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EFFECTIVE FUZZY SEMIGRAPHS

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Abstract

In this paper, effective fuzzy and various concepts in fuzzy semigraphs are defined and illustrated through examples. Some results on effective fuzzy semigraphs have been arrived.

1. Introduction

The theory of fuzzy graphs was introduced by Rosanfeld in 1975. characteristics of Fuzzy graphs were dealt by Azriei Rosanfeld [7]. Bhattacharya [1] contributed some useful remarks on fuzzy graphs. Some operations on fuzzy graphs were defined by J. N. Modeson and C. S. Peng [3]. The concept of semigraph was introduced by E. Sampath Kumar [2]. K. Radha [6] introduced the concept of Fuzzy semigraph. Fuzzy semigraphs have applications in road network, railway network and telecommunications. In this paper we have defined effective edge and effective fuzzy semigraphs.

2. Preliminaries

Definition 2.1. A graph G is a pair (V, E) where V is a non empty set of

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points which are called vertices and E is a set of ordered pairs of elements of V which are called edges of G.

Definition 2.2. A simple graph is an undirected graph without self loops and parallel edges.

Definition 2.3. A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge. A graph G is connected if there exists a path between every pair of vertices.

Definition 2.4 [7]**.** Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. Underlying crisp graph is denoted by $G^* : (V, E)$.

Definition 2.5. G is an effective fuzzy graph if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$.

Definition 2.6. G is a complete fuzzy graph if $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

Definition 2.7 [7]**.** If $\mu(xy) > 0$, then x and y are called neighbours, x and y are said to lie on the edge $e = xy$.

Definition 2.8. A path ρ in a fuzzy graph $G : (\sigma, \mu)$ is a sequence of distinct vertices $v_0, v_1, v_2, \ldots, v_n$ such that $\mu(v_{i-1}, v_i) > 0, 1 \leq i \leq n$. Here '$n$' is called the length of the path. The consecutive pairs (v_{i-1}, v_i) are called arcs of the path.

Definition 2.9 [7]**.** The strength of that path is defined as $\overset{n}{\underset{i=1}{\wedge}} \mu(v_{i-1}v_i)$ i.e., it is the Weight of the weakest edge. If u, v are connected by means of paths of length 'k', then $\mu^k(uv) = \sup\{\mu(uv_1)\mu(v_1v_2)\mu(v_2v_3)\ldots\mu(v_{k-1}v)/u, v_1, v_2, \ldots, v_{k-1} \in V, v_{i-1}v_i \in E\}$.

Definition 2.10 [7]**.** The strength of connectedness between u and v is $\mu^\infty(u, v) = \sup\{\mu^k(u, v)/k = 1, 2, 3, \ldots,\}$. A fuzzy graph G is connected if $\mu^\infty(u, v) > 0$ for all $u, v \in V$. An edge xy is said to be a strong edge if

$\mu(x, y) \geq \mu^{\infty}(x, y)$. A node x is said to be an isolated node if $\mu(x, y) = 0, \forall y = X$.

Definition 2.11 [8]**.** A semigraph is a pair (V, X), where V is a non-empty set of elements called vertices and X is a set of n-tuples called edges of distinct vertices for various $n \geq 2$ satisfying the conditions

1. Any two edges have at most one vertex in common

2. Two edges $E_1 = (u_1, u_2, \ldots, u_n), E_2 = (v_1, v_2, \ldots, v_m)$ are considered to be equal if and only if (a) $m = n$ (b) either $u_i = v_i$ for $i = 1$ to n or $u_i = v_{n-i+1}$ for $i = 1$ to n.

In the edge $E = (u_1, u_2, \ldots, u_n), u_1$ and u_n are called called the end vertices and all vertices in between them are called middle vertices. (m-vertices). If a middle vertex is an end vertex of some other edge, it is called middle end vertex.

Definition 2.12 [6]**.** Consider a semigraph $G^* : (V, X)$. A fuzzy semigraph on $G^* : (V, X)$ is defined as (σ, μ, η) where $\sigma : V \to [0, 1]$, $\mu : V \times V \to [0, 1]$, $\eta : X \to [0, 1]$ are such that

(i) $\mu(u, v) \leq \sigma(u) \wedge \sigma(v) \forall (u, v) \in V \times V$

(ii) $\eta(e) = \mu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \ldots \wedge \mu(u_{n-1}, u_n) \leq \sigma(u_1) \wedge \sigma(u_n)$ if $e = (u_1, u_2, \ldots, u_n), n \geq 2$ is an edge in G.

Here (σ, μ) is a fuzzy graph.

Example 2.13 [6]**.** Consider the fuzzy semigraph in Figure 2.1. Here v_1, v_3 and v_5 are end vertices v_2 is a middle vertex v_4 and v_6 are middle-end vertices. Here $E_1 = (v_1, v_6, v_5), E_2 = (v_1, v_2, v_3), E_3 = (v_3, v_4, v_5)$ and $E_4 = (v_4, v_6)$ are the edges of the semigraph.

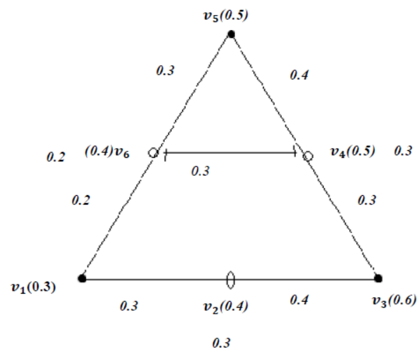

Figure 2.1. Fuzzy semi graph G :(σ, μ, η).

Definition 2.14 [6]**.** The fuzzy semigraph $H = (\gamma, \rho, \delta)$ is called a fuzzy subsemigraph of $G = (\sigma, \mu, \eta)$ if

(i) all the edges of H are subedges of G,

(ii) $\gamma(u) \leq \sigma(u)$ for all $u \in V$,

(iii) $\rho(uv) \leq \mu(uv)$ for all $(u, v) \in V \times V$,

(iv) $\delta(e) \leq \eta(e)$ for all $e \in X$.

Definition 2.15 [6]**.** The fuzzy subsemigraph $H = (\gamma, \rho, \delta)$ is called a spanning fuzzy subsemigraph of the fuzzy semi graph $G = (\sigma, \mu, \eta)$ if $\gamma(u) = \sigma(u)$ for all $u \in V$.

In this case the fuzzy semigraph and its spanning fuzzy semigraph differ in the weights of their edges. Spanning fuzzy sub semi graph $H = (\gamma, \rho, \delta)$ of the fuzzy semigraph $G : (\sigma, \mu, \eta)$ is given in Figure 2.

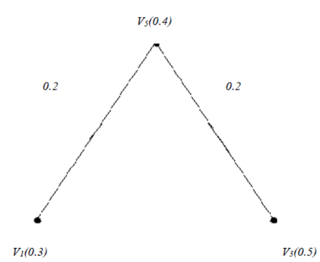

Figure 2.2. Fuzzy subsemigraph of the graph in 2.1.

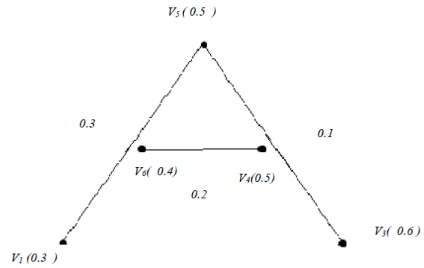

Figure 2.3. Spanning fuzzy subsemigraph of figure 2.1.

Definition 2.16 [6]. A subedge (fs-edge) of an edge $E = (v_1, v_2, v_3, \dots, v_n)$ is a k-tuple $(v_{i_1}, v_{i_2}, \dots, v_{i_k})$ where $1 \le i_1 < i_2 < \dots < i_k \le n$ or $1 \le i_k < i_{k-1} < \dots < i_1 \le n$.

Definition 2.17 [6]. A partial edge (fp-edge) of an edge $E = (v_1, v_2, v_3, \dots, v_n)$ is a $(j - i + n)$- tuple $E = (v_i, v_{i-1}, \dots, v_j)$ where $1 \le i \le n$.

3. Fuzzy Graphs Associated with Semigraphs

In this section, we define three fuzzy graphs associated with given fuzzy semigraph and discuss some of their properties.

Definition 3.1. End Vertex Fuzzy Graph (e-Fuzzy Graph) G_e :

Let $G : (\sigma, \mu, \eta)$ be a fuzzy semigraph $G^*(V, X)$. The fuzzy graph $G_e : (\sigma_e, \eta_e)$ with vertex set V in which two vertices are adjacent if and only if they are end vertices of an edge in G such that $\sigma_e(u) = \sigma(u)$ for every u in V and $\eta_e(uv) = \eta(uv)$ for every pair of end vertices u and v in G is called the end vertex fuzzy graph associated with G.

Definition 3.2. Adjacency Fuzzy Graph (a-Fuzzy Graph) G_a :

The fuzzy graph $G_a : (\sigma_a, \eta_a)$ with vertex set V in which two vertices are adjacent if and only if they are adjacent in G such that $\sigma_e(u) = \sigma(u)$ for every u in V and $\mu_e(uv) = \mu(uv_i) \wedge \mu(v_i v_{i+1}) \wedge, \dots, \wedge \mu(v_j v)$ for every pair of adjacent vertices u and v in G, where $(u, v_i, v_{i+1}, \dots, v_j, v)$ is an edge or a partial edge of G, is called the adjacency vertex fuzzy graph associated with G.

Definition 3.3. Consecutive adjacency fuzzy graph (*ca*-fuzzy graph) G_{ca} :

Let $G : (\sigma, \mu, \eta)$ be a fuzzy semigraph $G^*(V, X)$. The fuzzy graph $G_e : (\sigma_{ca}, \mu_{ca})$ with vertex set V in which two vertices are adjacent if and only if they are consecutively adjacent in G such that $\sigma_{ca}(u) = \sigma(u)$ for every u in V and $\mu_{ca}(uv) = \mu(uv)$ for every pair of consecutive adjacent vertices u and v in G is called the consecutive adjacency vertex fuzzy graph associated with G.

4. Various Concepts in Fuzzy Semigraphs

Definition 4.1. Two vertices in a fuzzy semigraph G are said to be adjacent if they belong to the same edge and are consecutively adjacent if in addition they are consecutive in order as well.

Definition 4.2. Any two edges in a fuzzy semigraph are adjacent if they have a vertex in common.

Definition 4.3. Any two edges in a fuzzy semigraph are said to be

(i) *ee*-adjacent if common vertex of the edges is end vertex in both the edges,

(ii) *em*-adjacent if common vertex of the edges is an end vertex of one edge and middle vertex of the other edge and

(iii) *mm*-adjacent if common vertex of the edges is a middle vertex in both the edges.

Definition 4.4. Cardinality of an edge in a semigraph is said to be k if the edge contains k number of vertices.

Definition 4.5. An edge in a semigraph G is said to be an s-edge if its cardinality $K \geq 3$.

Example 4.6. Consider Figure 2.1.

1. v_1 and v_5 are adjacent vertices since they belong to the same edge E_1, v_1 and v_5 are consecutively adjacent.

2. E_1 and E_3 are *ee*-adjacent. E_1 and E_4 are *em*-adjacent.

3. E_1, e_2 and E_3 have cardinality 3. E_4 has cardinality 2.

5. Effective Edges

Definition 5.1. An effective edge in a fuzzy semigraph.

An edge "*e*" of a fuzzy semigraph is called an effective edge if $\eta(e) = \eta(u_1, u_2, \ldots, u_n) = \sigma(u_1) \wedge \sigma(u_n)$ for $\forall e \in X$ and $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for every $uv \in E$. An edge "*e*" of a fuzzy semigraph is called an *e*-effective edge if $\eta(e) = \eta(u_1, u_2, \ldots, u_n) = \sigma(u_1) \wedge \sigma(u_n)$ for $n > 2$.

Definition 5.2. Effective fuzzy semigraph.

A fuzzy semigraph $G : (\sigma, \mu, \eta)$ is said to be an Effective Fuzzy semigraph if all the edges of G are effective edges. In other words $\eta(e) = \eta(u_1, u_2, \ldots, u_n) = \sigma(u_1) \wedge \sigma(u_n) \forall e \in X$ and $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for every edge $uv \in E$.

An effective fuzzy semigraph is shown in Figure 5.1.

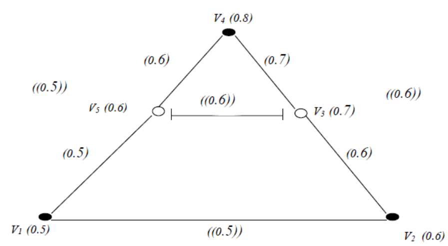

Figure 5.1. Effective semigraph.

Definition 5.3. Fuzzy effective subsemigraph.

A fuzzy subsemigraph H of a fuzzy semigraph $G : (\sigma, \mu, \eta)$ is said to be a fuzzy effective sub semigraph if all its edges are effective edges.

Example 5.4. Consider the fuzzy semigraph in Figure 5.2, the fuzzy subsemigraph of (5.2) is given in the figure 5.3.

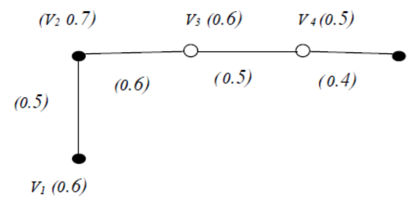

Figure 5.2. Fuzzy semigraph.

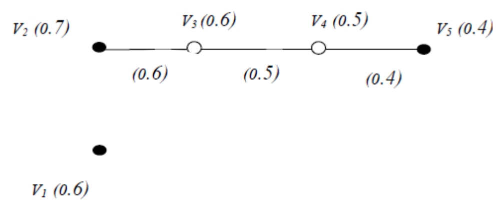

Figure 5.3. Fuzzy effective subsemigraph of the fuzzy semigraph in Figure 5.2.

6. Some Properties

Remark 6.1. Fuzzy sub semigraphs and spanning fuzzy sub semigraphs of an effective fuzzy semigraph need not be effective.

Theorem 6.2. *Induced subsemigraphs of an effective fuzzy semigraph are effective.*

Proof. Since membership values are preserved in induced fuzzy subsemigraphs, induced subsemigraphs of an effective fuzzy semigraph are effective.

Theorem 6.3. *End vertex fuzzy graph of an effective fuzzy semigraph is an effective fuzzy graph.*

Proof. In the end vertex fuzzy graph associated with G, $G_e : (\sigma_e, \eta_e)$ with vertex set V, two vertices are adjacent if and only if they are end vertices of an edge in G such that $\sigma_e(u) = \sigma(u)$ for every u in V and $\mu_e(uv) = \mu(uv)$ for every pair of end vertices u and v in G.

If G is an effective fuzzy semigraph, then $\eta(uv) = \sigma(u) \wedge \sigma(v)$ for every pair of end vertices u and v in G.

Therefore $\eta_e(uv) = \eta(uv) = \sigma(u) \wedge \sigma(v) = \sigma_e(u) \wedge \sigma_e(v)$ for every edge uv in G_e. Hence G_e is an effective fuzzy graph.

Theorem 6.4. *End vertex fuzzy graph of an e-effective fuzzy semigraph is an effective fuzzy graph.*

Proof. If G is an e-effective fuzzy semigraph, then $\eta(uv) = \sigma(u) \wedge \sigma(v)$ for every pair of end vertices u and v in G and hence the theorem follows.

Remark 6.4. Adjacency fuzzy graph (*ea*-Fuzzy Graph) of an e-effective fuzzy semigraph need not be effective.

Remark 6.5. Consecutive adjacency fuzzy graph of an e-effective fuzzy semigraph need not be effective.

Remark 6.6. Adjacency fuzzy graph of an effective fuzzy semigraph need not be effective.

Theorem 6.7. *If G is an effective fuzzy semigraph, then the consecutive adjacency fuzzy graph of G is an effective fuzzy graph.*

Proof. In the consecutive adjacency vertex fuzzy graph associated with G, $G_{ca} : (\sigma_{ca}, \mu_{ca})$ with vertex set V, two vertices are adjacent if and only if they are consecutively adjacent in G.

Also $\sigma_{ca}(u) = \sigma(u)$ for every u in V and $\mu_{ca}(uv) = \mu(uv)$ for every pair of consecutive adjacent vertices u and v in G.

Hence G is effective $\Rightarrow \mu(uv) = \sigma(u) \wedge \sigma(v)$

$$\Rightarrow \mu_{ca}(uv) = \sigma_{ca}(u) \wedge \sigma_{ca}(v)$$

$$\Rightarrow G_{ca} \text{ is effective.}$$

7. Conclusion

In this paper we have introduced the concept of effective fuzzy semigraph. Some properties of effectiveness of the three associated fuzzy graphs of an effective fuzzy semigraphs are also discussed which may be used for future studies and research. Neural networks and transportation networks can be modeled into fuzzy semigraphs and effective fuzzy semigraphs.

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