Advances and Applications in Mathematical Sciences
Volume 21, Issue 1, November 2021, Pages 399-404

DUAL OF FUZZY MODULAR LATTICE ORDERED *M*-GROUP

D. VIDYADEVI[1] and S. MEENAKSHI[2]

[1]Research Scholar
[2]Associate Professor
Department of Mathematics
Vels Institute of Science, Technology
and Advanced Studies
Chennai 600117, Tamilnadu, India
E-mail: vidyadayalan04@gmail.com
meenakshikarthikeyan8@gmail.com

Abstract

The idea of dual of fuzzy modular lattice ordered m-group is introduced in this article, and some of its properties are explored. We also examine at the homomorphic image, pre mage and direct product of the dual of fuzzy modular lattice ordered m-group, as well as some associated lattice properties.

1. Introduction

The set of all fuzzy normal subgroups of a group is a sub lattice of a lattice of all fuzzy subgroups of a given group and is modular, according to Ajmal [1]. Using the concept of fuzzy partial ordering, Nanda [5] created the concept of fuzzy lattice. L-fuzzy lattice ordered groups and sub l-groups were first studied by G. S. V. Satya Saibaba [2]. The fuzzy lattice ordered m-group was presented by M. U. Makandar and A. D. Lokhande. In an attempt to make a generalised study of fuzzy set theory by investigating L-fuzzy sets, J. A. Goguen [3] replaced the valuationi set [0, 1] with whole lattice. Dr. M. Madurai and V. Rajendran [4] changed the definition of fuzzy lattice and proposed the concept of fuzzy lattice of groups, as well as analyzing some of

2020 Mathematics Subject Classification: 06D72, 06C05, 06F15.
Keywords: Dual fuzzy m-group, dual fuzzy lattice ordered m-group, dual fuzzy modular lattice ordered m-group, pre-image and direct product.
Received May 17, 2021; Accepted June 7, 2021

its main features. Dr. SG. Karpagavalli and D. Vidyadevi [6] have extended to fuzzy modular lattice in ordered groups and also investigated some of its basic properties. This article introduces the concept of a dual of fuzzy modular lattice ordered m-group and investigates some of its features. Also examine at the homomorphic image and preimage of the dual of a fuzzy modular lattice ordered m-group, as well as some related lattice features.

2. Preliminaries

Definition 2.1[6]**.** Let G be a group, M be any set if,

(i) $mx \in G$.

(ii) $m(xy) = (mx)y = xmy$, for all $x, y \in G, m \in M$. Then G is called m group.

Definition 2.2. Let (X, μ_1) be a fuzzy set and G is a m-group G. A fuzzy set on $G, G \in \wp(X)$ is called a dual of fuzzy m-group if,

(i) $\mu(m(xy)) \leq \max\{\mu(mx), \mu(my)\}$

(ii) $\mu(mx^{-1}) \leq (mx)$, for all $x, y \in G, m \in M$.

Definition 2.3. Let $(X, \mu_1), G \in \wp(X), M \subset X$. A function μ on G is said to be a dual of fuzzy lattice ordered m-group if,

(i) $(G, \bullet)$ is a m-group.

(ii) $(G, \bullet, \leq)$ is a lattice ordered group.

(iii) $\mu(m(xy)) \leq \max\{\mu(mx), \mu(my)\}$

(iv) $\mu((mx)^{-1}) \leq \mu(mx)$

(v) $\mu(mx \vee my) \leq \max\{\mu(mx), \mu(my)\}$

(vi) $\mu(mx \wedge my) \leq \max\{\mu(mx), \mu(my)\}$, for all $mx, my \in G$.

4. Properties of Dual of Fuzzy Modular Lattice Ordered *m*-Group

Proposition 4.1. *Let G and G' be two dual of fuzzy modular lattice ordered m-groups and* $\theta : G \rightarrow G'$ *be a m-homomorphism defined by*

$\theta(mx) = m\theta(x)$. *If B is a dual of fuzzy modular lattice ordered m-group of G'* *then the pre-image* $\theta^{-1}(B)$ *is a dual of fuzzy modular lattice ordered m-group of G.*

Proof of Proposition 4.1. Assume B is a dual of fuzzy modular lattice ordered m-group of G'.

Let $mx, my, mz \in G$

(i) $\mu^{-1}_{\theta(B)}(m(xy)) = \mu_B\theta(m(xy)) = \mu_B(m\,\theta(xy)) = \mu_B(m\theta(x)\theta(y))$

$$\leq \max\{\mu_B(m\theta(x)), \mu_B(m\theta(y))\} \leq \max\{\mu_B(\theta(mx), \mu_B(\theta(my)))\}$$

$$\leq \max\{\mu^{-1}_{\theta(B)}(mx), \mu^{-1}_{\theta(B)}(my)\}$$

(ii) $\mu^{-1}_{\theta(B)}(mx)^{-1}) = \mu_B\,\theta((mx)^{-1}) = \mu_B(\theta(mx))^{-1}\mu_B\,(m\theta(x)^{-1} \leq \mu_B(m\theta(x))$

$$\leq \mu_B(\theta(mx)) \leq \mu^{-1}_{\theta(B)}(mx)$$

(iii) $\mu^{-1}_{\theta(B)}(mx \vee my) = \mu_B\theta(mx \vee my) = \mu_B\theta(mx) \vee \theta(my)$

$$\leq \max\{\mu_B\,\theta(mx), \mu_B\,\theta(my)\} \leq \max\{\mu^{-1}_{\theta(B)}(mx), \mu^{-1}_{\theta(B)}(my)\}$$

(iv) $\mu^{-1}_{\theta(B)}(mx \wedge my) = \mu_B\,\theta(mx \wedge my) = \mu_B\,\theta(mx) \wedge \mu_B\,\theta(my)$

$$\leq \max\{\mu_B\,\theta(mx), \mu_B\,\theta(my)\} \leq \max\{\mu^{-1}_{\theta(B)}(mx), \mu^{-1}_{\theta(B)}(my)\}$$

(v) $\mu^{-1}_{\theta(B)}(mx \wedge my) \vee \mu^{-1}_{\theta(B)}(mx \wedge my) = \mu_B\,\theta(mx \wedge my) \vee \mu_B\,\theta(mx \wedge mz)$

$\leq \max\{\mu_B\,\theta(mx), \mu_B\,\theta(my), \mu_B\theta(mx) \wedge \mu_B\theta(mz)\}$

$\leq \max\{\mu_B\,\theta(mx), \mu_B\,\theta(my) \vee \mu_B\,\theta(mx \wedge mz)\}$

$\leq \max\{\mu^{-1}_{\theta(B)}\theta(mx), \mu^{-1}_{\theta(B)}(my) \vee \mu^{-1}_{\theta(B)}(mx \wedge mz)\}$

Therefore $\theta^{-1}(B)$ is a dual of fuzzy modular lattice ordered m-group of G.

Proposition 4.2. *If* $\{A_i\}$ *is a family of dual of fuzzy modular lattice ordered* m-group *of G, then* $\cap A_i$ *is a dual of fuzzy modular lattice ordered m-group of G where* $\cap A_i = \{mx, \wedge \mu_{Ai}(mx)/mx \in G\}$.

Proof of Proposition 4.2. Let $mx, my, mz \in G$

(i) $(\bigcap \mu_{Ai})m(xy) = \wedge\mu_{Ai}m(xy) = \wedge\mu_{Ai}(mx\, my) \leq \wedge \max\{\mu_{Ai}(mx), \mu_{Ai}(my)\}$

$$\leq \max\{(\bigcap \mu_{Ai})(mx), (\bigcap \mu_{Ai})(my)\}$$

(ii) $(\bigcap \mu_{Ai})(mx)^{-1} = \wedge \bigcap \mu_{Ai}(mx)^{-1} \leq \wedge\mu_{Ai}(mx) \leq (\bigcap \mu_{Ai})(mx)$

(iii) $(\bigcap \mu_{Ai})(mx \vee my) = \wedge\mu_{Ai}(mx \vee my) \leq \wedge \max\{\mu_{Ai}(mx), \mu_{Ai}(my)\}$

$$\leq \max\{\bigcap \mu_{Ai}(mx), \bigcap \mu_{Ai}(my)\}$$

(iv) $(\bigcap \mu_{Ai})(mx \vee my) = \wedge \mu_{Ai}(mx \wedge my) \leq \wedge \max\{\mu_{Ai}(mx), \mu_{Ai}(my)\}$

$$\leq \max\{\bigcap \mu_{Ai}(mx), \bigcap \mu_{Ai}(my)\}$$

(v) $(\bigcap \mu_{Ai})(mx \wedge my) \vee (\bigcap \mu_{Ai})(mx \wedge my) \leq \wedge(\wedge \mu_{Ai})(mx \wedge my)$

$\vee(\wedge \mu_{Ai})(mx \wedge mz)$

$$\leq \wedge \max\{\mu_{Ai}(mx), \min\{\mu_{Ai}(my), \mu_{Ai}(mx \wedge mz)\}$$

Proposition 4.3. *If A is a fuzzy set in G and* θ *is a m-homomorphism of G, then the fuzzy set* $A^{\theta} = \{\langle mx; \mu_A^{\theta}(mx)\rangle, mx \in G\}$ *is a dual of fuzzy modular lattice ordered m-group*.

Proof of Proposition 4.3. Let $mx, my, mz \in G$

(i) $\mu_A^{\theta}(m(xy)) = \mu_A\theta(m(xy)) = \mu_A m\theta(xy) = \mu_A^{\theta} m(\theta(x)\theta(y))$

$$\leq \max\{\mu_A m\,\theta(x), \mu_A m\theta(y)\} \leq \max\{\mu_A\theta(mx), \mu_A\theta(my)\}$$

$$\leq \max\{\mu_A^{\theta}(mx), \mu_A^{\theta}(my)\}$$

(ii) $\mu_A^{\theta}(mx)^{-1} = \mu_A\theta(mx)^{-1} = \mu_A(\theta(mx))^{-1} = \mu_A(m\theta(x))^{-1}$

$$\leq \mu_A\theta(mx) \leq \mu_A^{\theta}(mx)$$

(iii) $\leq \mu_A^{\theta}(mv \vee my) = \mu_A\theta(mx \vee my) = \mu_A\theta(mx) \vee \theta(my)$

$$\leq \max\{\mu_A\theta(mx), \mu_A\theta(my) \leq \max\{\mu_A^{\theta}(mx), \mu_A^{\theta}(my)\}$$

(iv) $\mu_A^{\theta}(mx \wedge my) = \mu_A\theta(mx \wedge my) = \mu_A\theta(mx) \wedge \theta(mx)$

$$\leq \max\{\mu_A\theta(mx), \mu_A\theta(my)\} \leq \max\{\mu_A^{\theta}(mx), \mu_A^{\theta}(my)\}$$

(v) $\mu_A^{\theta}(mx \wedge my) \vee \mu_A^{\theta}(mx \wedge my) = \mu_A\theta(mx \wedge my) \vee \mu_A\theta(mx \wedge mz)$

$= (\mu_A\theta(mx) \wedge \theta(my)) \vee (\mu_A\theta(mx) \wedge \theta(mz))$

$\leq \max\{\mu_A\theta(mx), \mu_A\theta(my) \vee \mu_A\theta(mx \wedge mz)\}$

$\leq \max\{\mu_A\theta(mx), \mu_A\theta(my) \vee \mu_A\theta(mx \wedge mz)\}$

Therefore A^{θ} is a dual of fuzzy modular lattice ordered m-group of G.

Proposition 4.4. *Let $f: G \to G'$ be a lattice group m-homomorphism and A is a fuzzy set in dual of fuzzy modular lattice ordered m-group of G' then $f^{-1}(A)$ is a dual of fuzzy modular lattice ordered m-group of G.*

Proof of Proposition 4.4. Let $mx, my, mz \in G$ and A be a dual of fuzzy modular lattice ordered m-group of G'.

(i) $f^{-1}(A)(m(xy)) = Af(m(xy)) = A(f(mx)f(my)) = A(mf(x)mf(y))$

$$= A(mf(x)f(y)) \leq \max\{A(mf(x)), A(mf(y))\}$$

$$\leq \max\{A(f(mx)), A(f(my))\} \leq \max\{f^{-1}(A)(mx), f^{-1}(A)(my)\}$$

(ii) $f^{-1}(A)((mx)^{-1}) = Af((mx)^{-1}) = A(f(mx)^{-1}) = A(mf(x)^{-1} \leq A(mf(x))$

$$\leq A(f(mx)) \leq f^{-1}(A)(mx)$$

(iii) $f^{-1}(A)(mx \vee my) = Af(mx \vee my) = A(f(mx) \vee f(my))$

$$= A(mf(x) \vee mf(y))$$

$$\leq \max\{A(mf(x)), A(mf(y))\} \leq \max\{A(f(mx)), A(f(my))\}$$

$$\leq \max\{f^{-1}(A)(mx), f^{-1}(A)(my)\}$$

(iv) $f^{-1}(A)(mx \wedge my) = Af(mx \vee my) = A(f(mx) \wedge f(my))$

$$= A(mf(x) \wedge mf(y))$$

$$\leq \max\{A(mf(x)), A(mf(y))\} \leq \max\{A(f(mx)), A(f(my))\}$$

$$\leq \max\{f^{-1}(A)(mx), f^{-1}(A)(my)\}$$

(v) $f^{-1}(A)(mx \wedge my) \vee f^{-1}(A)(mx \wedge my) = A(mx \wedge my) \vee A(mx \wedge mz)$

$$= A(mf(x \wedge y) \vee mf(x \wedge z)) \leq \max\{A(f(mx)), A(f(my) \vee f(m(x \wedge z)))\}$$

$$\leq \max\{f^{-1}(A)(mx), f^{-1}(A)(my) \vee f^{-1}(A)(mx \wedge mz)\}$$

Therefore $f^{-1}(A)$ is a dual of fuzzy modular lattice ordered m-group of G.

5. Direct Product of Dual of Fuzzy Modular Lattice Ordered *M*-Group

Definition 5.1. Let A_i be a fuzzy set of G_i for $i = 1, 2, \ldots, n$. Then the product $A_i (i = 1, 2, \ldots, n)$ is the function $A_1 \times A_2 \times \ldots \times A_n; G_1 \times G_2 \times \ldots \times G_n \to L$ defined by $(A_1 \times A_2 \times \ldots \times A_n) m(x_1 \times x_2, \ldots, x_n) = \max \{A_1(mx_1), A_2(mx_2), \ldots, A_n(mx_n)\}$.

Proposition 5.2. *The direct product of dual of fuzzy modular lattice ordered m groups is a dual of fuzzy modular lattice ordered m-group.*

References

[1] N. Ajmal and K. V. Thomas, The lattice of fuzzy subgroups and fuzzy normal subgroups, Inform. Sci. 76 (1994), 1-11.

[2] G. S. V. Satya Saibaba, Fuzzy lattice ordered groups, South East Asian Bulletin of Mathematics 32 (2008), 749-766.

[3] J. A. Goguen, *L*-fuzzy sets, J. Math Anal. Appl. 18 (1967), 145-174.

[4] M. Marudai and V. Rajendran, Characterization of fuzzy lattices on a group with respect to *T*-norms, International Journal of Computer Applications 8(8) (2010), 8-15.

[5] S. Nanda, Fuzzy lattices, Bulletin Calcutta Math. Soc. 81 (1989), 1-2.

[6] SG. Karpagavalli and D. Vidyadevi, Fuzzy modular lattice ordered *m*-group, AIP Conference Proceedings (2020), 020-001.