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1 MODULO 3 GRACEFUL LABELING IN CERTAIN CLASSES OF GRAPHS

P. VIJAYAKUMAR, K. THULUKKANAM and K. THIRUSANGU

COE, GIET University
Gunupur, Odisha-765022, India
E-mail: vijaysanthosh2002@gmail.com

Department of Mathematics
Dr. Ambedkar Govt. Arts College
Vyasarpadi, Chennai-600039, India
E-mail: thulukkanamk@gmail.com

Department of Mathematics
S.I.V.E.T College, Gowrivakkam
Chennai-600073, India
E-mail: kthirusangu@gmail.com

Abstract

In this chapter we present algorithm and prove the existence of 1 modulo 3 graceful labeling in the extended duplicate graph of path graph, the extended duplicate graph of comb graph, the extended duplicate graph of twig graph, the extended duplicate graph of star graph, the extended duplicate graph of Bistar graph, the extended duplicate graph of double star graph.

1. Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling) [1]. In the intervening years various labeling of graphs have been investigated in over 2000 papers. E. Sampathkumar introduced the concept of duplicate

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graph and studied the characterization of the duplicate graphs [3]. The concept of one modulo N graceful labeling was introduced by V. Ramachandran and C. Sekar. They have proved that the Supersubdivision of Ladder is one modulo N graceful for all positive integers N [2]. 1 (mod 3) graceful labeling was introduced by V. Swaminathan et al. Confining themselves to $N = 3$ in the definition of one modulo N graceful for all positive integers N given by Ramachandran and Sekar.

2. Main Results

Definition 2.1[2]**.** A graph G is said to be one modulo N graceful (where N is a positive integer) if there is a function Φ from the vertex set of G to $\{0, 1, N, (N+1), 2N, (2N+1), \cdots, N(q-1), N(q-1)+1\}$ in such a way that (i) Φ is $1-1$ (ii) Φ induces a bijection Φ^* from the edge set of G to $\{1, N+1, 2N+1, \cdots, N(q-1)+1\}$ where $\Phi^*(uv) = |\Phi(u) - \Phi(v)|$.

Definition 2.2 [2]**.** A graph G is said to be one modulo 3 graceful, if there is a function Φ from the vertex set of G to $\{0, 1, 3, 4, 6, 7, \cdots, 3(q-1), 3(q-1)+1\}$ in such a way that (i) Φ is $1-1$ (ii) Φ induces a bijection Φ^* from the edge set of G to $\{1, 4, 7, \cdots, (3q-2)\}$ where $\Phi^*(uv) = |\Phi(u) - \Phi(v)|$.

Definition 2.3 [3]**.** Let $G(V, E)$ be simple graph. A duplicate graph of G is $DG = (V_1, E_1)$, where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \emptyset$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is defined as The edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.

Algorithm 2.1. (One modulo three graceful labeling – *EDG* (P_m)) $V \leftarrow \{n_1, n_2, \cdots n_{m+1}, n'_2, \cdots, n'_{m+1}\}$ $E \leftarrow \{\ell_1, \ell_2, \cdots, \ell_{m+1}, \ell'_1, \ell'_2, \cdots, \ell'_m\}$.

Case (i). When m is odd

Fix $n_1 \leftarrow 3m+3$, $n'_1 \leftarrow 3m+1$.

For $1 \le k \le \frac{m+1}{2} n_{2k} \leftarrow 3k - 3, n'_{2k} \leftarrow 3m + 3k + 1 n_{2k+1} \leftarrow 3m - 3k + 6.$

For $1 \le k \le \frac{m-1}{2} n'_{2k+1} \leftarrow 6m - 3k + 4.$

Case (ii). When m is even

Fix $n_1 \leftarrow 3m + 3$, $n'_1 \leftarrow 4$.

For $1 \le k \le \frac{m}{2} n_{2k} \leftarrow 3k - 3$, $n'_{2k} \leftarrow 3m + 3k + 1 n_{2k+1} \leftarrow 3m - 3k + 6$, $n'_{2k+1} \leftarrow 6m - 3k + 4$.

Theorem 2.1. *The extended duplicate graph of path graph $EDG(P_m)$, $m \ge 2$ admits* 1 *mod* 3 *graceful labeling*.

Proof. Case (i): When m is odd

Using the algorithm 2.1, $2m + 2$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 6m + 1 (= 3q - 2)$. Using the induced function ϕ^* defined as $\Phi^*(uv) = |\Phi(u) - \Phi(v)|$, the $m - 1$ edges namely $\ell_2, \ell'_3, \ell_4, \ell'_5, \ell_6, \ell'_7, \ldots, \ell_{m-3}, \ell'_{m-2}, \ell'_{m-1}, \ell_m$ receive labels $6m + 1 = (3q - 2), 6m - 2, 6m - 5, 6m - 8, 6m - 11, 6m - 14, \cdots, 3m + 16, 3m + 13$ $3m + 10, 3m + 7$ respectively, the $m - 1$ edges namely $\ell_1, \ell'_2, \ell_3, \ell'_4, \ell_5, \ell'_6, \cdots, \ell'_{m-3}, \ell_{m-2}, \ell'_{m-1}, \ell_m$ receive labels $1, 4, 7, 10, 13, 16, \ldots, 3m - 11, 3m - 8, 3m - 5, 3m - 2$ respectively and the three edges ℓ_1, ℓ'_1, ℓ_m receive labels $1, 3m + 1, 3m + 4$ respectively. Thus the $2m + 1$ edges are labeled with $1, 4, 7, 10, \cdots, 6m + 1 = (3q - 2)$.

Case (ii). When m is even.

Using the algorithm 2.1 $2m + 2$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 6m + 1 (= 3q - 2)$. Using the induced function ϕ^* defined in case (i) above, the $m - 1$ edges namely $\ell_2, \ell'_3, \ell_4, \ell'_5, \ell_6, \ell'_7, \ldots, \ell_{m-3}, \ell_{m-2}, \ell'_{m-1}, \ell_m$ receive labels $6m + 1 (= 3q - 2), 6m - 2, 6m - 5, 6m - 8, 6m - 11, 6m - 14, 6m - 17, \ldots, 3m + 16$ $3m + 13, 3m + 10, 3m + 7$ respectively, the $m - 1$ edges namely $\ell'_2, \ell_3, \ell'_4, \ell_5, \ell'_6, \ell_7, \ldots, \ell_{m-3}, \ell'_{m-2}$,

ℓ'_{m-1}, ℓ'_{m} receive labels $7, 10, 13, 16, 19, 22, \ldots, 3m-8, 3m-5, 3m-2, 3m+1$ respectively and the three edges ℓ_1, ℓ'_1, ℓ_{m+1} receive labels 1, 4, $3m+4$ respectively. Thus the $2m+1$ edges are labeled with 1, 4, 7, 10, $\cdots$, $6m+1(=3q-2)$.

Hence the extended duplicate graph of path graph $EDG(P_m)$, $m \geq 2$ admits 1 mod 3 graceful labeling.

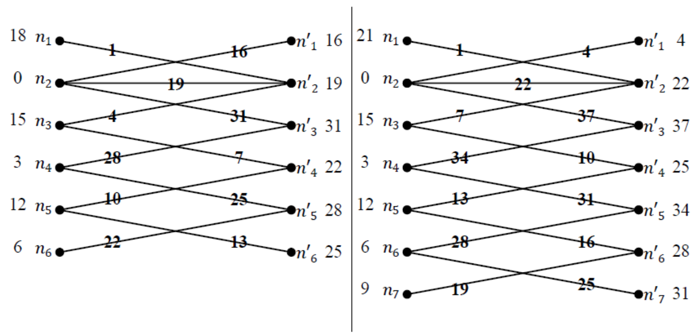

Figure 2.1. One modulo three Graceful Labeling in $EDG(P_5)$ and $EDG(P_6)$.

Algorithm 2.2. (One modulo three graceful labeling $-EDG(CB_m)$**)**

$V \leftarrow \{n_1, n_2, \cdots n_{2m}, n'_1, n'_2, \cdots, n'_{2m}\}$ $E \leftarrow \{\ell_1, \ell_2, \cdots, \ell_{2m}, \ell'_1, \ell'_2, \cdots, \ell'_{2m-1}\}$.

Case (i). When m is odd

Fix $n_1 \leftarrow 0$, $n'_1 \leftarrow 6m-2$.

For $1 \leq k \leq \dfrac{m+1}{2}$ $n_{4k} \leftarrow 6k-3$, $n'_{4k} \leftarrow 6m+6k-5 n_{4k-1} \leftarrow 6m-6k$, $n'_{4k-1} \leftarrow 12m-6k-2 n_{4k+1} \leftarrow 6k$, $n'_{4k+1} \leftarrow 6m+6k-2$.

For $1 \leq k \leq \dfrac{m+1}{2}$ $n_{4k-2} \leftarrow 6m-6k+3$, $n'_{4k-2} \leftarrow 12m-6k+1$

Case (ii). When m is even

Fix $n_1 \leftarrow 0$, $n'_1 \leftarrow 6m-2$.

For $1 \leq k \leq \dfrac{m}{2}$ $n_{4k-2} \leftarrow 6m-6k+3$, $n'_{4k-2} \leftarrow 12m-6k+1$

$n_{4k-1} \leftarrow 6m - 6k, n'_{4k-1} \leftarrow 12m - 6k - 2$

$n_{4k} \leftarrow 6k - 3,\ n'_{4k} \leftarrow 6m + 6k - 5.$

For $1 \leq k \leq \frac{m-2}{2}\, n_{4k+1} \leftarrow 6k, n'_{4k+1} \leftarrow 6m + 6k - 2.$

Theorem 2.2. *The extended duplicate graph of comb graph* $EDG(CB_m)$, $m \geq 2$ *admits* 1 *mod* 3 *graceful labeling*.

Proof. Case (i): When m is odd.

Using the algorithm 2.2, $4m$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 12m - 5 = (3q - 2)$. Using the induced function ϕ^* defined in Theorem 2.1 the $2m - 2$ edges namely $\ell'_1, \ell'_2, \ell_3, \ell_4, \ell'_5, \ell'_6, \ell_7, \ell_8, \ldots, \ell'_{2m-5}, \ell'_{2m-6}, \ell_{2m-3}, \ell_{2m-2}$ receive labels $1, 4, 7, 10, 13, 16, 19, 22, \ldots, 6m - 17, 6m - 14, 6m - 11, 6m - 8$ respectively, the $2m - 2$ edges namely $\ell_1, \ell_2, \ell'_3, \ell'_4, \ell_5, \ell_6, \ell'_7, \ell'_8, \ldots, \ell_{2m-5}, \ell_{2m-4}, \ell'_{2m-3}, \ell'_{2m-2}$ receive labels $12m - 5 = (3q - 2), 12m - 8, 12m - 11, 12m - 14, 12m - 17, 12m - 20, 12m - 23, 12m - 26, \cdots, 6m + 13, 6m + 10, 6m + 7, 6m + 4$ respectively and the three edges $\ell_{2m-1}, \ell_{2m}, \ell'_{2m-1}$ receive labels $6m + 1, 6m - 2, 6m - 5$ respectively. Thus the $4m - 1 (= q)$ edges are labeled with $1, 4, 7, 10, 13, \cdots, 12m - 5 = (3q - 2)$.

Case (ii). When m is even.

Using the algorithm 2.2, $4m$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 12m - 5 = (3q - 2)$. Using the induced function ϕ^* defined in Theorem 3.3.1, the $2m - 2$ edges namely $\ell_1, \ell'_2, \ell_3, \ell_4, \ell'_5, \ell'_6, \ell_7, \ell_8, \ldots, \ell'_{2m-5}, \ell'_{2m-4}, \ell_{2m-3}, \ell_{2m-2}$ receive labels $1, 4, 7, 10, 13, 16, 19, 22, 25, 28, \ldots, 6m + 3, 6m + 10, 6m + 7, 6m + 4$ respectively, the $2m - 2$ edges namely $\ell_1, \ell_2, \ell'_3, \ell'_4, \ell_5, \ell_6, \ell'_7, \ell'_8 \cdots, \ell_{2m-5}, \ell_{2m-4}, \ell'_{2m-3}, \ell'_{2m-2}$ receive labels $12m - 5 = (3q - 2), 12m - 8, 12m - 11, 12m - 14, 12m - 17, 12m - 20, 12m - 23, 12m - 26, \cdots, 6m + 13, 6m + 10, 6m + 7, 6m + 4$ respectively and the three edges $\ell'_{2m-1}, \ell_{2m}, \ell_{2m-1}$ receive labels $6m + 1, 6m - 2, 6m - 5$ respectively.

Thus the $4m - 1 = (q)$ edges are labeled with $1, 4, 7, 10, 13, \ldots, 12m - 5 = (3q - 2)$.

Hence the extended duplicate graph of comb graph $EDG(CB_m)$ admits 1 mod 3 graceful labeling.

Illustration:

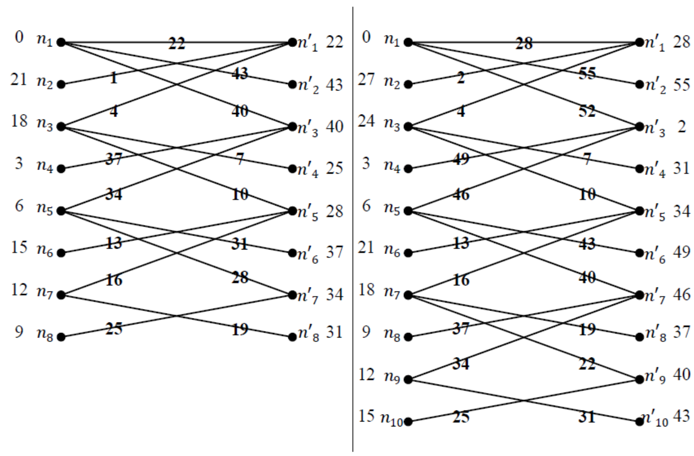

Figure 2.2. One modulo three Graceful Labeling in $EDG(CB_4)$ and $EDG(CB_5)$.

Algorithm 2.3. (One modulo three graceful labeling $EDG(T_m)$)

$V \leftarrow \{n_1, n_2, \cdots n_{3m+2}, n'_1, n'_2, \cdots, n'_{3m+2}\}$

$E \leftarrow \{\ell_1, \ell_2, \cdots, \ell_{3m+2}, \ell'_1, \ell'_2, \cdots, \ell'_{3m+1}\}$

Case (i). When m is odd

Fix $n_1 \leftarrow 9m + 3,\ n'_1 \leftarrow 1,\ n_2 \leftarrow 0,\ n'_2 \leftarrow 9m + 7$

For $1 \leq k \leq \dfrac{m+1}{2}$ $n_{6k} \leftarrow 9m - 9k + 9,\ n'_{6k-3} \leftarrow 18m - 9k + 16$

$n_{6k-2} \leftarrow 9m - 9k + 6,\ n'_{6k-2} \leftarrow 18m - 9k + 13$

$n_{6k-1} \leftarrow 9m - 9k + 3,\ n'_{6k-1} \leftarrow 18m - 9k + 10$

For $1 \leq k \leq \frac{m-1}{2}$ $n_{6k} \leftarrow 6k - 3$, $n'_{6k} \leftarrow 9m + 9k + 1$

$n_{6k+1} \leftarrow 6k$, $n'_{6k+1} \leftarrow 9m + 9k + 4$

$n_{6k+2} \leftarrow 6k + 3$, $n'_{6k+2} \leftarrow 9m + 9k + 7$

Case (ii). When m is even

Fix $n_1 \leftarrow 9m + 3$, $n'_1 \leftarrow 1$, $n_2 \leftarrow 0$, $n'_2 \leftarrow 9m + 7$

For $1 \leq k \leq \frac{m}{2}$ $n_{6k-3} \leftarrow 9m - 9k - 9$, $n'_{6m-3} \leftarrow 18m - 9k + 6$

$n_{6k-2} \leftarrow 9m - 9k - 12$, $n'_{6k-2} \leftarrow 18m - 9k + 13$

$n_{6k-1} \leftarrow 9m - 9k - 15$, $n'_{6k-1} \leftarrow 18m - 9k + 10$

$n_{6k} \leftarrow 9k - 6$, $n'_{6k} \leftarrow 9m - 9k - 9$

$n_{6k+1} \leftarrow 9k - 3$, $n'_{6k+12} \leftarrow 9m + 9k - 6$

$n_{6k+2} \leftarrow 9k$, $n'_{6k+2} \leftarrow 9m + 9k - 3$.

Theorem 2.3. *The extended duplicate graph of twig graph* $EDG(T_m)$, $m \geq 2$ *admits* 1 *mod* 3 *graceful labeling*.

Proof. Case (i): When m is odd

Using the algorithm 2.3, $6m + 4$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 18m + 7 = (3q - 2)$. Using the induced function ϕ^* defined in Theorem 2.1, the $3m$ edges namely $\ell_2, \ell_3, \ell_4, \ell'_5, \ell'_6, \ell'_7, \ell_8, \ell_9, \ell_{10}, \ell'_{11}, \ell'_{12}, \ell'_{13} \cdots, \ell_{3m-7}, \ell_{3m-6}, \ell_{3m-5}, \ell_{3m-4}, \ell'_{3m-3}, \ell'_{3m-2}, \ell'_{3m-1}$ receive labels $18m + 7 = (3q - 2), 18m + 4, 18m + 1, 18m - 2, 18m - 5, 18m - 8, 18m - 11$ $18m - 14, 18m - 17, 18m - 20, 18m - 23, 18m - 26, \cdots, 9m + 34, 9m + 31, 9m + 28, 9m + 25, 9m + 22, 9m + 19, 9m + 16, 9m + 13, 9m + 10$ respectively, the $3m$ edges namely $\ell'_2, \ell'_3, \ell'_4, \ell_5, \ell_6, \ell_7, \ell'_8, \ell'_9, \ell'_{10}, \ell_{11}, \ell_{12}, \ell_{13}, \cdots, \ell'_{3m-7}, \ell'_{3m-6}, \ell'_{3m-5}, \ell_{3m-4}, \ell_{3m-3}, \ell_{3m-2}, \ell'_{3m-1}, \ell'_{3m}, \ell'_{3m+1}$ receive labels 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, $\cdots$, $9m - 20$, $9m - 17$, $9m - 14$, $9m - 11$, $9m - 8$, $9m - 5$, $9m - 2$, $9m - 1$, $9m - 4$ respectively and the three edges

ℓ_1, ℓ'_1, ℓ_{2m+1} receive labels 1, 4, $9m+7$ respectively. Thus the $6m+3=(q)$ edges are labeled with 1, 4, 7, $\cdots$, $18m+7=(3q-2)$.

Case (ii). When m is even

Using the algorithm 2.3, $6m+4$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 18m+7=(3q-2)$. Using the induced function ϕ^* defined in Theorem 2.1, the $3m$ edges namely $\ell_2, \ell_3, \ell_4, \ell'_5, \ell'_6, \ell'_7, \ell_8, \ell_9, \ell_{10}, \ell'_{11}, \ell'_{12}, \ell'_{13}, \ldots, \ell'_{3m-4}, \ell'_{3m-3}, \ell'_{3m-2}, \ell'_{3m-1}, \ell'_{3m}, \ell'_{3m+1}$ receive labels $18m+7=(3q-2)$, $18m+4$, $18m+1$, $18m-2$, $18m-5$, $18m-8$, $18m-11$, $18m-14$, $18m-17$, $18m-20$, $18m-23$, $18m-26$, $\cdots$, $9m+25$, $9m+22$, $9m+19$, $9m+16$, $9m+13$, $9m+10$ respectively, the $3m$ edges namely $\ell_2, \ell_3, \ell_4, \ell'_5, \ell'_6, \ell'_7, \ell_8, \ell_9, \ell_{10}, \ell'_{11}, \ell'_{12}, \ell'_{13}, \cdots, \ell'_{3m-4}, \ell'_{3m-3}, \ell'_{3m-2}, \ell_{3m-1}, \ell_{3m}, \ell_{3m+1}$ receive labels $7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, \ldots, 9m-11, 9m-8, 9m-5, 9m-2, 9m+1, 9m+4$ respectively and the three edges $\ell_1, \ell'_1, \ell_{3m+2}$ receive labels 4, 1, $9m+7$ respectively. Thus the $6m+3$ edges are labeled with $1, 4, 7, 10, \cdots, 18m+7=(3q-2)$.

Hence the extended duplicate graph of Twig graph $EDG(T_m)$ admits 1 mod 3 graceful labeling.

Illustration:

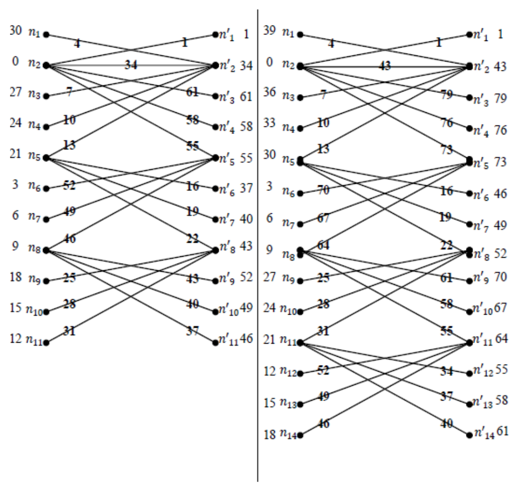

Figure 2.3. One modulo three Graceful Labeling in $EDG(T_3)$ and $EDG(T_4)$.

Algorithm 2.4. (One modulo three graceful labeling $-EDG(S_m)$)

$V \leftarrow \{n_1, n_2, \cdots n_m, n'_1, n'_2, \cdots, n'_m\}$

$E \leftarrow \{\ell_1, \ell_2, \cdots, \ell_m, \ell'_1, \ell'_2, \cdots, \ell'_{m-1}\}$

Fix $n_1 \leftarrow 0$, $n'_1 \leftarrow 6m-5$

For $1 \leq k \leq m-1 n_{k+1} \leftarrow 3m$, $n'_{k+1} \leftarrow 3m-2$.

Theorem 2.4. *The extended duplicate graph of star graph $EDG(S_m)$, $m \geq 3$ admits 1 mod 3 graceful labeling.*

Proof. Using the algorithm 2.4, the $2m$ vertices are labeled with $0, 1, 3, 4, 6, 7, \ldots, 6m-5 = (3q-2)$. Using the induced function ϕ^* defined in Theorem 2.1, the $m-1$ edges namely $\ell_1, \ell_2, \ell_3, \ell_4, \cdots, \ell_{m-2}, \ell_{m-1}$ receive labels $1, 4, 7, 10, \cdots, 3m-8, 3m-5$ respectively and the $m-1$ edges namely $\ell'_{m-1}, \ell'_{m-2}, \ell'_{m-3}, \cdots, \ell'_2, \ell'_1$ receive labels $3m-2, 3m+1, 3m+4, \ldots, 6m-11, 6m-8$ respectively and the edge ℓ_m receives label $6m-5 = (3q-2)$. Thus the $2m-1$ edges are labeled with $1, 4, 7, \cdots, 6m-8, 6m-5 = (3q-2)$.

Hence the extended duplicate graph of star graph $EDG(S_m)$, $m \geq 3$ admits 1 mod 3 graceful labeling.

Illustration:

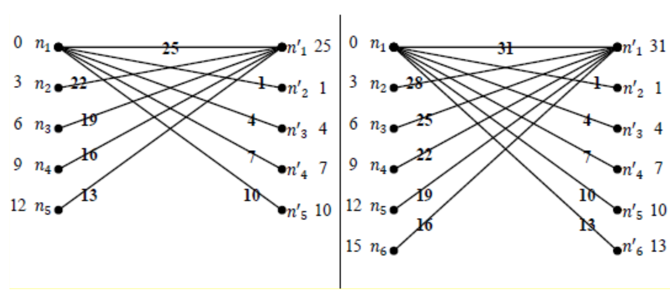

Figure 2.4. One modulo three graceful Labeling in $EDG(S_5)$, and $EDG(S_6)$.

Algorithm 2.5. (One modulo three graceful labeling $-EDG(BS_{m,m})$))

$V \leftarrow \{n_1, n_2, \cdots n_{2m+2}, n'_1, n'_2, \cdots, n'_{2m+2}\}$

$E \leftarrow \{\ell_1, \ell_2, \cdots, \ell_{2m+2}, \ell'_1, \ell'_2, \cdots, \ell'_{2m+1}\}$

For $1 \leq k \leq m+2$ $n_k \leftarrow 3k-3$, $n'_{m+3-k} \leftarrow 12m-3k+10$

For $1 \leq k \leq m$

$n_{m+k+2} \leftarrow 9m + 3k + 6$, $n'_{m+k+2} \leftarrow 9m - 3k + 4$.

Theorem 2.5. *The extended duplicate graph of Bi-star graph* $EDG(BS_{m,m})$, $m \geq 2$ *admits* 1 *mod* 3 *graceful labeling.*

Proof. Using the algorithm 2.5, the $4m+4$ vertices are labeled with $0, 1, 3, 4, 6, 7, \cdots, 12m-7 = (3q-2)$. Using the induced function ϕ^* defined in Theorem 2.1, the m edges namely $\ell'_{2m+1}, \ell'_{2m}, \ell'_{2m-1}, \cdots, \ell'_{m+2}$ receive labels $1, 4, 7, 10, \cdots, 3m-2$ respectively, the m edges namely $\ell'_{2m+1}, \ell'_{2m}, \ell'_{2m-1}, \ldots, \ell'_{m+2}$ receive labels $3m+1, 3m+4, 3m+7, \cdots, 6m-2$ respectively, the $m+1$ edges namely $\ell'_{m+1}, \ell'_m, \ell'_{m-1}, \cdots, \ell'_2, \ell'_1$ receive labels $6m+1, 6m+4, 6m+7, \cdots, 9m-2, 9m+1$ respectively, the edge ℓ_{2m+2} receives label $9m+4$ and the $m+1$ edges namely $\ell_1, \ell_2, \ell_3, \cdots, \ell_m, \ell_{m+1}$ receive labels $9m+7, 9m+10, 9m+13, \cdots,$ $12m+4, 12m+7 = (3q-2)$ respectively. Thus the $4m+3 = (q)$ edges are labeled with $1, 4, 7, 10, 13, \cdots, 12m+7 = (3q-2)$.

Hence the extended duplicate graph of bi-star graph $EDG(BS_{m,m})$, $m \geq 2$ admits 1 mod 3 graceful labeling.

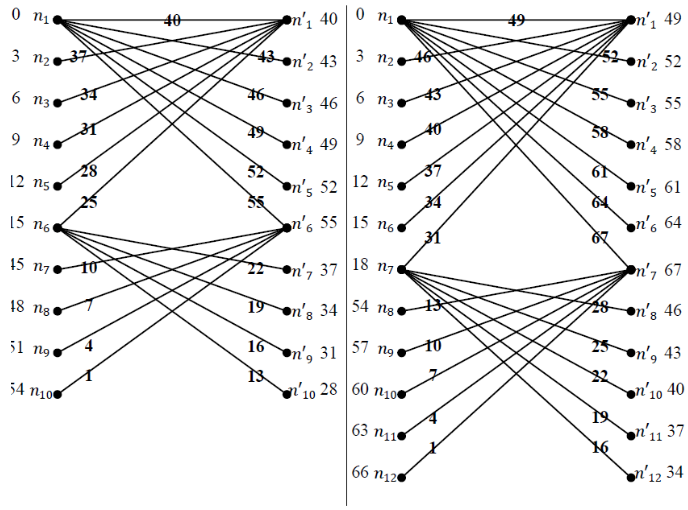

Figure 2.5. One modulo three graceful Labeling in $EDG(S_{4,4})$, and $EDG(S_{m,m})$.

Algorithm 2.6. (One modulo three graceful labeling $EDG(DS_{m,m})$)

$V \leftarrow \{n_1, n_2, \cdots n_{2m+1}, n'_1, n'_2, \cdots, n'_{2m+1}\}$

$E \leftarrow \{\ell_1, \ell_2, \cdots, \ell_{2m+1}, \ell'_1, \ell'_2, \cdots, \ell'_{2m}\}$

For $1 \le k \le m+1$ $n_k \leftarrow 3k-3$, $n'_{m+2-k} \leftarrow 12m-3k+4$

For $1 \le k \le m$

$n_{m+k+1} \leftarrow 9m-3k+3$, $n'_{m+k+1} \leftarrow 9k-5$.

Theorem 2.6. *The extended duplicate graph of double star graph $EDG(DS_{m,m})$, $m \ge 2$ admits 1 mod 3 graceful labeling.*

Proof. Using the algorithm 2.6, the $4m+2$ vertices are labeled using $0, 1, 3, 4, 6, 7, \cdots, 12m+1 = (3q-2)$. Using the induced function ϕ^* defined in Theorem 2.1, the m edges namely $\ell_m, \ell_{m-1}, \ell_{m-2}, \cdots, \ell_2, \ell_1$ receive labels $12m+1 = (3q-2), 12m-2, 12m-5, \cdots, 9m+7, 9m+4$ respectively, the edge ℓ_{2m+1} receives the label $9m+1$, the m edges namely $\ell'_1, \ell'_2, \ell'_3, \cdots, \ell'_{m-1}, \ell'_m$ receive labels $9m-2, 9m-5, 9m-8, \cdots, 6m+4, 6m+1$ respectively, the m

edges namely $\ell_{m+1}, \ell_{m+2}, \ell_{m+3}, \cdots, \ell_{2m}$ receive labels $1, 7, 13, \cdots, 6m-11$, $6m-5$ respectively and the m edges namely $\ell'_{m+1}, \ell'_{m+2}, \ell'_{m+3}, \cdots, \ell'_{2m}$ receive labels $4, 10, 16, \cdots, 6m-8, 6m-2$ respectively. Thus the $4m+1=(q)$ edges are labeled with $1, 4, 7, \cdots, 12m+1=(3q-2)$.

Hence the extended duplicate graph of double star graph $EDG(DS_{m,m})$, $m \geq 2$ admits 1 mod 3 graceful labeling.

Illustration:

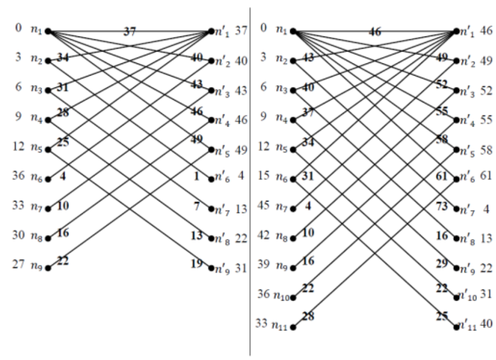

Figure 2.6. One modulo three graceful Labeling in $EDG(S_{4,4})$, and $EDG(S_{5,5})$.

3. Conclusion

In this chapter it is proved that the extended duplicate graph of path graph, the extended duplicate graph of comb graph, the extended duplicate graph of twig graph, the extended duplicate graph of star graph, the extended duplicate graph of Bistar graph, the extended duplicate graph of double star graph are one modulo three graceful.

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