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STRUCTURE OF ROUGH NEUTROSOPHIC HYPERIDEALS OF Γ-SEMIHYPERRINGS

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Abstract

The aim of the present paper is to introduce the notion of Rough neutrosophic hyperideals of Γ-semihyperrings. Several theorems, related properties and in particular, some structural characteristics of each are given in this paper.

1. Introduction

Pawlak [11] introduced the notion of rough sets in 1982. It is derived from elementary research on logical properties of information system. Numerous models applied this concept those were algebra, graph theory, probability

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theory, topology, pattern recognition etc., The notion of Fuzzy sets were introduced by Zadeh [15] in 1965. The notion of intuitionistic fuzzy sets is introduced by Atanassov [1] in 1986. The neutrosophic set theory is the generalisation of intuitionistic fuzzy set is introduced by Smarandache [12]. It gives the picture of the objective world more practical and realistic. Marty [9] initiated the theory of algebraic hyperstructures which is a branch of classical algebraic theory. Various fuzzy models in hyperstructures have been established by many authors [2] [3] [8] [10]. Davvaz [5] and Vougiouklis [13] [14] presented the notion of Semihyperring in which addition and multiplication are Hyperoperation. Dehkordi and Davvaz [7] introduced the concept of Γ-Semihyperrings. Debabrata Mandal [6] introduced the notion of Neutrosophic Hyperideals of Γ-Semihyperrings.

Inspired by this concept, in this paper the notion of Rough neutrosophic Ideals of Γ-Semihyperring is defined and its basic properties, related theorems are discussed.

2. Preliminaries

Refer [4] [5] [6] [7] [12] for basic definitions and concepts used in this work.

3. Rough Neutrosophic Hyperideals of Γ semihyperrings

In this section we will study some results on lower and upper approximations of neutrosophic hyperideals (NHI) of Γ semihyperring. Also we introduce Rough neutrosophic left (right) hyper ideals (RNLHI, RNRHI) of $\mathcal{R}$. Throughout this paper let $\mathcal{R}$ be the semihyperring, Φ be a congruence relation on $\mathcal{R}$, $\mathcal{S}$ be the subset of $\mathcal{R}$.

Definition 3.1. A NLHI is said to be RNLHI of $\mathcal{R}$ if it is both upper RNLHI and lower RNLHI of $\mathcal{R}$.

Definition 3.2. A NLHI is said to be upper (lower) RNLHI of $\mathcal{R}$ if it's upper (lower) approximation is NLHI.

Definition 3.3. A NRHI is said to be RNRHI of $\mathcal{R}$ if it is both upper RNRHI and lower RNRHI of $\mathcal{R}$.

Definition 3.4. A NRHI is said to be upper (lower) RNRHI of $\mathcal{R}$ if it's upper (lower) approximation is NRHI.

Theorem 3.5. *A RNS* $\Phi(E)$ *of* $\mathcal{R}$ *is a RNLHI* (*RNRHI*) *of* $\mathcal{R}$ *iff any level subsets*

$$\begin{bmatrix} \Phi^*(E_l^t) = \{x \in S \mid \Phi^*(E^t(x)) \geq l,\ l \in [0, 1]\} \\ \Phi^*(E_l^i) = \{x \in S \mid \Phi^*(E^i(x)) \geq l,\ l \in [0, 1]\} \\ \Phi^*(E_l^f) = \{x \in S \mid \Phi^*(E^f(x)) \leq l,\ l \in [0, 1]\} \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \Phi_*(E_l^t) = \{x \in S \mid \Phi_*(E^t(x)) \geq l,\ l \in [0, 1]\} \\ \Phi_*(E_l^i) = \{x \in S \mid \Phi_*(E^i(x)) \geq l,\ l \in [0, 1]\} \\ \Phi_*(E_l^f) = \{x \in S \mid \Phi_*(E^f(x)) \leq l,\ l \in [0, 1]\} \end{bmatrix} \text{ are NLHI(NRHI) of } \mathcal{R}.$$

Proof. Consider RNS of $\mathcal{R}$ is upper NLHI (NRHI) of $\mathcal{R}$. Then not all of E_l^t, E_l^i, E_l^f are equal to zero. Let $v, \omega \in E_l = (E_l^t, E_l^i, E_l^f)$ and $s \in \mathcal{R}$. Then

$$\inf_{c \in v + \omega} \{\Phi^*(E^t(c))\} \geq \min\{\Phi^*(E^t(v)), \Phi^*(E^t(\omega))\} \geq \min\{l, l\} = l$$

$$\inf_{c \in v + \omega} \{\Phi^*(E^i(c))\} \geq \left[\frac{\Phi^*(E^i(v)) + \Phi^*(E^i(\omega))}{(2)}\right] \geq \Phi^*\left(\frac{l + l}{2}\right) = l$$

$$\sup_{c \in v,\, \omega} \{\Phi^*(E^f(c))\} \leq \max\{\Phi^*(E^f(v)), \Phi^*(E^f(\omega))\} \leq \max\{l, l\} = l$$

$$\Rightarrow v + \omega \subseteq E_l^t, E_l^i, E_l^f, \text{ i.e. } v + \omega \subseteq E_l.$$

Also $\inf_{c \in syx} \{\Phi^*(E^t(c))\} \geq \Phi^*(E^t(v) \geq l;$ $\inf_{c \in syx} \{\Phi^*(E^i(c))\} \geq \Phi^*(E^i(x) \geq l;$ $\sup_{c \in syx} \{\Phi^*(E^f(c))\} \leq \Phi^*(E^f(x) \leq l.$

Hence $syx \subseteq \Phi^*(E_l)$ and $\Phi^*(E_l)$ is a NLHI (NRHI) of $\mathcal{R}$. Conversely, $\Phi^*(E_l)$ is a NLHI (NRHI) of S. Assume $\Phi^*(E)$ is not a upper NLHI (NRHI).

Then any one of the following inequality is true for $v, \omega \in S$.

(i) $\inf_{c \in v+\omega} \{\Phi^*(E^t(c))\} < \min\{\Phi^*(E^t(v)), \Phi^*(E^t(\omega))\}$

(ii) $\inf_{c \in v+\omega} \{\Phi^*(E^i(c))\} < \left(\dfrac{\Phi^*(E^t(v)), \Phi^*(E^t(\omega))}{(2)}\right)$

(iii) $\sup_{c \in v+\omega} \{\Phi^*(E^f(c))\} > \max(\{\Phi^*(E^f(v)), \Phi^*(E^f(\omega))\})$

For (i) take $l_1 = \frac{1}{2}[\inf_{c \in v+\omega} \Phi^*(E^t(c) + \min\{\Phi^*(E^t(v)), \Phi^*(E^t(\omega))\}]$. then $\inf_{c \in v+\omega} \Phi^*(E^t(c)) < l_1 < \min\{\Phi^*(E^t(v)), \Phi^*(E^t(\omega))\} \Rightarrow v, \omega \in E^t_{l_1}$, but $v + \omega \notin E^t_{l_1}$. Which contradicts.

For (ii) take $l_2 = \frac{1}{2}[\inf_{c \in v+\omega} \Phi^*(E^i(c) + \min\{\Phi^*(E^i(v)), \Phi^*(E^i(\omega))\}]$. then $\inf_{c \in v+\omega} \Phi^*(E^i(c)) < l_2 < \dfrac{\{\Phi^*(E^i(v)) + \Phi^*(E^i(\omega))\}}{2} \Rightarrow v, \omega \in E^i_{l_2}$, but $v + \omega \notin E^i_{l_2}$. Which contradicts.

For (iii) take $l_3 = \frac{1}{2}(\sup_{c \in v+\omega} [[\Phi^*(E^f(c)] + \max[\{\Phi^*(E^f(v)), \Phi^*(E^f(\omega))\}]])$.

$[\sup_{c \in v+\omega} \{[\Phi^*(E^f(c))]\} > l_3 > \max[\{\Phi^*(E^f(v)), \Phi^*(E^f(\omega))\}] \Rightarrow v, \omega \in E^f_{l_3}$, but $v + \omega \notin E^f_{l_3}$. Which contradicts. So, for all three cases it is a contradiction that $\Phi^*(E_l)$ is a NLHI (NRHI) of $\mathcal{R}$.

Similarly we can prove for lower NLHI (NRHI) of $\mathcal{R}$. Hence $\Phi(E)$ of $\mathcal{R}$ is a RNLHI (RNRHI) of $\mathcal{R}$. Hence Proved.

Definition 3.6. If E and F be any two RNS of $\mathcal{S}$. Then the $\cap$ of E and F is given by

$$\begin{bmatrix} \Phi^*[(E^t \cap F^t)(\kappa)] = \min\{\Phi^*(E^t(\kappa)), \Phi^*(F^t(\kappa))\} \\ \Phi^*[(E^i \cap F^i)(\kappa)] = \min\{\{\Phi^*(E^i(\kappa)), \Phi^*(F^i(\kappa))\}\} \\ \Phi^*[(E^f \cap F^f)(\kappa)] = \max\{\Phi^*(E^f(\kappa)), \Phi^*(F^f(\kappa))\} \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \Phi_*[(E^t \cap F^t)(\kappa)] = \min\{\Phi_*(E^t(\kappa)), \Phi_*(E^t(\kappa))\} \\ \Phi_*[(E^i \cap F^i)(\kappa)] = \min\{\{\Phi_*(E^i(\kappa)), \Phi_*(E^i(\kappa))\}\} \\ \Phi_*[(E^f \cap F^f)(\kappa)] = \max\{\Phi_*(E^f(\kappa)), \Phi_*(E^f(\kappa))\} \end{bmatrix} \forall \kappa \in S,$$

Theorem 3.7. *A non-empty collection of $\cap$ of RNHI is a RNHI of $\mathcal{R}$.*

Proof. Let $\{\Phi^*(E_{i_1}) \neq \phi : i_1 \in I\}$ be upper NLHI (NRHI) of $\mathcal{S}$ and $a, b \in \mathcal{S}, \gamma \in \Gamma$. Then

$$\inf_{c \in a+b} (\bigcap_{i_1 \in l} \Phi^*(E^t_{i_1})(c) \inf_{c \in a+b} \inf_{i_1 \in I} \Phi^*(E^t_{i_2})(c) \geq \inf_{i_1 \in I} \{\min\{\Phi^*(E^t_{i_1}(a)), \Phi^*(E^t_{i_1}(b))\}\}$$

$$= \min\{\inf_{i_1 \in I} \Phi^*(E^t_{i_1}(a)), \inf_{i_1 \in I} \Phi^*(E^t_i(b))\} = \min\{\bigcap_{i_1 \in I} \Phi^*(E^t_{i_1}(a)), \bigcap_{i_1 \in I} \Phi^*(E^t_{i_1}(b))\}$$

$$\inf_{c \in a+b} (\bigcap_{i_1 \in l} \Phi^*(E^i_{i_1})(c) \inf_{c \in a+b} \inf_{i_1 \in I} \Phi^*(E^i_{i_2})(c) \geq \left[\inf_{i_1 \in I} \left(\frac{\Phi^*(E^i_{i_1}(a)), \Phi^*(E^i_{i_1}(b))}{2} \right) \right]$$

$$= \left(\frac{\inf_{i_1 \in I} \Phi^*(E^t_{i_1}(a)), \inf_{i_1 \in I} \Phi^*(E^t_i(b))}{2} 0 \right) = \left(\frac{\bigcap_{i_1 \in I} \Phi^*(E^t_{i_1}(a)), \bigcap_{i_1 \in I} \Phi^*(E^t_{i_1}(b))}{2} \right)$$

This

$$\sup_{c \in a+b} (\bigcap_{i_1 \in l} \Phi^*(E^f_{i_1})(c) \sup_{c \in a+b} \sup_{i_1 \in I} \Phi^*(E^f_{i_2}(c)) \leq \sup_{i_1 \in I} \{\max\{\Phi^*(E^f_{i_1}(a)), \Phi^*(E^f_{i_1}(b))\}\}$$

$$= \max\{\sup_{i_1 \in I} \Phi^*(E^f_{i_1}(a)), \inf_{i_1 \in I} \Phi^*(E^f_i(b))\} = \max\{\bigcap_{i_1 \in I} \Phi^*(E^f_{i_1}(a)), \bigcap_{i_1 \in I} \Phi^*(E^f_{i_1}(b))\}$$

$$\inf_{c \in \delta\gamma x} (\bigcap_{i_1 \in l} \Phi^*(E^t_{i_1}(c)) \inf_{c \in \delta\gamma x} \inf_{i_1 \in I} \Phi^*(E^t_{i_1}(c)) \geq \inf_{i_1 \in I} \Phi^*(E^t_{i_1}(x)), \Phi^*(E^t_{i_1}(x))\}\}$$

$$\inf_{c \in \delta\gamma x} (\bigcap_{i_1 \in l} \Phi^*(E^i_{i_1}(c)) \inf_{c \in \delta\gamma x} \inf_{i_1 \in I} \Phi^*(E^i_{i_2}(c)) \geq \inf_{i_1 \in I} \Phi^*(E^i_{i_1}(x)), \Phi^*(E^i_{i_1}(x))\}\}$$

$$\sup_{c \in \delta\gamma x} (\bigcap_{i_1 \in l} \Phi^*(E^f_{i_1}(c)) \sup_{c \in \delta\gamma x} \sup_{i_1 \in I} \Phi^*(E^f_{i_2}(c)) \leq \sup_{i_1 \in I} \Phi^*(E^f_{i_1}(x)), \Phi^*(E^f_{i_1}(x))\}\}$$

implies $\bigcap_{i_1 \in l} \Phi^*(E_{i_1})$ is a upper NLHI(NRHI) of $\mathcal{R}$. Similarly we can prove for lower NLHI (NRHI) of $\mathcal{R}$. Hence $\bigcap_{i_1 \in l} \Phi^*(E_{i_1})$ is a RNLHI (RNRHI) of $\mathcal{R}$.

Definition 3.8. If E and F be two RNS of $\mathcal{S}$. Then the Cartesian Product of E, F is given by

$$\begin{bmatrix} \Phi^*[(E^t \times F^t)(\rho, \omega)] = \min\{\Phi^*(E^t(\rho)), \Phi^*(F^t(\omega))\} \\ \Phi^*[(E^i \times F^i)(\rho, \omega)] = \left(\dfrac{[\Phi^*(E^i(\rho)), \Phi^*(F^i(\omega))]}{(2)}\right) \\ \Phi^*[(E^f \times F^f)(\rho, \omega)] = \max\{\Phi^*(E^f(\rho)), \Phi^*(F^f(\omega))\} \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \Phi_*[(E^t \times F^t)(\rho, \omega)] = \min\{\Phi_*(E^t(\rho)), \Phi_*(F^t(\omega))\} \\ \Phi_*[(E^i \times F^i)(\rho, \omega)] = \left(\dfrac{[\Phi_*(E^i(\rho)), \Phi_*(F^i(\omega))]}{(2)}\right) \\ \Phi_*[(E^f \times F^f)(\rho, \omega)] = \max\{\Phi_*(E^f(\rho)), \Phi_*(F^f(\omega))\} \end{bmatrix} \text{ for all } \rho, \omega \in \mathcal{R}.$$

Theorem 3.9. *Let E and F be the two RNHI of $\mathcal{R}$. Then Cartesian product $E \times F$ is a RNHI of $\mathcal{R} \times \mathcal{R}$.*

Proof. Consider $(\Delta_1, \Delta_2), (\nabla_1, \nabla_2) \in \mathcal{R} \times \mathcal{R}$.

$$\inf\{\underset{(c_1, c_2) \in (\Delta_1, \Delta_2) + (\nabla_1, \nabla_2)}{\Phi^*(E^t \times F^t)(c_1, c_2)}\} = \inf_{c_1 \in (\Delta_1 + \nabla_1),\, c_2 \in (\Delta_2, \nabla_2)} \Phi^*(E^t \times F^t)(c_1, c_2)$$

$$= \inf_{c_1 \in (\Delta_1 + \nabla_1),\, c_2 \in (\Delta_2, \nabla_2)} \min\{\Phi^*(E^t(c_1)), \Phi^*(F^t(c_2))\}$$

$$\geq \min\{\min\{\Phi^*(E^t(\Delta_1)), \Phi^*(E^t(c_1 \in (\nabla_1)))\}, \min\{\Phi^*(F^t(\Delta_2)), \Phi^*(F^t(\nabla_2))\}\}$$

$$= \min(\{\min\{\Phi^*(E^t(\Delta_1)), \Phi^*(F^t(\Delta_2))\}, \min\{\Phi^*(E^t(\nabla_1)), \Phi^*(F^t(\nabla_2))\}\})$$

$$= \min(\{\Phi^*(E^t \times F^t)(\Delta_1, \Delta_2), \Phi^*(E^t \times F^t)(\nabla_1, \nabla_2)\})$$

$$\inf\{\underset{(c_1, c_2) \in (\Delta_1, \Delta_2) + (\nabla_1, \nabla_2)}{\Phi^*(E^i \times F^i)(c_1, c_2)}\} = \inf_{c_1 \in (\Delta_1 + \nabla_1),\, c_2 \in (\Delta_2, \nabla_2)} \Phi^*(E^i \times F^i)(c_1, c_2)$$

$$= \inf_{c_1 \in (\Delta_1 + \nabla_1),\, c_2 \in (\Delta_2, \nabla_2)} \left(\frac{\Phi^*(E^i(c_1)), \Phi^*(F^i(c_2))}{(2)}\right)$$

$$\geq \left(\frac{1}{2}\left\{\frac{\Phi^*(E^i(\Delta_1)), \Phi^*(E^i(\nabla_1))}{(2)} + \frac{\Phi^*(F^i(\Delta_2)), \Phi^*(F^i(\nabla_2))}{(2)}\right\}\right)$$

$$= \left(\frac{1}{2}\left\{\frac{\Phi^*(E^i(\Delta_1)), \Phi^*(E^i(\Delta_2))}{2} + \frac{\Phi^*(F^i(\nabla_2)), \Phi^*(F^i(\nabla_2))}{2}\right\}\right)$$

$$= (1/2\{\Phi^*(E^i \times F^i)(\Delta_1, \Delta_2), \Phi^*(E^i \times F^i)(\nabla_1, \nabla_2)\})$$

$$\sup_{(c_1, c_2) \in (\Delta_1, \Delta_2) + (\nabla_1, \nabla_2)}\{\Phi^*(E^f \times F^f)(c_1, c_2)\} = \inf_{c_1 \in (\Delta_1 + \nabla_1),\, c_2 \in (\Delta_2, \nabla_2)} \Phi^*(E^f \times F^f)(c_1, c_2)$$

$$= (\sup_{c_1 \in (\Delta_1 + \nabla_1),\, c_2 \in (\Delta_2, \nabla_2)} \max\{\Phi^*(E^f(c_1)), \Phi^*(F^f(c_2))\}$$

$$\leq \max\{\max\{\Phi^*(E^f(\Delta_1)), \Phi^*(E^f(\nabla_1))\}, \max\{\Phi^*(F^f(\Delta_2)), \Phi^*(F^f(\nabla_2))\}\}$$

$$= \max(\{\max\{\Phi^*(E^f(\Delta_1)), \Phi^*(F^f(\Delta_2))\}, \max\{\Phi^*(E^f(\nabla_1)), \Phi^*(F^f(\nabla_2))\}\})$$

$$= (\{\max\{\Phi^*(E^f \times F^f)(\Delta_1, \Delta_2), \Phi^*(E^f \times F^f)(\nabla_1, \nabla_2)\}\})$$

$$\inf_{(c_1, c_2) \in (\Delta_1, \Delta_2)\gamma(\nabla_1, \nabla_2)}\{\Phi^*(E^t \times F^t)(c_1, c_2)\} = \inf_{(c_1, c_2) \in (\Delta_1\gamma\nabla_1), (\Delta_2\gamma\nabla_2)}\{\Phi^*(E^t \times F^t)(c_1, c_2)\}$$

$$= \inf_{c_1 \in (\Delta_1\gamma\nabla_1),\, c_2 \in (\Delta_2\gamma\nabla_2)} \min\{\Phi^*(E^t(c_1)), \Phi^*(F^t(c_2))\} \geq \min\{\Phi^*(E^t(\nabla_1)), \Phi^*(E^t(\nabla_2))\}$$

$$= (\Phi^*(E^t \times F^t)(\nabla_1, \nabla_2))$$

$$\inf_{(c_1, c_2) \in (\Delta_1, \Delta_2)\gamma(\nabla_1, \nabla_2)}\{\Phi^*(E^i \times F^i)(c_1, c_2)\} = \inf_{(c_1, c_2) \in (\Delta_1\gamma\nabla_1), (\Delta_2\gamma\nabla_2)}\{\Phi^*(E^i \times F^i)(c_1, c_2)\}$$

$$= \inf_{c_1 \in (\Delta_1\gamma\nabla_1),\, c_2 \in (\Delta_2\gamma\nabla_2)} \left(\frac{\Phi^*(E^i(c_1)), \Phi^*(F^i(c_2))}{2}\right) \geq \frac{\Phi^*(E^i(\nabla_1)), \Phi^*(E^i(\nabla_2))}{2}$$

$$= (\Phi^*(E^i \times F^i)(\nabla_1, \nabla_2))$$

$$\sup_{(c_1, c_2) \in (\Delta_1, \Delta_2)\gamma(\nabla_1, \nabla_2)}\{\Phi^*(E^f \times F^f)(c_1, c_2)\} = \inf_{(c_1, c_2) \in (\Delta_1\gamma\nabla_1), (\Delta_2\gamma\nabla_2)}\{\Phi^*(E^f \times F^f)(c_1, c_2)\}$$

$$= \sup_{c_1 \in (\Delta_1\gamma\nabla_1),\, c_2 \in (\Delta_2\gamma\nabla_2)} \max\{\Phi^*(E^f(c_1)), \Phi^*(F^f(c_2))\} \leq \max\{\Phi^*(E^f(\nabla_1)), \Phi^*(E^f(\nabla_2))\}$$

$= (\Phi^*(E^f \times F^f)(\nabla_1, \nabla_2))$

implies $E \times F$ is a upper NLHI(NRHI) of $\mathcal{R} \times \mathcal{R}$. Similarly we can prove $E \times F$ is a lower NLHI(NRHI) of $\mathcal{R} \times \mathcal{R}$. Hence $E \times F$ is a RNHI of $(\mathcal{R} \times \mathcal{R})$.

Definition 3.10. If E, F be two RNS of $\mathcal{R}$. Then Composition of E, F is defined

$$\left[\begin{array}{l} \Phi^*(E^t \circ F^t)(z) = \begin{cases} \sup\{\min\{\Phi^*(E^t(x_i)), \Phi^*(F^t(y_i))\}\} \\ z \in \sum_{i=1}^{n} x_i \gamma_i y_i \\ 0, \text{ otherwise} \end{cases} \\ \Phi^*(E^i \circ F^i)(z) = \begin{cases} \sup\left(\dfrac{\Phi^*(E^i(a)), \Phi^*(F^i(b))}{(2)}\right) \\ z \in \sum_{i=1}^{n} x_i \gamma_i y_i \\ 0, \text{ otherwise} \end{cases} \\ \Phi^*(E^f \circ F^f)(z) = \begin{cases} \inf\{\max\{\Phi^*(E^f(x_i)), \Phi^*(F^f(y_i))\}\} \\ z \in \sum_{i=1}^{n} x_i \gamma_i y_i \\ 0, \text{ otherwise} \end{cases} \end{array}\right] \text{ and}$$

$$\left[\begin{array}{l} \Phi_*(E^t \circ F^t)(z) = \begin{cases} \sup\{\min\{\Phi_*(E^t(x_i)), \Phi_*(F^t(y_i))\}\} \\ z \in \sum_{i=1}^{n} x_i \gamma_i y_i \\ 0, \text{ otherwise} \end{cases} \\ \Phi_*(E^i \circ F^i)(z) = \begin{cases} \sup\left(\dfrac{\Phi_*(E^i(a)), \Phi_*(F^i(b))}{(2)}\right) \\ z \in \sum_{i=1}^{n} x_i \gamma_i y_i \\ 0, \text{ otherwise} \end{cases} \\ \Phi_*(E^f \circ F^f)(z) = \begin{cases} \inf\{\max\{\Phi_*(E^f(x_i)), \Phi_*(F^f(y_i))\}\} \\ z \in \sum_{i=1}^{n} x_i \gamma_i y_i \\ 0, \text{ otherwise} \end{cases} \end{array}\right]$$

$\forall z, x_i, y_i \in \mathcal{R}$; for $i = 1, 2, \ldots, n$.

Theorem 3.11. *If* E, F *be two RNHI of* $\mathcal{R}$, *then* $E \circ F$ *is a RNHI of* $\mathcal{R}$.

Proof. Given E and F be RNHI of $\mathcal{R}$, $a, b \in \mathcal{R}$. Assume that, $a + b$ is not equal to $\sum_{i=1}^{n} x_i \gamma_i y_i$, for $x_i \gamma_i y_i \in \mathcal{R}$ and $\gamma_i \in \Gamma$. Then

$$\inf_{c \in a+b} \{(\Phi^*(E^t \circ F^t)(z))\} = \sup_{c \in a+b \in \sum_{i=1}^{n} x_i \gamma_i y_i} \{\min_i \{\Phi^*(E^t(x_i)), \Phi^*(F^t(y_i))\}\}$$

$$\geq \sup_{a \in \sum_{i=1}^{n} p_i \gamma_i q_i,\, b \in \sum_{i=1}^{n} p_i \gamma_i q_i} \{\min_i \{\Phi^*(E^t(p_i)), \Phi^*(F^t(q_i)), \Phi^*(E^t(r_i)), \Phi^*(F^t(s_i))\}\}$$

$$= \min\{\sup_{a \in \sum_{i=1}^{n} p_i \gamma_i q_i} \{\min_i \{\Phi^*(E^t(p_i)), \Phi^*(F^t(q_i)), \sup_{b \in \sum_{i=1}^{n} p_i \gamma_i q_i} \Phi^*(E^t(r_i)), \Phi^*(F^t(s_i))\}\}$$

$$= \min\{(\Phi^*(E^t \circ F^t)(a)), (\Phi^*(E^t \circ F^t)(b))\}$$

$$\inf_{c \in a+b} \{(\Phi^*(E^i \circ F^i)(z))\} = \sup_{c \in a+b \in \sum_{i=1}^{n} x_i \gamma_i y_i} \sum_{i=1}^{n} \left(\frac{\Phi^*(E^i(x_i)), \Phi^*(F^t(y_i))}{2n} \right)$$

$$\geq \sup_{a \in \sum_{i=1}^{n} p_i \gamma_i q_i,\, b \in \sum_{i=1}^{n} p_i \gamma_i q_i} \sum_{i=1}^{n} \left(\frac{\Phi^*(E^i(p_i)), \Phi^*(F^i(q_i)), \Phi^*(E^i(r_i)), \Phi^*(F^i(s_i))}{4n} \right)$$

$$\geq \frac{1}{2} [\sup_{a \in \sum_{i=1}^{n} p_i \gamma_i q_i} \sum_{i=1}^{n} \frac{\Phi^*(E^i(p_i)), \Phi^*(F^i(q_i))}{2n}, \sup_{b \in \sum_{i=1}^{n} p_i \gamma_i q_i} \sum_{i=1}^{n} \frac{\Phi^*(E^i(r_i)), \Phi^*(F^i(s_i))}{2n}]$$

$$= \frac{\Phi^*(E^i \circ F^i)(a), \Phi^*(E^i \circ F^i)(b)}{2}$$

$$\sup_{c \in a+b} \{(\Phi^*(E^f \circ F^f)(z))\} = \inf_{c \in a+b \in \sum_{i=1}^{n} x_i \gamma_i y_i} \{\min_i \{\Phi^*(E^f(x_i)), \Phi^*(F^f(y_i))\}\}$$

$$\leq \inf_{a \in \sum_{i=1}^{n} p_i \gamma_i q_i,\, b \in \sum_{i=1}^{n} p_i \gamma_i q_i} \{\min_i \{\Phi^*(E^f(p_i)), \Phi^*(F^f(q_i)), \Phi^*(E^f(r_i)), \Phi^*(F^f(s_i))\}\}$$

$$= \min\{\inf_{a \in \sum_{i=1}^{n} p_i \gamma_i q_i} \{\min_i \{\Phi^*(E^f(p_i)), \Phi^*(F^f(q_i)), \inf_{b \in \sum_{i=1}^{n} p_i \gamma_i q_i} \Phi^*(E^f(r_i)), \Phi^*(F^f(s_i))\}\}$$

$$= \min[\{(\Phi^*(E^f \circ F^f)(a)), (\Phi^*(E^f \circ F^f)(b))\}]$$

$$\inf_{c \in a\gamma b}\{(\Phi^*(E^t \circ F^t)(z))\} = \sup_{c \in a\gamma b \in \sum_{i=1}^n x_i\gamma_i y_i}\{\min_i\{\Phi^*(E^t(x_i)), \Phi^*(F^t(y_i))\}\}$$

$$\geq \sup_{c \in a\gamma b \in \sum_{i=1}^n a\delta_i r_i \gamma_i s_i}\{\min_i\{\Phi^*(E^t(a\delta_i r_i)), \Phi^*(F^t(s_i))\}\}$$

$$\geq \sup_{b \in \sum_{i=1}^n p_i\gamma_i q_i}\{\min_i\{\Phi^*(E^t(r_i)), \Phi^*(F^t(s_i))\}\} = \Phi^*((E^t \circ F^t)(b))$$

$$\inf_{c \in a\gamma b}\{(\Phi^*(E^i \circ F^i)(z))\} = \sup_{c \in a\gamma b \in \sum_{i=1}^n x_i\gamma_i y_i}\sum_{i=1}^n\left(\frac{\Phi^*(E^i(x_i)), \Phi^*(F^i(y_i))}{(2n)}\right)$$

$$\geq \sup_{c \in a\gamma b \in \sum_{i=1}^n a\delta_i r_i \gamma_i s_i}\sum_{i=1}^n\left(\frac{\Phi^*(E^i(a\delta_i r_i)), \Phi^*(F^i(s_i))}{2n}\right)$$

$$\geq \sup_{b \in \sum_{i=1}^n p_i\gamma_i q_i}\sum_{i=1}^n\frac{\Phi^*(E^i(r_i)), \Phi^*(F^i(s_i))}{2n} = \Phi^*((E^i \circ F^i)(b))$$

$$\sup_{c \in a\gamma b}\{(\Phi^*(E^f \circ F^f)(z))\} = \inf_{c \in a\gamma b \in \sum_{i=1}^n x_i\gamma_i y_i}\left[\{\max_i\{\Phi^*(E^f(x_i)), \Phi^*(F^f(y_i))\}\}\right]$$

$$\geq \inf_{c \in a\gamma b \in \sum_{i=1}^n a\delta_i r_i \gamma_i s_i}\max_i\{\Phi^*(E^f(ar_i)), \Phi^*(F^f(s_i))\}$$

$$\geq \inf_{b \in \sum_{i=1}^n p_i\gamma_i q_i}\{\min_i\{\Phi^*(E^f(r_i)), \Phi^*(F^f(s_i))\}\} = \Phi^*((E^f \circ F^f)(b))$$

Similarly we can prove for lower NLHI (NRHI) of $\mathcal{R}$. Hence $E \circ F$ is a RNHI of $\mathcal{R}$.

4. Conclusion

The concept Rough Neutrosophic Hyperideals of Γ-semihyperrings and some properties, structural characteristics have been analysed in this paper. These results can be extended to other properties such Rough prime neutrosophic Hyperideal, Rough neutrosophic Bi-hyperideal and so on.

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