Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4483-4491
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A NOVEL METHOD FOR SOLVING TYPE-2 INTUITIONISTIC FUZZY TRANSPORTATION PROBLEMS USING SINGULARLY PERTURBED DELAY DIFFERENTIAL EQUATIONS

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Abstract

In this paper, a transportation problem having uncertainty in cost is considered. To deal with uncertainty, triangular intuitionistic fuzzy numbers is used. A method to solve type-2 Intuitionistic Fuzzy Transportation Problems (IFTP-2), where the weights of the trader are provided in the form of Singularly Perturbed Delay Differential Equations (SPDDEs) is proposed. The Trader weights are computed using the numerical solutions with a finite difference scheme on Shish kin mesh of SPDDEs. Finally, a numerical example is provided to prove the efficiency of the method.

1. Introduction

"Bellman and Zadeh [3] popularized fuzzy theory in the context of optimization problems'. 'The idea of Intuitionistic Fuzzy Set (IFS) was

2020 Mathematics Subject Classification: 65L05, 34A07.

Keywords: Triangular Intuitionistic fuzzy numbers, Intuitionistic Fuzzy Transportation problem, Singular perturbation problem, Delay differential equation, Shishkin mesh, Finite difference scheme.

Received March 24, 2022; Accepted April 19, 2022

suggested by Atanassov [2]'. 'Hussain and Kumar [4], Singh and Yadav [9] and Ali Mahmoodirad et al. [1] solved transportation problem by using Intuitionistic Fuzzy Numbers (IFNs)'. Malley [5] and Nayfeh [6] introduced Singular Perturbation Problems (SPP). "Paramasivam et al. [7] proposed a linear second-order SPP with piecewise-uniform Shishkin mesh which was used to develop the numerical methods for the same'. 'Robinson et al. [8] proposed a new approach to solve MAGDM problems'. Here, a new method to solve type-2 Intuitionistic Fuzzy Transportation Problems (IFTP-2) is proposed using the Singularly Perturbed Delay Differential Equations (SPDDES) for finding the weighting vector and the method is illustrated by anumerical example.

2. Preliminaries

In this section, basic definitions of fuzzy theory which are needed for this paper are given in (Atanassov [2], Singh and Yadav [9]).

Definition 1. An Intuitionistic Fuzzy subset $\widetilde{F}^I = \{(\mho, \mu_{\widetilde{F}^I}(v), \nu_{\widetilde{F}^I}(\mho))/\mho \in U\}$ of the real line R is called an IFN if

(i) $\mathcal{F}\mho \in R$ such that $\nu_{\widetilde{F}^I}(\mho) = 1$ and $\mu_{\widetilde{F}^I}(\mho) = 0$

(ii) $\mu_{\widetilde{F}^I}$ and $\nu_{\widetilde{F}^I}$ are piecewise continuous mappings from R to the closed interval [0, 1] and the relation $0 \leq \mu_{\widetilde{F}^I}(v) + \nu_{\widetilde{F}^I}(v) \leq 1\mho \in U$ holds.

Definition 2. A Triangular Intuitionistic Fuzzy Number (TIFN) $\widetilde{F}^I$ with membership function $\mu_{\widetilde{F}^I}$ and non-member ship function $\nu_{\widetilde{F}^I}$ is defined as an IFS in R where

$$\mu_{\widetilde{F}^I}(\mho) = \begin{cases} \dfrac{\mho - \rho_1}{\rho_2 - \rho_1}, & \text{if } \rho_2 < \mho < \rho_2 \\ \dfrac{\rho_3 - v}{\rho_3 - \rho_2}, & \text{if } \rho_2 < \mho < \rho_2 \\ 0, & \text{othrewise} \end{cases}$$

$$\nu_{\widetilde{F}^I}(\mho) = \begin{cases} \dfrac{\rho_2 - \upsilon}{\rho_2 - \rho_1'}, & \text{if } \rho_2' < \mho < \rho_2 \\ \dfrac{\mho - \rho_2}{\rho_3' - \rho_2}, & \text{if } \rho_2 < \mho < \rho_2' \\ 1, & \text{othrewise} \end{cases}$$

Where $\rho_1' \leq \rho_1 < \rho_2 < \rho_3 \leq \rho_3'$. This TIFN is denoted by $\widetilde{F}^I = (\rho_1, \rho_2, \rho_3; \rho_1', \rho_2, \rho_3')$

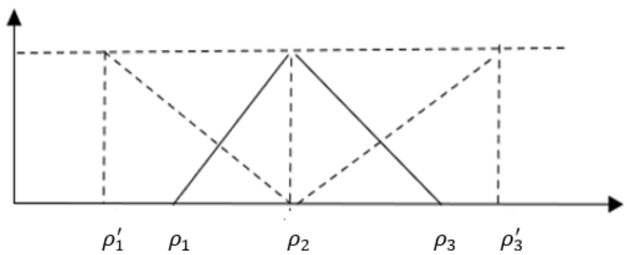

Figure 1. Membership and non-membership function of TIFN.

Definition 3. Arithmetic operations on TIFNS. Let $\widetilde{F}_1^I = (\tau_1, \tau_2, \tau_3; \tau_1', \tau_2, \tau_3')$ and $\widetilde{F}_1^I = (\upsilon_1, \upsilon_2, \upsilon_3; \upsilon_1', \upsilon_2, \upsilon_3')$ be two TIFNs. Then the following operations can be done. Addition:

$$\widetilde{\mathcal{F}}_1^I \oplus \widetilde{\mathcal{F}}_2^I = (\tau_1 + \upsilon_1, \tau_2 + \upsilon_2, \tau_3 + \upsilon_3; \tau_1' + \upsilon_1', \tau_2 + \upsilon_2, \tau_3' + \upsilon_3')$$

Subtraction:

$$\widetilde{\mathcal{F}}_1^I \ominus \widetilde{\mathcal{F}}_2^I = (\tau_1 - \upsilon_3, \tau_2 - \upsilon_2, \tau_3 - \upsilon_3; \tau_1' - \upsilon_3', \tau_2 - \upsilon_2, \tau_3' - \upsilon_3')$$

Multiplication:

$\widetilde{\mathcal{F}}_1^I \otimes \widetilde{\mathcal{F}}_2^I = (f_1, f_2, f_3, f_1', f_2, f_3')$, where $f_1 = \min\{\tau_1\upsilon_1, \tau_1, \upsilon_3, \tau_3\upsilon_1, \tau_3\upsilon_3\}$ $f_2 = \tau_2\upsilon_2; f_3 = \max\{\tau_1\upsilon_1, \tau_1, \upsilon_3, \tau_3\upsilon_1, \tau_3\upsilon_3\}$

$$f_1' = \min\{\tau_1'\upsilon_1', \tau_1', \upsilon_3', \tau_3'\upsilon_1', \tau_3'\upsilon_3'\}$$

$$f_1' = \min\{\tau_1'\upsilon_1', \tau_1', \upsilon_3', \tau_3'\upsilon_1', \tau_3'\upsilon_3'\}$$

Scalar Multiplication:

$$k\widetilde{\mathcal{F}}_1^I = (k\tau_1,\, k\tau_2,\, k\tau_3',\, k\tau_1',\, k\tau_2,\, k\tau_3') : k > 0$$

$$k\widetilde{\mathcal{F}}_1^I = (k\tau_3,\, k\tau_2,\, k\tau_1; k\tau_3',\, k\tau_2,\, k\tau_1') : k < 0$$

3. Intuitionistic Fuzzy Transportation Problem of Type-2 (IFTP-2)

In this section, a Type-2 Intuitionistic Fuzzy Transportation Problem with m origins and n destinations is considered.

Let $\widetilde{c}_{ij}^I$ be the "IF transportation cost of transporting one unit of the product from the i^{th} origin to the j^{th} destination", a_i be the "quantity available at the i^{th} origin", a_i be the "quantity needed at the j^{th} destination", X_{ij} be the "quantity transported from the i^{th} origin to the j^{th} destination. Then the IFTP-2 is given by

$$Min\, \widetilde{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n} \widetilde{c}_{ij}^I \times X_{ij}$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = a_i,\ i = 1, 2, 3, \ldots, m$$

$$\sum_{i=1}^{m} X_{ij} = b_j,\ j = 1, 2, 3, \ldots$$

$$X_{ij} \ge 0;\ i = 1, 2, 3, \ldots, m;\ j = 1, 2, 3, \ldots n. \tag{I}$$

4. Singularly Perturbed Delay Differential Equations (SPDDEs)

"A singularly perturbed delay differential equation is an ordinary differential equation in which the highest derivative is multiplied by a small parameter and involving at least one delay term". Delay Differential Equations (DDEs) with constant lags $\tau_j > 0$ for $j = 1, 2, \ldots, k$ have the form: $y' = f(t, y(t), y(t-\tau_1), \ldots, y(t-\tau_k))$. An initial value $y(0) = \emptyset(0)$ is not enough to define a unique solution of $y' = f(t, y(t), y(t-\tau_1), \ldots, y(t-\tau_k))$. on an interval $a \le t \le b$. The function $y(t) = \emptyset(t)$ must be specified for $t \le a$ so that $y(t-\tau_j)$ is defined when $\le t \le a + \tau_j$. The function $\emptyset(t)$ is called the history of the solution. "A hybrid finite difference scheme with an appropriate piecewise uniform Shishkin-type mesh is suggested to solve a second-order SPDDE of reaction-diffusion type". Consider

$$Lu(x) = -\varepsilon u''(x) + a(x)u(x) + b(x)u(x-1) = f(x) \text{ on } (0, 2) \tag{1}$$

with $u = \emptyset$ on $[-1, 0]$ and $u(2) = l$ here $\emptyset$ is sufficiently smooth on $[-1, 0]$. (2)

The Novel Hybrid Scheme for SPDDEs is given as:

$$D_0^+U(x_j) = \frac{3U(x_{j+1}) - 4U(x_j) + U(x_{j-1})}{2h_{j+1}}; D_0^-U(x_j)$$

$$= \frac{3U(x_j) - 4U(x_{j-1}) + U(x_{j-2})}{2h_j}$$

This is used to compute numerical approximations to the solution of (1) and (2).

5. Weight Determination for Solving IFTP-2 Using SPDDEs

The decision maker represents weighting vector in the form of the following Singularly Perturbed delay differential Equation

$$-\varepsilon u''(x) + (2+x)u(x) - u(x-1) = 0$$

$$\text{for } x \in (0, 2), u(x) = 1 \text{ for } x \in [-1, 0], u(2) = 1.$$

Four distinct points based on the parameter τ are identified and the numerical solutions at those points are chosen and normalized for obtaining the weighting vector. Details of the weighting vector from the decision maker are given in the following table.

Table 1. Numerical solution of $-\varepsilon u''(x)+(2+x)u(x)-u(x-1)=0$.

X	Values of U(X)	Weight Vector
0.2500000	0.4461796	0.426800856
0.7500000	0.3637271	0.347929484
1.2500000	0.1381052	0.132106931
1.7500000	0.09739275	0.093162729

As a result, the decision maker's weighting vector is constructed as $w=(0.426800856,\ 0.347929484,\ 0.132106931,\ 0.093162729)^T$.

6. The Proposed Approach to Find the Solution of the IFTP-2

The proposed approach is as follows:

Step 1. Considering triangular intuitionistic fuzzy parameters, the balanced IFTP-2(1) is expanded as

$$\widetilde{Z}^I=\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}^1,\ c_{ij}^2,\ c_{ij}^3,\ c_{ij}^4,\ c_{ij}^2,\ c_{ij}^5)\times X_{ij}^* \tag{II}$$

Subject to

$$\sum_{i=1}^{n}X_{ij}=a_i,\ i=1,2,\ldots,n$$

$$\sum_{i=1}^{m}X_{ij}=b_j,\ j=1,2,\ldots,n$$

$$X_{ij}\geq 0,\ i=1,2,\ldots,m;\ j=1,2,\ n \tag{III}$$

Step 2. For the above Fuzzy Linear programming problem, the multi objective linear programming problem (MOLPP) with fuzzy coefficients can be formulated as

$$Minimize(f_1(x),\ f_2(x),\ \ldots,f_k(x))$$

Subject to (III) where, $f_i : R^n \to R^i$, where R is the set of all real numbers and R^n is the n-dimensional Euclidean space.

By considering the weighting factor, the MOLPP is defined as

$$Minimizew(x)_{x \in X} = \sum_{m=1}^{k} w_m f_m(x)$$

Subject to (III).

Step 3. Convert the multi-objective linear programming problem with fuzzy coefficients obtained from step 2 to the crisp linear programming problem, by giving weights obtained from Singular Perturbation Problem given in section 5 and then find the optimal solution of crisp linear programming problem using LINGO Software. And then the intuitionistic fuzzy objective function value of IFTP is calculated as follows

$$\widetilde{Z}^I \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4, c_{ij}^2, c_{ij}^5) \times X_{ij}^*$$

7. Numerical Example

Consider the following IFTP having four origins and four destinations. Here transportation costs from origins to destinations are TIFNs. The transportation cost per unit is known as in Table 2 below. Availabilities and demands are taken to be crisp numbers. The trader wishes to determine the optimal allocation which minimizes total transportation cost using the Singularly Perturbed delay differential problem given in section 5 for finding the weights (Singh and Yadav [9]).

Table 2. IFTP-2.

	D_1	D_2	D_3	D_4	a_i
S_1	(2, 4, 5; 1, 4, 6)	(2, 5, 7; 1, 5, 8)	(4, 6, 8; 3, 6, 9)	(4, 7, 8; 3, 7, 9)	11
S_2	(4, 6, 8; 3, 6, 9)	(3, 7, 12, 2, 7, 13)	(10, 15, 20; 8, 15, 22)	(11, 12, 13, 10, 12, 14)	11
S_3	(3, 4, 6; 1, 4, 8)	(8, 10, 13; 5, 10, 16)	(2, 3, 5; 1, 3, 6)	(6, 10, 14; 5, 10, 15)	11

S_4	(2, 4, 6; 1, 4, 7)	(3, 9, 10; 2, 9, 12)	(3, 6, 10; 2, 6, 12)	(3, 4, 5, 2, 4, 8)	12
b_j	16	10	8	11	45

The problem is balanced. Using the Steps of the proposed method, the optimal solution is obtained as follows.

Table 3. Optimal solution.

Destinations Sources	D_1	D_2	D_3	D_4
S_1	(2, 4, 5; 1, 4,6)11	(2, 5, 7, 1, 5, 8)	(4, 6, 8, 3, 6, 9)	(4, 7, 8; 3, 7, 9)
S_2	(4, 6, 8, 3; 6, 9)1	(3, 7, 12; 2, 7, 13) 10	(10, 15, 20; 8, 15, 22)	(11, 12, 13, 10, 12, 14)
S_3	(3, 4, 6; 1, 4, 8)3	(8, 10, 13; 5, 10, 16)	(2, 3, 5; 1, 3, 6)8	(6, 10, 14; 5, 10, 15)
S_4	(2, 4, 6; 1, 4, 7) 1	(3, 9, 10; 2, 9, 12)	(3, 6, 10; 2, 6, 12)	(3, 4, 5; 2, 4, 8) 11

The Intuitionistic fuzzy objective function value of IFTP is $\widetilde{Z}^I$ = (116, 204, 302, 68, 204, 372).

8. Conclusion

In this paper, a new method is proposed to solve IFTP -2. Here the weights of decision maker (trader) are determined from SPDDE which has been solved through numerical methods with a finite difference scheme on Shishkin mesh and normalized and utilized. Finally, a numerical example is provided to illustrate the proposed method.

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