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GEODETIC PARAMETERS OF SOME SPECIAL GRAPHS

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Abstract

In this paper, we initiate the results on geodetic parameters of some special graphs such as helm graph H_n, dutch windmill graph D_n^k, tadpole graph $T_{n,k}$.

1. Introduction

Throughout this paper all graphs consider here are simple, undirected connected graph. We define $I[u, v]$ to the set of all vertices lying on some $u-v$ geodesic of G and for a nonempty subset S of $V(G)$, $I[S] = \cup_{u, v\in S} I[u, v]$. A set S of vertices of G is called a geodetic set in G if $I[S] = V(G)$ and a geodetic set of minimum cardinality is its geodetic number of G and denoted it by $g(G)$. A vertex v is an extreme vertex of a graph G if the subgraph induced by its neighbours is complete. A vertex v of a graph G is said to be an antipodal vertex of u in G if $d(u, v) = diam(G)$. The wheel W_n is defined to be the join $C_{n-1} + K_1$ where $n > 3$. The vertex corresponding to K_1 is known as the apex vertex and the vertices corresponding to cycle are known as the rim vertex. In this paper we find the split geodetic number of a

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graph was studied by in [11]. A geodetic set S of graph G is a split geodetic set, if the induced subgraph $\langle V(G) - S \rangle$ is disconnected. The split geodetic number $g_s(G)$ of G is the minimum cardinality of a split geodetic set. A strong split geodetic number of a graph was studied in [4]. A geodetic set $S \subseteq V(G)$ is a strong split geodetic set of G, if S is a geodetic set and induced subgraph $\langle V - S \rangle$ is totally disconnected. The strong split geodetic number of G, is denoted by $g_{ss}(G)$, is the minimum cardinality of a strong split geodetic set of G. The non-split geodetic was introduced by Tejaswini and Venkanagouda M Gowda in [9]. A set $S \subseteq V(G)$ is a non-split geodetic set in G, if S is a geodetic set and $\langle V - S \rangle$ is connected, non-split geodetic number $g_{ns}(G)$ of G is the minimum cardinality of a non-split geodetic set of G. A set $S \subseteq V(G)$ is said to be doubly connected geodetic set, if S is a geodetic set and both induced subgraphs $\langle S \rangle$ and $\langle V - S \rangle$ are connected. The minimum order of a doubly connected geodetic set g_{dc}-set of G is called doubly connected geodetic number of a graph G and it is denoted by $g_{dc}(G)$, and it was studied in [5]. A geodetic set S of vertices of G is an independent geodetic set if S is an independent set and $I[S] = V$, is denoted by $ig(G)$, it was introduced in [2]. A set $S \subseteq V(G)$ is a total geodetic set in G, if the subgraph $G[S]$ induced by S has no isolated vertices. The minimum cardinality of a total geodetic set is the total geodetic number $g_t(G)$, is discussed in [1]. A geodetic set $S \subseteq V(G)$ of a graph G is a restrained geodetic set, if the subgraph $G[V \setminus S]$ has no isolated vertex. The minimum cardinality of a restrained geodetic set, denoted $g_r(G)$. A $g_r(G)$-set is a restrained geodetic set of cardinality $g_r(G)$, and it was formalized by Abdollahadeh Ahangar, Samodivkin, Sheikholeslami and Abdollah Khodkar in [3]. A geodetic set $S \subseteq V(G)$ is said to be total outer-independent geodetic set if $\langle S \rangle$ has no isolated vertex and $\langle V - S \rangle$ is an independent set. The minimum order of a total outer-independent geodetic set g_{toi}-set of G is called the total outer-independent geodetic number of G and it is denoted by $g_{toi}(G)$, it was introduced in [10] of some special graph respectively.

We follow the notations, standard terminology and undefined terms related to the graph theory we refer [6], [7] and [8].

Definition 1.1. The helm graph is the graph obtained from an wheel graph by adjoining a pendant edge at each vertex of the cycle and it is denoted by H_n is as shown in figure.

Definition 1.2. The tadpole graph, also called a dragon graph, is the graph obtained by joining a cycle graph C_n to a path P_n with a bridge and it is denoted by $T_{n,\, k}$.

Definition 1.3. The dutch windmill graph is the graph obtained by taking k copies of the cycle C_n with a vertex in common and it is denoted by D_n^k.

Lemma 1.4. *Every geodetic set of a graph contains its extreme vertices*.

2. Geodetic Parameters on Helm Graph

Theorem 2.1. *For any helm graph H_n of order $n \geq 3$, $g(H_n) = n$.*

Proof. It is easy to verify that the pendant vertices of H_n forms a geodetic set. Hence, $g(H_n) = n$.

Illustration 2.2. Consider the helm graph H_6 as shown in figure 1. The empty vertices is its geodetic set.

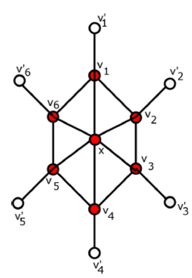

Figure 1. H_6.

Let $V(H_6) = \{v_1, v_2, \ldots, v_6, v'_1, v'_{2,} \ldots, v'_6, x\}$, the geodetic set $S = (v'_1, v'_2, \ldots, v'_6\}$. Thus, $g(H_6) = 6$.

Corollary 2.3. *For helm graph H_n of order $n \geq 3$, $g_{ns}(H_n) = n$.*

Corollary 2.4. *For helm graph H_n of order $n \geq 3$, $g_r(H_n) = n$.*

Theorem 2.5. *If* H_n *is the helm graph of order* $n \geq 3$, *then* $g_s(H_n) = \Delta(H_n) + d - 1$, *where d is the diameter.*

Proof. Let $V(H_n) = \{v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_n', x\}$ with $\deg(x) = \Delta(H_n) = n$ and diameter $d = 4$. We have by Theorem 2.1, $S = \{v_1', v_2', \ldots, v_n'\}$ be the g-set such that $\langle V(H_n) - S \rangle$ is connected. Let us consider the set $S' = \{x, v_i, v_j\}$, where $\deg(v_i) = 3 = \deg(v_j)$ and $v_1, v_j \notin E(H_n)$, then $S_1 = S \cup S'$ be the g_s-set which makes $\langle V(H_n) - S_1 \rangle$ disconnected. Clearly, S_1 is a g_s-set. Therefore, $g_s(H_n) = | S_1 | = \Delta(H_n) + d - 1$.

Theorem 2.6. *For the helm graph* H_n, $n \geq 4$,

$$g_{ss}(H_n) = \begin{cases} \dfrac{3n+2}{2}, & \text{if } n \text{ is even} \\ \dfrac{3(n+1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let $V(H_n) = \{v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_n', x\}$, $\deg(v_i') = 1, \deg(v_i) = 3$ for all $i \in \{1, 2, 3, \ldots, n\}$. By Corollary 2.3, $S = \{v_i', v_2', \ldots, v_n'\}$ be the g_{ns}-set and $\langle V(H_n) - S \rangle$ is connected. Let $S \cup S'$ forms a g_{ss}-set, where S' is defined by the following cases.

Case (i). Suppose n is even. Consider $S' = \{x\} \cup \{v_2, v_4, v_6, \ldots, v_n\}$ be the vertex set containing rim vertex and v_{2j} be the non-adjacent vertices, then $S \cup S'$ which makes $\langle V(H_n) - S \rangle$ totally disconnected. Therefore, $g_{ss}(H_n) = | S \cup S' | = n + 1 + \frac{n}{2} = \frac{3n+2}{2}$.

Case (ii). Suppose n is odd. Consider $S' = \{x\} \cup \{v_2, v_4, v_6, \ldots, v_{n-1}\} \cup \{v_n\}$, then $S \cup S'$ which makes $\langle V(H_n) - S \rangle$ totally disconnected. Therefore, $g_{ss}(H_n) = | S \cup S' | = n + 1 + \frac{n+1}{2} = \frac{3(n+1)}{2}$.

Theorem 2.7. *If* H_n *is the helm graph of order* $n \geq 3$, *then* $g_t(H_n) = n + | N(x) |$, *where* $N(x)$ *are the neighborhood vertices of apex vertex x.*

Proof. Let $V(H_n) = \{v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_n', x\}$ and by Theorem 2.1, $S = \{v_1', v_2', \ldots, v_n'\}$ be the geodetic set such that $\langle S \rangle$ has isolated vertex.

Consider $S' = \{v_1', v_2', \ldots, v_n'\} = N(x)$ and $\{v_i, v_i'\} \in E(H_n)$. Then $S_1 = S \cup S'$ which forms $\langle S_1 \rangle$ has no isolated vertex. Clearly, S_1 is a g_t-set. Therefore, $g_t(H_n) = |S_1| = |S| = |S'| = n + |N(x)|$.

Corollary 2.8. *For any helm graph* H_n, $n \geq 3$, $g_c(H_n) = g_t(H_n)$.

Corollary 2.9. *For any helm graph* H_n, $n \geq 3$, $g_{dc}(H_n) = g_t(H_n)$.

Theorem 2.10. *For helm graph* H_n *of order* $n \geq 3$, $g_r(H_n) = g_{ns}(H_n) = n$.

Proof. We have from Theorem 2.1, $S = \{v_1', v_2', \ldots, v_n'\}$ be the geodetic set of H_n and $\langle V(H_n) - S \rangle$ has no isolated vertex, which is connected. Hence, the set S itself satisfy the condition of both restrained geodetic set and nonsplit geodetic set. Therefore, $g_r(H_n) = g_{ns}(H_n) = |S| = n$.

3. Geodetic Parameters on Tadpole Graph

Theorem 3.1. *For the tadpole graph* $T_{n,k}$, $n \geq 3$,

$$g(T_{n,k}) = \begin{cases} 2, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let $V(T_{n,k}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_k\}$. To prove this result we have two cases.

Case (i). Suppose n is even. Consider $S = \{v_i, u_k\}$, where $d\{v_i, u_k\} = \operatorname{diam}(T_{n,k})$, $\deg(v_i) = 2$ and $\deg(u_k) = 1$. Clearly, $I[S] = V[T_{n,k}]$. Thus, S is the g-set. Therefore, $g(T_{n,k}) = |S| = 2$.

Case (ii). Suppose n is odd. Consider $S = \{v_i, v_j, u_k\}$, where $\deg(v_i) = 2 = \deg(v_j)$, $\{v_i, v_j\} \in E(T_{n,k})$, $\deg(u_k) = 1$, $d(v_i, u_k) = \operatorname{diam}(T_{n,k}) = d(v_j, u_k)$ such that $I(S) = V[T_{n,k}]$. Clearly, S is the g-set. Therefore, $g(T_{n,k}) = |S| = 3$.

Illustration 3.2. The tadpole graph $T_{3,3}$ as shown in figure 2

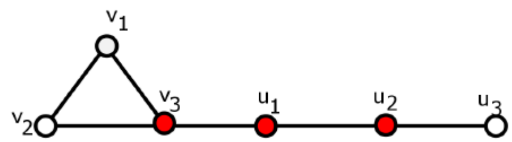

Figure 2. $T_{3,3}$.

Let $V(T_{3,3}) = \{v_1, v_2, v_3, u_1, u_2, u_3\}$. The geodetic set $S = \{v_1, v_2, u_3\}$ and if we remove S from $V(T_{3,3})$, then $\langle V(T_{3,3}) - S\rangle$ will be connected. Hence, S itself be the g-set and g_{ns}-set. Therefore, $g(T_{3,3}) = g_{ns}(T_{3,3}) = 3$.

Corollary 3.3. *For the tadpole graph* $T_{n,k}$, $n \geq 3$, $g_{ns}(T_{n,k}) = g(T_{n,k})$.

Corollary 3.4. *For the tadpole graph* $T_{n,k}$, $n \geq 3$, $g_r(T_{n,k}) = g(T_{n,k})$.

Theorem 3.5. *For the tadpole graph* $T_{n,k}$, $n \geq 3$, $g_s(T_{n,k}) = g(T_{n,k}) + 1$.

Proof. Let $V(T_{n,k}) = \{\{\bigcup_{i=1}^{n} v_i\} \cup \{\bigcup_{i=1}^{k} u_k\}\}$. By Theorem 3.1,

$$S = \begin{cases} \{v_i, u_k\} & \text{if } n \text{ is even} \\ \{v_i, v_j, u_k\} & \text{if } n \text{ is odd} \end{cases}$$

be the g-set and $\langle V(T_{n,k}) - S\rangle$ is connected. Consider $S' = \{x\}$ be any arbitrary vertex which is not adjacent to vertex set in S, then the set $S_1 = S \cup S'$ forms a g_s-set of $T_{n,k}$ and $\langle V(T_{n,k}) - S_1\rangle$ is disconnected. Therefore, $g_s(T_{n,k}) = |S_1| = |S + S'| = g(T_{n,k}) + 1$.

Theorem 3.6. *Let* $T_{n,k}$ *be the tadpole graph of order* $n \geq 4$, $g_{tr}(T_{n,k}) = 4$.

Proof. Let $V(T_{n,k}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_k\}$. To prove this result we have two cases.

Case (i). If n is even, then $S = \{v_i, u_k\}$ be the g-set such that $\langle S\rangle$ has isolated vertex. Let $S' = \{v_i, u_{k-1}\}$, where $v_j, v_i \in E(T_{n,k})$ and u_{k-1}, $u_k \in E(T_{n,k})$. Consider $S_1 = S \cup S'$. Clearly, $\langle S_1\rangle$ has no isolated vertex. By Theorem 3.1, $|S| = 2$. Therefore, $g_t(T_{n,k}) = |S_1| = 4$.

Case (ii). If n is odd, then $S = \{v_i, v_j, u_k\}$ be the g-set such that $\langle S \rangle$ has isolated vertex. Consider $S' = \{u_{k-1}\}$, where $\{u_{k-1}, u_k\} \in E(T_{n,k})$ and $S_1 = S \cup S'$ which makes $\langle S_1 \rangle$ has no isolated vertex. By Theorem 3.1, $|S| = 3$. Therefore, $g_t(T_{n,k}) = |S_1| = 4$.

Corollary 3.7. *Let $T_{n,k}$ be the tadpole graph of order $n \geq 4$, $g_{tr}(T_{n,k}) = 4$.*

Proof. Since every g_t-set of $T_{n,k}$ is the g_{tr}-set, we have by Theorem 3.6, $g_{tr}(T_{n,k}) = 4$.

Theorem 3.8. *For the tadpole graph $T_{n,k}$ with $n \geq 4$,*

$$g_c(T_{n,k}) = \begin{cases} \frac{n}{2} + 1 + k, & \text{if } n \text{ is even} \\ \frac{n+1}{2} + 1 + k, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let $V(T_{n,k}) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_k\}$. To prove this result we have two cases.

Case (i). For n is even, let $S = \{v_i, u_k\}$ be the g-set such that $\langle S \rangle$ is not connected. Consider $S_1 = S \cup \{\bigcup_{l=1}^{\frac{n}{2}} v_l\} \cup \{\bigcup_{j=1}^{k-1} u_j\}$, where v_l and u_j are the adjacent vertices. So that $\langle S_1 \rangle$ is connected. Clearly, S_1 forms g_c-set. Hence, $g_c(T_{n,k}) = |S_1| = |S + \{\bigcup_{l=1}^{\frac{n}{2}} v_i\} + \{\bigcup_{j=1}^{k-1} u_j\}| = \frac{n}{2} + 1 + k.$

Case (ii). For n is odd, let $S = \{v_i, v_j, u_k\}$ be the g-set such that $\langle S \rangle$ is not connected. Consider $S_1 = S \cup \{\bigcup_{l=1}^{\frac{n-1}{2}} v_l\} \cup \{\bigcup_{j=1}^{k-1} u_j\}$, where v_l and u_j are the adjacent vertices. So that $\langle S_1 \rangle$ is connected. Clearly, S_1 forms g_c-set. Hence, $g_c(T_{n,k}) = |S_1| = |S + \{\bigcup_{l=1}^{\frac{n-1}{2}} v_l\} + \{\bigcup_{j=1}^{k-1} u_j\}| = \frac{n+1}{2} + 1 + k.$

Corollary 3.9. *For the tadpole graph $T_{n,k}$ with $n \geq 4$, $g_c(T_{n,k}) = g_{dc}(T_{n,k})$.*

4. Geodetic Parameter on Dutch Windmill Graph

Theorem 4.1. *Let d_n^k be the dutch windmill graph of order $n \geq 4$, then*

$$g(D_n^k) = \begin{cases} k, & \text{for } n \equiv 0(\bmod 2) \\ 2k, & \text{for } n \equiv 1(\bmod 2). \end{cases}$$

Proof. Let $V(D_n^k) = \{\{\cup_{i=1}^{n-1} v_{1i}\} \cup \{\cup_{i=1}^{n-1} u_{2i}\}, \ldots, \cup \{\cup_{i=1}^{n-1} v_{ki}\}\}$, where v_{1n} a common vertex is consider the following cases.

Case (i). For n is even, consider $S = \{v_{1j}, v_{2j}, \ldots, u_{kj}\}$ such that $d(v_{1j}, v_{2j}) = d(v_{2j}, v_{3j}) = \cdots = d(u_{k-1j}, v_{kj}) = \operatorname{diam}(D_n^k)$. Clearly, $I[S] = V(D_n^k)$. Thus, S is a g-set. Therefore, $g(D_n^k) = |S| = k$.

Case (ii). For n is odd, consider $S = \{v_{1j}, v_{1q}, v_{2j}, v_{2q}, \ldots, u_{kj}, u_{kq}\}$ be the vertex set with $d((v_i)_j, (v_l)_j) = d((v_i)_q, (v_l)_q) = \operatorname{diam}(D_n^k), (v_i)_j, (v_i)_q \in E(D_n^k)$. Clearly, $I[S] = V(D_n^k)$. Thus, S is a g-set. Therefore, $g(D_n^k) = |S| = 2k$.

Corollary 4.2. *For the dutch windmill graph D_n^k of order $n \geq 4$, $g_r(D_n^k) = g(D_n^k)$.*

Corollary 4.3. *For the dutch windmill graph D_n^k of order $n \geq 4$, $g_{ns}(D_n^k) = g(D_n^k)$.*

Illustration 4.4. The dutch windmill graph D_6^3 as shown in figure 3

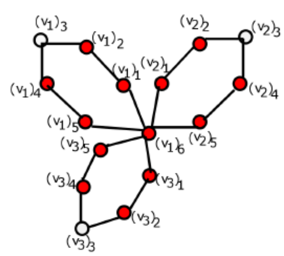

Figure 3. D_6^3.

In this D_6^3 graph having vertex set $V(D_6^3) = \{(v_1)_1, (v_1)_2, \ldots, (v_1)_6, (v_2)_1, (v_2)_2, \ldots, (v_2)_5, (v_3)_2, \ldots, (v_3)_5\}$ and it has a subset $S = \{(v_1)_3, (v_2)_3, (v_3)_3\}$ vertices. Every vertex in S is antipodal. Therefore, S is geodetic set. Clearly, $g(D_6^3) = 3$.

Theorem 4.5. *Let D_n^k be the dutch windmill graph of order $n \geq 4$, then*

$$g_s(D_n^k) = \begin{cases} k+1, & \text{for } n \text{ is even} \\ 2k+1, & \text{for } n \text{ is odd.} \end{cases}$$

Proof. Consider the graph D_n^k, we have the following cases.

Case (i). If n is even, then $S = \{v_{1j}, v_{2j}, \ldots, u_{kj}\}$ be the g-set such that $\langle V(D_n^k) - S \rangle$ is connected. Let $S' = \{v1n\}$, where $v1n$ be any arbitrary vertex in $V(D_n^k)$ not adjacent to vertices in S. Clearly, $S_1 = S \cup S'$ which makes $\langle V(D_n^k) - S \rangle$ disconnected. Therefore, S is a g_s-set. Hence, $g_s(D_n^k) = k + 1$.

Case (ii). If n is odd, then $S = \{v_{1j}, v_{1q}, v_{2j}, v_{2q}, \ldots, v_{kj}, v_{kq}\}$ be the g-set such that $\langle V(D_n^k) - S \rangle$ is connected. Let $S' = \{v1n\}$, where $v1n$ be the arbitrary vertex in $V(D_n^k)$ not adjacent to S. Hence, $S_1 = S \cup S'$ which makes $\langle V(D_n^k) - S \rangle$ disconnected. Therefore, S is a g_s-set. Clearly, $g_s(D_n^k) = k + 1$.

Theorem 4.6. *For the dutch windmill graph D_n^k of order $n \geq 4$, $g_t(D_n^k) = 2k$.*

Proof. Consider the graph D_n^k, we have the following cases.

Case (i). If n is even, then $S = \{v_{1j}, v_{2j}, \ldots, v_{kj}\}$ be the g-set such that $\langle S \rangle$ has isolated vertex. Consider $S' = \{v_{1j-1}, v_{2j-1}, \ldots, u_{kj-1}\} = \{v_{1j+1}, v_{2j+1}, \ldots, u_{kj+1}\}$ be the vertex set such that v_{ij} is adjacent to v_{ij-1} or v_{ij+1}. Consider $S_1 = S \cup S'$ forms a g_t-set. Clearly, $\langle S_1 \rangle$ has no isolated vertex. By Theorem 4.1, $|S| = k$. Hence, $|S_1| = |S \cup S'| = 2k$.

Case (ii). If n is odd, then $S = \{v_{1j}, v_{1q}, v_{2j}, v_{2q}, \ldots, v_{kj}, v_{kq}\}$ be the g-set and $\langle S \rangle$ has no isolated vertex. Hence, S itself forms a g_t-set. By Theorem 4.1, $|S| = k$. Hence, $g_t(D_n^k) = 2k$.

Corollary 4.7. *For the dutch windmill graph* D_n^k *of order* $n \geq 4$, $g_{tr}(D_n^k) = g_t(D_n^k)$.

Theorem 4.8. *If* D_n^k *is the dutch windmill graph of order* $n \geq 4$, *then*

$$g_c(D_n^k) = \begin{cases} \frac{kn}{2} + 1, & \text{for } n \text{ is even} \\ \frac{k(n+1)}{2} + 1, & \text{for } n \text{ is odd.} \end{cases}$$

Proof. Let $V(D_n^k) = \{\{\cup_{i=1}^{n} v_{1i}\} \cup \{\cup_{i=1}^{n-1} u_{2i}\}, \ldots, \cup \{\cup_{i=1}^{n-1} v_{ki}\}\}$, and $|V(D_n^k)| = k(n-1)+1$. To prove these results we have following cases.

Case (i). For n is even, let $S = \{v_{1j}, v_{2j}, \ldots, u_{kj}\}$ be the g-set such that $\langle S \rangle$ has isolated vertex. Consider $S_1 = S \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{1j}\} \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{2j}\} \cup \ldots \cup \{ \cup_{i=1}^{\frac{n-2}{2}} v_{ki}\} \cup \{v_{in}\}$, where v_{1n} is a common vertex, which forms a g_c-set. By Theorem 4.1, $|S| = k$. Clearly, $g_c(D_n^k) = |S| + |\{\cup_{i=1}^{\frac{n-2}{2}} v_{1j}\} \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{2j}\} \cup \ldots \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{ki}\} \cup \{v_{in}\}| = k + k\left(\frac{n-2}{2}\right) + 1 = \frac{kn}{2} + 1$.

Case (ii). For n is odd, let $S = \{v_{1j}, v_{1q}, v_{2j}, v_{2q}, \ldots, v_{kj}, v_{kq}\}$ be the g-set such that $\langle S \rangle$ is disconnected. Consider $S_1 = S \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{1j}\} \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{2j}\} \cup \ldots \cup \{\cup_{i=1}^{\frac{n-2}{2}} v_{ki}\} \cup \{v_{in}\}$, where v_{1n} is a common vertex, which forms a g_c-set. By Theorem 4.1, $|S| = 2k$. Clearly, $g_c(D_n^k) = |S| + |\{\cup_{i=1}^{\frac{n-3}{2}} v_{1j}\} \cup \{\cup_{i=1}^{\frac{n-3}{2}} v_{2j}\} \cup \ldots \cup \{\cup_{i=1}^{\frac{n-3}{2}} v_{ki}\} \cup \{v_{in}\}| = 2k + k\left(\frac{n-3}{2}\right) + 1 = k\left(\frac{n+1}{2}\right) + 1$.

Theorem 4.9. *If* d_n^k *is the dutch windmill graph of order* $n \geq 4$, *then*

$$g_c(D_n^k) = \begin{cases} \dfrac{(2k-1)n - 2k + 4}{2}, & \textit{for } n \textit{ is even} \\ \dfrac{(2k+1)n - 2k + 3}{2}, & \textit{for } n \textit{ is odd.} \end{cases}$$

Proof. Let $V(D_n^k) = \{\{\bigcup_{i=1}^{n} v_{1i}\} \cup \{\bigcup_{i=1}^{n-1} u_{2i}\}, \ldots, \cup \{\bigcup_{i=1}^{n-1} v_{ki}\}\}$, here v_{1n} is a common vertex and $|V(D_n^k)| = k(n-1)+1$. To prove this result we have following cases.

Case (i). For n is even, consider $A = \{v_{i1}, v_{i2}, \ldots, v_{i((n-2)/2)}\}$, be the adjacent vertices of C_1 for fixed i. Let $S = V(D_n^k) - A$ be the g_{dc}-set with his $\langle S \rangle$ and $\langle V(D_n^k) - S \rangle$ are connected. Clearly, $g_{dc}(D_n^k) = |S|$ $= |V(D_n^k) - A| = \dfrac{(2k-1)n - 2k + 4}{2}$.

Case (ii). For n is odd, consider $A = \{v_{i1}, v_{i2}, \ldots, v_{i((n-3)/2)}\}$, where $i \in \{1, 2, \ldots, k\}$, be the adjacent vertices of C_1 for fixed i. Let $S = V(D_n^k) - A$ be the g_{dc}-set. Clearly, $g_{dc}(D_n^k) = |S| = |V(D_n^k) - A| = \dfrac{(2k-1)n - 2k + 3}{2}$.

5. Conclusion

In this paper, we established geodetic number, split geodetic number, non-split geodetic number, restrained geodetic number, connected geodetic number, doubly connected geodetic number, total geodetic number and total outer independent geodetic number of some special graphs.

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