \frac{5}{22} & \frac{3}{22} & \frac{0}{22} & \frac{6}{22} & \frac{5}{22} \\ \frac{1}{18} & \frac{3}{18} & \frac{4}{18} & \frac{3}{18} & \frac{2}{18} & \frac{5}{18} & \frac{0}{18} \\ \frac{1}{11} & \frac{0}{11} & \frac{1}{11} & \frac{2}{11} & \frac{4}{11} & \frac{3}{11} & \frac{0}{11} \end{bmatrix}$$

$$P_{ij} = \begin{bmatrix} 0.000 & 0.250 & 0.000 & 0.250 & 0.500 & 0.000 & 0.000 \\ 0.100 & 0.300 & 0.200 & 0.100 & 0.000 & 0.200 & 0.100 \\ 0.091 & 0.227 & 0.318 & 0.227 & 0.136 & 0.000 & 0.000 \\ 0.030 & 0.091 & 0.151 & 0.212 & 0.242 & 0.121 & 0.151 \\ 0.045 & 0.090 & 0.227 & 0.136 & 0.000 & 0.272 & 0.227 \\ 0.056 & 0.167 & 0.222 & 0.167 & 0.111 & 0.278 & 0.000 \\ 0.091 & 0.000 & 0.091 & 0.182 & 0.364 & 0.272 & 0.000 \end{bmatrix}$$

To construct the future rainfall state generation is provided by transition matrix.

The subsequent steps are carried out:

Step 1: The cumulative summation Transition probability matrix is evaluated with the cumulative summation of each row which leads to the following matrix:

$$P_{cum} = \begin{bmatrix} 0.000 & 0.250 & 0.250 & 0.500 & 1.000 & 1.000 & 1.000 \\ 0.100 & 0.400 & 0.600 & 0.700 & 0.700 & 0.900 & 1.000 \\ 0.091 & 0.318 & 0.636 & 0.863 & 1.000 & 1.000 & 1.000 \\ 0.030 & 0.121 & 0.370 & 0.629 & 0.777 & 0.925 & 1.000 \\ 0.045 & 0.231 & 0.385 & 0.539 & 0.616 & 0.847 & 1.000 \\ 0.056 & 0.223 & 0.445 & 0.612 & 0.723 & 1.000 & 1.000 \\ 0.091 & 0.091 & 0.182 & 0.364 & 0.728 & 1.000 & 1.000 \end{bmatrix}$$

Step 2: By using algorithm of C++, generating uniform random number between the number 0 and 1.

Step 3: The cumulative probability of the before state but less than or equal to the cumulative probability of the next state .For the observations that the consecutive states are obtained when the random number is higher than the P_{cum}. The number of rainfall states S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, S_{16} and S_{17}. are reveals that the class interval as presented in Table 1 and the rainfall states are measured using the following rule:

$$\text{State} = \begin{cases} S_{11} & \text{if } \quad \rho < 0.054 \\ S_{12} & \text{if } 0.054 \leq \rho \leq 0.102 \\ S_{13} & \text{if } 0.102 \leq \rho \leq 0.266 \\ S_{14} & \text{if } 0.266 \leq \rho \leq 0.319 \\ S_{15} & \text{if } 0.319 \leq \rho \leq 0.620 \\ S_{16} & \text{if } 0.620 \leq \rho \leq 0.868 \\ S_{17} & \text{if } 0.868 \leq \rho \leq 1.000 \end{cases}$$

Where ρ is generated random number.

The Table-2 is clearly reveals that states change with changing the probability with some class interval. The climatic factor that beneficial to Aviation co; to make short term probabilistic prediction for seasonal conditions. The variability of rainfall and its pattern of extreme way, any desired number of rainfall states can be generated. The study of states S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, S_{16} and S_{17} are the class interval value as it appeared in Table-1., so when we generate the synthetic series for these states we can get the states of rainfall not the exact value (amount in mm) of rainfall. The use of modified water balance models offers many advantages in evaluating the regional impacts of global climate change [3]. Concerns over climate change caused by increasing concentration of C_{O2} and other trace gases in the atmosphere has increased in recent years. A major effect of climate change may be alterations in regional hydrologic cycles and changes in regional water availability. The states will be decomposed according to the rule' high or low precipitation are very important for the agriculture as well as the economy of the country.

Table 2. Synthetic series of generated Annual rainfall Uniform Random Number.

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Uniform Random Number	**0.284**	**0.324**	**0.982**	**0.941**	**0.560**	**0.701**	**0.812**	**0.061**	**0.360**	**0.443**
Random States	S_{14}	S_{15}	S_{17}	S_{17}	S_{15}	S_{16}	S_{16}	S_{12}	S_{15}	S_{15}

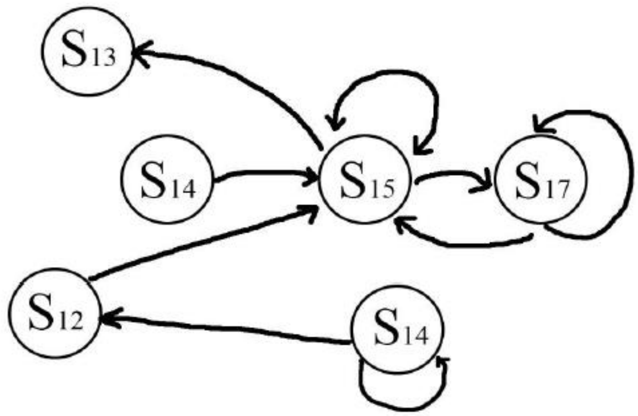

Figure 4. Forecast Strategy of Annual Rainfall from the period (2012-2020).

5. Conclusion

We conclude that the long term prediction of the primary source of data the transition probability matrix represents the weather model in which the trend of the following year is estimated from the period 1901-2020 (for the past 120 years). The annual rainfall obtained from multi-step transition Markov chain and fuzzy heptagonal numbers, and random generated numbers, sates are unstable or uncertainty of the climatic factor. Finally, overall 10% of the system predicted on which annual rainfall a periodic with uncertainty. For 10% prediction, cycles of length shows that the model described only to forecast with high accuracy. But the long range forecasting based on this model does not give more accuracy. The year 2009 and 2010 actual rainfall values do not match with our model. The analysis of extreme yearly rainfall shows that Markov Chain approach provides on alternative of modelling future variation in annual rainfall. Markov modeling is one of the tools that can be utilized to improvement of planner with assessing the annual rainfall.

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