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THE FUZZY QUEUE WITH SINGLE WORKING VACATION SERVING AT THE SLOWER RATE DURING THE START-UP PERIOD

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Abstract

We consider *FM*/*FM*/1 queue with single working vacation serving at the slower rate during start-up period. We obtain model in fuzzy environment as the number of customer in the system, the membership function of the mean queue length, the membership function of the mean sojourn time, probability of the server is in close-up period, probability of the server is in start-up period, probability of the server is in working vacation period and probability of the server is in regular busy period. Also numerical study for all this performance measures is provided.

1. Introduction

Vacation queue have been introduced by many authors for their having various applications in the communication, computer networks and production management and so forth. In 2002, Servi and Finn introduced a class of semi-vacation, called working vacation (*WV*), during which the customers are served with a slower rate rather than completely stopping serving. And they first analyzed an $M/M/1$ queue with this class of semi-vacation policy, defined by $M/M/1/MV$, and obtain the number of customer in the system and the waiting time distribution. Later Liu, Xu and Tian (2002) gave simple explicit expressions of distribution for the stationary

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queue length and waiting time which have intuitionistic probability sense. Recently, Tian, Zhao and Wang [4] study an $M/M/1$ queue with single working vacation and they also introduced a start-up period. In addition, there is a slow rate of service during the start-up period. Many authors worked and generalized the work of $M/G/1$ queue with multiple working vacations. Baba [1] investigated a $GI/M/1$ queue with multiple working vacations. In this paper, we study the fuzzy queue with single working vacation serving at slower rate during the start-up period. We obtain the membership function of the mean queue length, the membership function of the mean sojourn time, probability of the server is in close-up period, probability of the server is in start-up period, probability of the server is in working vacation period and probability of the server is in regular busy period. The numerical study carried out for all this performance measures.

2. The Fuzzy Model

In this section the fuzzy arrival rate, fuzzy service rate, the server serves at a slower rate, the server serves at a normal rate, server at slower rate in the vacation and vacation of time are assumed to be fuzzy numbers $\bar{\lambda}, \bar{\gamma}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3$ and $\bar{\theta}$ respectively. Now

$$\bar{\lambda} = \{x, \mu_{\bar{\lambda}}(x); x \in S(\bar{\lambda})\},$$

$$\bar{\gamma} = \{y_1, \mu_{\bar{\gamma}}(y_1); y_1 \in S(\bar{\gamma})\},$$

$$\bar{\gamma}_1 = \{y_2, \mu_{\bar{\gamma}_1}(y_2); y_2 \in S(\bar{\gamma}_1)\},$$

$$\bar{\gamma}_2 = \{y_3, \mu_{\bar{\gamma}_2}(y_3); y_3 \in S(\bar{\gamma}_2)\},$$

$$\bar{\gamma}_3 = \{s, \mu_{\bar{\gamma}_3}(s); s \in S(\bar{\gamma}_3)\},$$

$$\bar{\theta} = \{z, \mu_{\bar{\theta}}(z); z \in S(\bar{\theta})\}.$$

Where, $S(\bar{\lambda}), S(\bar{\gamma}), S(\bar{\gamma}_1), S(\bar{\gamma}_2), S(\bar{\gamma}_3)$ and $S(\bar{\theta})$ are the universal set of the fuzzy arrival rate, fuzzy service rate, the server serves at a slower rate, the server serves at a normal rate, server at slower rate in the vacation and vacation of time respectively.

Define $f(x, y_1, y_2, y_3, s, z)$ as the system performance measure related to the above defined fuzzy queuing model which depends on the fuzzy membership function $f(\bar{\lambda}, \bar{\gamma}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3, \bar{\theta})$. Applying Zadeh's extension principle [7] the membership function of the performance measure $f(\bar{\lambda}, \bar{\gamma}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3, \bar{\theta})$ can be defined as,

$$\mu_{\bar{f}(\bar{\lambda}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3\ \bar{\theta})}(H) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}) \\ y_2 \in S(\bar{\gamma}_1) \\ y_3 \in S(\bar{\gamma}_2) \\ s \in S(\bar{\gamma}_3) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}}(y_1), \mu_{\bar{\gamma}_1}(y_2), \mu_{\bar{\gamma}_2}(y_3),$$

$$\mu_{\bar{\gamma}_3}(s), \mu_{\bar{\theta}}(z))/H = f(x, y_1, y_2, y_3, s, z\}\}. \tag{1}$$

If the α-cuts of $f(\bar{\lambda}, \bar{\gamma}, \bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3, \bar{\theta})$ degenerates to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number. We obtain the membership function of some performance measures namely the membership function of the mean queue length, the membership function of the mean sojourn time, probability of the server is in close-up period, probability of the server is in start-up period, probability of the server is in working vacation period and probability of the server is in regular busy period. For the system, in terms of this membership function are

$$\mu_{\overline{EL}}(A) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\gamma}) \\ y_2 \in S(\bar{\gamma}_1) \\ y_3 \in S(\bar{\gamma}_2) \\ s \in S(\bar{\gamma}_3) \\ z \in S(\bar{\theta})}} \{\min\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}}(y_1), \mu_{\bar{\gamma}_1}(y_2), \mu_{\bar{\gamma}_2}(y_3), \mu_{\bar{\gamma}_3}(s), \mu_{\bar{\theta}}(z))/A\}\}. \tag{2}$$

where, $$A = \left[\frac{4y_1^2}{(3y_1 - x + z - G)} + \frac{2y_3 z}{3y_3 - x + s - H}\frac{[x + z + y_1 - G]}{2y_1}\right.$$

$$\left. + \frac{xz[3y_1 - x + z - G]}{2y_1 y_2}\frac{[2y_1 y_2]^2}{[y_2 - x]^2[3y_1 - x + z - G]^2}\right] \times \pi_{00}$$

$$G = \sqrt{(x + z + y_1)^2 - 4xy_1},\ H = \sqrt{(x + s + y_3)^2 - 4xy_3}$$

$$\mu_{\overline{E(w)}}(B)=\sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma})\\ y_2\in S(\bar{\gamma}_1)\\ y_3\in S(\bar{\gamma}_2)\\ s\in S(\bar{\gamma}_3)\\ z\in S(\bar{\theta})}}\{\min\{(\mu_{\bar{\lambda}}(x),\ \mu_{\bar{\gamma}}(y_1),\ \mu_{\bar{\gamma}_1}(y_2),\ \mu_{\bar{\gamma}_2}(y_3),\ \mu_{\bar{\gamma}_3}(s),\ \mu_{\bar{\theta}}(z))/B\}\}.$$

where, $B=\left[\dfrac{z[3y_3-x+s-H]2y_3}{s[x+s+y_3-H]}\right]\times\pi_{00}$

$$\mu_{\overline{P_0}}(C)=\sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma})\\ y_2\in S(\bar{\gamma}_1)\\ y_3\in S(\bar{\gamma}_2)\\ s\in S(\bar{\gamma}_3)\\ z\in S(\bar{\theta})}}\{\min\{(\mu_{\bar{\lambda}}(x),\ \mu_{\bar{\gamma}}(y_1),\ \mu_{\bar{\gamma}_1}(y_2),\ \mu_{\bar{\gamma}_2}(y_3),\ \mu_{\bar{\gamma}_3}(s),\ \mu_{\bar{\theta}}(z))/C\}\}. \quad (4)$$

where, $C=\dfrac{z}{s}\times\pi_{00}$

$$\mu_{\overline{P_1}}(D)=\sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma})\\ y_2\in S(\bar{\gamma}_1)\\ y_3\in S(\bar{\gamma}_2)\\ s\in S(\bar{\gamma}_3)\\ s\in S(\bar{\beta})\\ z\in S(\bar{\theta})}}\{\min\{(\mu_{\bar{\lambda}}(x),\ \bar{\mu}_{\gamma}(y_1),\ \bar{\mu}_{\gamma_1}(y_2),\ \bar{\mu}_{\gamma_2}(y_3),\ \mu_{\bar{\gamma}_3}(s),\ \mu_{\bar{\theta}}(z))/D\}\}. \quad (5)$$

where, $D=\dfrac{2y_1}{3y_1-x+z-G}$

$$\mu_{\overline{P_2}}(E)=\sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma})\\ y_2\in S(\bar{\gamma}_1)\\ y_3\in S(\bar{\gamma}_2)\\ s\in S(\bar{\gamma}_3)\\ z\in S(\bar{\theta})}}\{\min\{(\mu_{\bar{\lambda}}(x),\ \mu_{\bar{\gamma}}(y_1),\ \mu_{\bar{\gamma}_1}(y_2),\ \mu_{\bar{\gamma}_2}(y_3),\ \mu_{\bar{\gamma}_3}(s),\ \mu_{\bar{\theta}}(z))/E\}\}. \quad (6)$$

where, $E=\dfrac{2s}{3s-x+z.m}$

$$\mu_{\overline{P_3}}(F)=\sup_{\substack{x\in S(\bar{\lambda})\\ y_1\in S(\bar{\gamma})\\ y_2\in S(\bar{\gamma}_1)\\ y_3\in S(\bar{\gamma}_2)\\ s\in S(\bar{\gamma}_3)\\ z\in S(\bar{\theta})}}\{\min\{(\mu_{\bar{\lambda}}(x),\ \mu_{\bar{\gamma}}(y_1),\ \mu_{\bar{\gamma}_1}(y_2),\ \mu_{\bar{\gamma}_2}(y_3),\ \mu_{\bar{\gamma}_3}(s),\ \mu_{\bar{\theta}}(z))/F\}\}. \quad (7)$$

$$\text{where, } F = \left[\frac{1}{\lambda} \frac{\dfrac{z\left[x + z + y_1 - \sqrt{(x + z + y_1)^2 - 4xy_1}\right]}{2y_1}}{\left[y_2 - y_2 x + zy_2 + y_1 y_2 - y_2\sqrt{(x + z + y_1)^2 - 4xy_1}\right]} \right.$$

$$+ \left(\frac{z[2y_3 - x + s + y_1 - H]}{2y_2} \times \frac{2y_1 y_2}{[y_2 - x] z[2y_1 - x + z + y_1 - G]} \right)$$

$$+ \frac{x2Y_1 + 2zy_1 - [x + z + y_1 - G]y_2}{y_1}$$

$$\left. + \frac{z[x + z + y_1 - G]}{x + z + y_1 + G} - \frac{z[3y_3 - x + s - H]}{y_3 - x + s - H} \right] \times \pi_{00}.$$

Using the fuzzy analysis technique explained, we can find the membership of $\overline{E(L)}$, $\overline{E(w)}$, $\overline{P_0}$, $\overline{P_1}$, $\overline{P_2}$ and $\overline{P_3}$ as a function of the parameter α, thus the α-cut approach can be used to develop the membership function of $\overline{E(L)}$, $\overline{E(w)}$, $\overline{P_0}$, $\overline{P_1}$, $\overline{P_3}$, and $\overline{P_3}$ respectively.

3. Performance of Measure

The membership function of the mean queue length

The membership value $\mu_{\overline{E(L)}}(A)$ is the superimum of minimum over

$$\{(\mu_{\bar{\lambda}}(x), \mu_{\bar{\gamma}}(y_1), \mu_{\bar{\gamma}_1}(y_2), \mu_{\bar{\gamma}_2}(y_3), \mu_{\bar{\gamma}_3}(s), \mu_{\bar{\theta}}(z)) : A = f(x, y_1, y_2, y_3, s, z)\},$$

to satisfying $\mu_{\overline{E(L)}}(A) = \alpha,\ 0 < \alpha \leq 1$

We consider the following six cases:

Case (i)

$$\mu_{\bar{\lambda}}(x) = \alpha,\ \mu_{\bar{\gamma}}(y_1) \geq \alpha,\ \mu_{\overline{\gamma_1}}(y_2) \geq \alpha,\ \mu_{\bar{\gamma}_2}(y_3) \geq \alpha,\ \mu_{\overline{\gamma_3}}(s) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha$$

Case (ii)

$$\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\gamma}}(y_1) = \alpha,\ \mu_{\overline{\gamma_1}}(y_2) \geq \alpha,\ \mu_{\bar{\gamma}_2}(y_3) \geq \alpha,\ \mu_{\overline{\gamma_3}}(s) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha$$

Case (iii)

$$\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\gamma}}(y_1) \geq \alpha,\ \mu_{\overline{\gamma_1}}(y_2) = \alpha,\ \mu_{\bar{\gamma}_2}(y_3) \geq \alpha,\ \mu_{\overline{\gamma_3}}(s) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha$$

Case (iv)

$$\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\gamma}}(y_1) \geq \alpha,\ \mu_{\overline{\gamma_1}}(y_2) \geq \alpha,\ \mu_{\bar{\gamma}_2}(y_3) = \alpha,\ \mu_{\overline{\gamma_3}}(s) \geq \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha$$

Case (v)

$$\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\gamma}}(y_1) \geq \alpha,\ \mu_{\overline{\gamma_1}}(y_2) \geq \alpha,\ \mu_{\bar{\gamma}_2}(y_3) \geq \alpha,\ \mu_{\overline{\gamma_3}}(s) = \alpha,\ \mu_{\bar{\theta}}(z) \geq \alpha$$

Case (vi)

$$\mu_{\bar{\lambda}}(x) \geq \alpha,\ \mu_{\bar{\gamma}}(y_1) \geq \alpha,\ \mu_{\overline{\gamma_1}}(y_2) \geq \alpha,\ \mu_{\bar{\gamma}_3}(y_3) \geq \alpha,\ \mu_{\overline{\gamma_3}}(s) \geq \alpha,\ \mu_{\bar{\theta}}(z) = \alpha.$$

For case (i) the lower and upper bound of α-cuts of $\overline{E(L)}$ can be obtained through the corresponding parametric non-linear programs, Similarly, we can calculate the lower and upper bounds of the α-cuts of $\overline{E(L)}$ for the case (ii), (iii), (iv), (v) and (iv). By considering all the cases simultaneously, the lower and upper bounds of the α-cuts of $\overline{E(L)}$ can be written as

$$[E(L)]_\alpha^L = \min_{\Omega} \{[A]\} \text{ and } [E(L)]_\alpha^U = \max_{\Omega} \{[A]\}$$

such that $x_\alpha^L \leq x \leq x_\alpha^U$, $y_{1\alpha}^L \leq y_1 \leq y_{1\alpha}^U$, $y_{2\alpha}^L \leq y_2 \leq y_{2\alpha}^U$, $y_{3\alpha}^L \leq y_3 \leq y_{3\alpha}^U$, $s_\alpha^L \leq s \leq s_\alpha^U$, $z_\alpha^L \leq z \leq z_\alpha^U$.

If both $(E(L))_\alpha^L$ and $(E(L))_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(A) = [(E(L))_\alpha^L]^{-1}$ and $R(A) = [(E(L))_\alpha^U]^{-1}$ can be derived from, which the membership function $\mu_{\overline{E(L)}}(A)$ can be constructed as

$$\mu_{\overline{E(L)}}(A) = \begin{cases} L(A),\ (E(L))_{\alpha=0}^L \leq A \leq (E(L))_{\alpha=0}^U \\ 1,\ (E(L))_{\alpha=1}^L \leq A \leq (E(L))_{\alpha=1}^U \\ R(A),\ (E(L))_{\alpha=1}^L \leq A \leq (E(L))_{\alpha=0}^U \end{cases} \tag{8}$$

In the same way we get the following results.

The membership function of the mean sojourn time

$$\mu_{\overline{E(W)}}(B) = \begin{cases} L(B), \ (E(W))_{\alpha=0}^{L} \leq B \leq (E(W))_{\alpha=0}^{U} \\ 1, \ (E(W))_{\alpha=1}^{L} \leq B \leq (E(W))_{\alpha=1}^{U} \\ R(B), \ (E(W))_{\alpha=1}^{L} \leq B \leq (E(W))_{\alpha=0}^{U} \end{cases} \quad (9)$$

Probability of the server is in close-up period

$$\mu_{\overline{P_0}}(C) = \begin{cases} L(C), \ (P_0(C))_{\alpha=0}^{L} \leq C \leq (P_0(C))_{\alpha=0}^{U} \\ 1, \ (P_0(C))_{\alpha=1}^{L} \leq C \leq (P_0(C))_{\alpha=1}^{U} \\ R(C), \ (P_0(C))_{\alpha=1}^{L} \leq C \leq (P_0(C))_{\alpha=0}^{U} \end{cases} \quad (10)$$

Probability of the server is in start-up period

$$\mu_{\overline{P_1}}(D) = \begin{cases} L(D), \ (P_1(D))_{\alpha=0}^{L} \leq D \leq (P_1(D))_{\alpha=0}^{U} \\ 1, \ (P_1(D))_{\alpha=1}^{L} \leq D \leq (P_1(D))_{\alpha=1}^{U} \\ R(D), \ (P_1(D))_{\alpha=1}^{L} \leq D \leq (P_1(D))_{\alpha=0}^{U} \end{cases} \quad (11)$$

Probability of the server is in working vacation period

$$\mu_{\overline{P_2}}(E) = \begin{cases} L(E), \ (P_2(E))_{\alpha=0}^{L} \leq E \leq (P_2(E))_{\alpha=0}^{U} \\ 1, \ (P_2(E))_{\alpha=1}^{L} \leq E \leq (P_2(E))_{\alpha=1}^{U} \\ R(E), \ (P_2(E))_{\alpha=1}^{L} \leq E \leq (P_2(E))_{\alpha=0}^{U} \end{cases} \quad (12)$$

Probability of the server is in regular busy period

$$\mu_{\overline{P_3}}(F) = \begin{cases} L(F), \ (P_3(F))_{\alpha=0}^{L} \leq F \leq (P_3(F))_{\alpha=0}^{U} \\ 1, \ (P_3(F))_{\alpha=1}^{L} \leq F \leq (P_3(F))_{\alpha=1}^{U} \\ R(F), \ (P_0(F))_{\alpha=1}^{L} \leq F \leq (P_3(F))_{\alpha=0}^{U} \end{cases}. \quad (13)$$

4. Numerical Study

The membership function of the mean queue length

Suppose the fuzzy arrival rate $\bar{\lambda}$, fuzzy service rate $\bar{\gamma}$, server serves at a slower rate $\overline{\gamma_1}$, server serves at a normal rate $\overline{\gamma_2}$ server at slower in the vacation $\overline{\gamma_3}$ and the vacation of time $\bar{\theta}$ are assumed to be trapezoidal fuzzy numbers described by: $\bar{\lambda} = [1, 2, 3, 4]$, $\bar{\gamma} = [6, 7, 8, 9]$, $\overline{\gamma_1} = [11, 12, 13, 14]$, $\overline{\gamma_2} = [26, 27, 28, 29]$, $\overline{\gamma_3} = [31, 32, 33, 34]$ and $\bar{\theta} = [51, 52, 53, 54]$ per hour respectively: Then

$$\lambda(\alpha) = \min_{x \in S(\bar{\lambda})}\{x \in S(\bar{\lambda}), G(x) \geq \alpha\}, \max_{x \in S(\bar{\lambda})}\{x \in S(\bar{\lambda}), G(x) \geq \alpha\},$$

where,

$$G(x) = \begin{cases} x - 1, 1 \leq x \leq 2 \\ 1, 2 \leq x \leq 3 \\ 4 - x, 3 \leq x \leq 4. \end{cases}$$

That is, $\lambda(\alpha) = [1 + \alpha, 4 - \alpha]$, $\gamma(\alpha) = [6 + \alpha, 9 - \alpha]$, $\gamma_1(\alpha) = [11 + \alpha, 14 - \alpha]$, $\gamma_2(\alpha) = [26 + \alpha, 29 - \alpha]$, $\gamma_3(\alpha) = [31 + \alpha, 34 - \alpha]$ and $\theta(\alpha) = [51 + \alpha, 54 - \alpha]$. It is clear that, when $x = x_\alpha^U$, $y_1 = y_{1_\alpha}^U$, $y_2 = y_{2_\alpha}^U$, $y_3 = y_{3_\alpha}^U$, $s = s_\alpha^U$, $z = z_\alpha^U$, A attains its maximum value and when $x = x_\alpha^L$, $y_1 = y_{1_\alpha}^L$, $y_2 = y_{2_\alpha}^L$, $y_3 = y_{3_\alpha}^L$, $s = s_\alpha^U$, $z = z_\alpha^L$, A attains its minimum value.

From the generated for the given input value of $\bar{\lambda}$, $\bar{\gamma}$, $\overline{\gamma_1}$, $\overline{\gamma_2}$, $\overline{\gamma_3}$, $\bar{\theta}$ with we infer that:

(i) For fixed values of x, y_1, y_2 and y_3, s A decreases as z increase.

(ii) For fixed values of y_1, y_2, y_3 and s, z, A decreases as x increase.

(iii) For fixed values of y_2, y_3, s and z, x A decreases as y_1 increase.

(iv) For fixed values of y_3, s, z, x and y_1, A decreases as y_2 increase.

(v) For fixed values of s, z, x, y_1 and y_2, A decreases as y_3 increase.

(vi) For fixed values of z, x, y_1 and y_2, y_3 A decreases as s increase.

The smallest value of $E(L)$ occurs, when x-takes its lower bound. That is, $x = 1 + \alpha$, y_1, y_2, y_3, s and z take their upper bounds given by $y_1 = 9 - \alpha$, $y_2 = 14 - \alpha$, $y_3 = 29 - \alpha$, $s = 34 - \alpha$ and $z = 54 - \alpha$ respectively. The maximum value of $E(L)$ occurs when $x = 4 - \alpha$, $y_1 = 6 + \alpha$, $y_2 = 11 + \alpha$, $y_3 = 26 + \alpha$, and $s = 31 + \alpha$, $z = 51 + \alpha$.

If both $(E(L))_{\alpha}^{L}$ and $(E(L))_{U}^{\alpha}$ are invertible with respect to 'α' then the left shape function $L(A) = [(E(L))]^{-1}$ and right shape function $R(A) = [(E(L))_{\alpha}^{L}]^{-1}$ can be obtained and from, which the membership function $\mu_{\overline{E(L)}}(A)$ can be constructed as

$$\mu_{\overline{E(L)}}(A) = \begin{cases} L(A), & A_1 \leq A \leq A_2 \\ 1, & A_2 \leq A \leq A_3 \\ R(A), & A_3 \leq A \leq A_4 \end{cases}. \tag{14}$$

For the given set of input values, the values of A_1, A_2, A_3 and A_4 evaluated using C program are 20.4845, 30.7262, 43.6364 and 60.5124 respectively. The correspond membership function is

$$\mu_{\overline{E(L)}}(A) = \begin{cases} L(A), & 20.4845 \leq A \leq 30.7262 \\ 1, & 30.7262 \leq A \leq 43.6364 \\ R(A), & 43.6364 \leq A \leq 60.5124 \end{cases}. \tag{15}$$

In the same way we get the following results.

The membership function of the mean sojourn time

$$\mu_{\overline{EW}}(B) = \begin{cases} L(B), & 0.5930 \leq B \leq 0.6062 \\ 1, & 0.6062 \leq B \leq 0.6841 \\ R(B), & 0.6841 \leq B \leq 0.7359 \end{cases}. \tag{16}$$

Probability of the server is in close-up period

$$\mu_{\overline{P_0}}(C) = \begin{cases} L(C),\ 0.3846 \le C \le 0.6077 \\ 1,\ 0.6077 \le C \le 0.9154 \\ R(C),\ 0.1954 \le C \le 1.9158 \end{cases}. \tag{17}$$

Probability of the server is in start-up period

$$\mu_{\overline{P_1}}(D) = \begin{cases} L(D),\ 0.1939 \le D \le 0.3127 \\ 1,\ 0.3127,\ D \le 0.6080 \\ R(D),\ 0.6080 \le D \le 0.9414 \end{cases}. \tag{18}$$

Probability of the server is in working vacation period

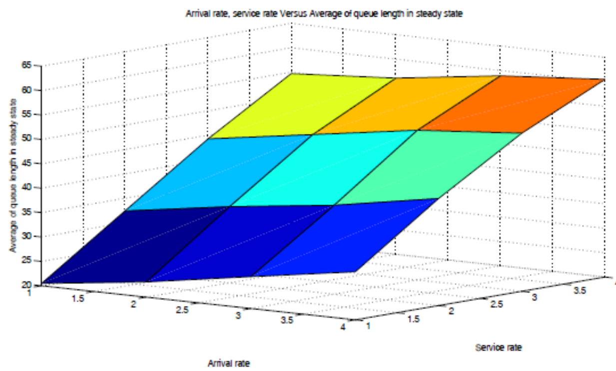
$$\mu_{\overline{P_2}}(E) = \begin{cases} L(E),\ 0.2841 \le E \le 0.5681 \\ 1,\ 0.5681 \le E \le 0.7651 \\ R(E),\ 0.7651 \le E \le 0.9591 \end{cases}. \tag{19}$$

Probability of the server is in regular busy period

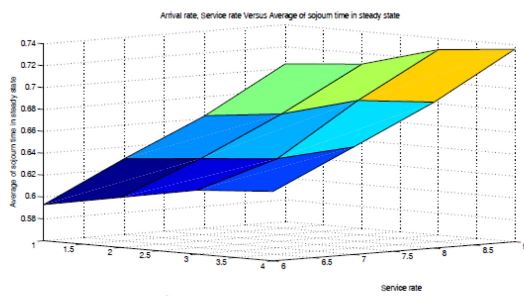
$$\mu_{\overline{P_3}}(F) = \begin{cases} L(F),\ 0.2391 \le F \le 0.4856 \\ 1,\ 0.4856 \le F \le 0.6846 \\ R(F),\ 0.6846 \le F \le 0.9619 \end{cases}. \tag{20}$$

Further by fixing the vacation rate $\overline{\theta}$ by crisp value 52.3 and taking arrival rate $\overline{\lambda} = [1, 2, 3, 4]$, service rate $\overline{\gamma} = [6, 7, 8, 9]$ both trapezoidal fuzzy numbers and the values of the average queue length are generated and from the figure 1. It is observed that as $\overline{\lambda}$ increases the average queue length increases for the fixed value of the service rate. Whereas for fixed value of arrival rate, the average queue length decreases as service rate increases. It is also observed from the data generated that, the membership value of the average queue length is 63, when the ranges of arrival rate, service rate, and the queue length lie in the intervals (2, 3.4), (2.3, 4.6) respectively.

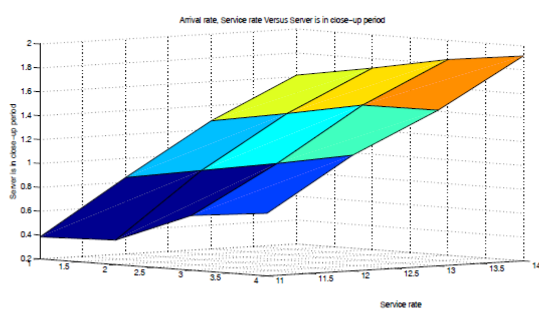
The graph of sojourn time drawn in figure 2 shows that, as arrival rate increases waiting time also increases, while the waiting time decreases as the service rate decreases. It is also observed from the data generated that, the membership value of the sojourn time is 0.73, when the ranges of all units are lie in the intervals (3.5, 4), (6.3, 7.6) respectively.



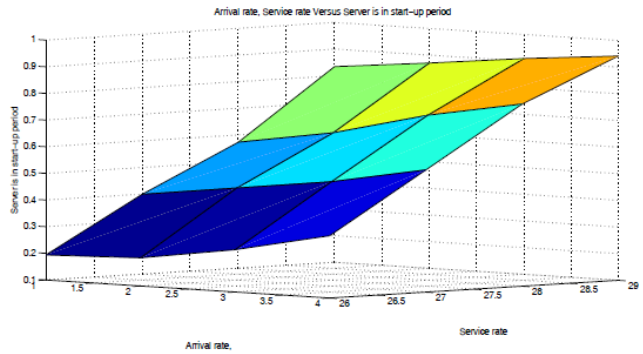
arrival rate, service rate versus membership function of the mean queue length



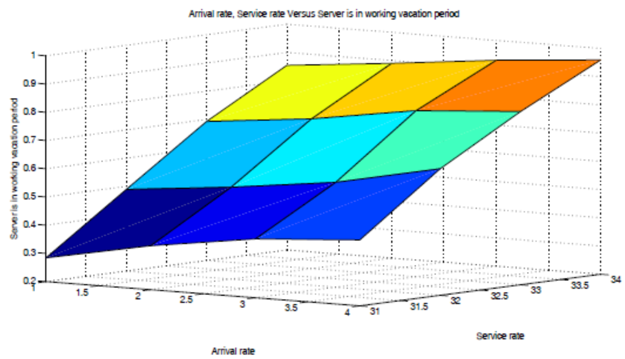
arrival rate, service rate versus membership function of the mean sojourn time



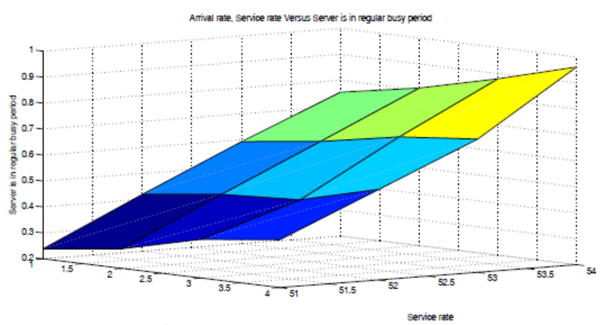
arrival rate, service rate versus probability of the server is in close-up period



arrival rate, service rate versus probability of the server is in start-up period



arrival rate, service rate versus probability of the server is in working vacation period



arrival rate, service rate versus probability of the server is in regular busy period.

5. Conclusion

In this paper, we obtained the number of customers in the system, the membership function of the mean queue length, the membership function of the mean sojourn time, probability of the server is in close-up period, probability of the server is in start-up period, probability of the server is in working vacation period and probability of the server is in regular busy period. We have obtained numerical results to all the performance measures for this fuzzy queue. The examples for this paper are Refrigerators, Ac and Iron box etc.

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