



ON NEUTROSOPHIC θ -QUOTIENT FUNCTIONS

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Abstract

We continue the study on neutrosophic θ -closed set and neutrosophic θ -quotient, neutrosophic strongly θ -quotient and neutrosophic quasi θ -open functions are presented in neutrosophic topological spaces. Characterizations as well as properties are also discussed.

1. Introduction

Fuzzy set hypothesis is presented and considered as a mathematical tool for managing vulnerabilities where every component had a level of participation, membership function (mf) or truth (t), by Zadeh [15]. The degree of non-membership function (nmf), the falsehood (f), was introduced by Atanassov [2] in an intuitionistic fuzzy set. Coker [3] developed the notion

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of intuitionistic fuzzy topology. Neutrality (i), the degree of indeterminacy, as an independent notion, was introduced by Smarandache [11, 12, 13]. He likewise characterized the neutrosophic set (NS) on three parts $(t, f, i) =$ (truth, falsehood, indeterminacy). Salama et al. [11, 12] converted Neutrosophic crisp set into neutrosophic topological spaces (NTS). The introduction of NS opened a broad variety of exploration in requisites of NTS decision making problems. A. A. Salama et al. in [10] introduced neutrosophic closed sets and continuous functions. R. Dhavaseelan et al. [4] presented generalized neutrosophic closed sets. Neutrosophic semi-open, and α -open are presented in [14]. In [8] the authors developed the notion of neutrosophic θ -closure operator and utilizing this, neutrosophic θ -closed set is elucidated. Neutrosophic θ -continuous, neutrosophic strongly θ -continuous including neutrosophic weakly continuous mappings are characterized concerning the operator defined as applications of neutrosophic θ -closure and neutrosophic θ -closed sets.

This paper is devoted to the continuation study on neutrosophic θ -closed [8] set and presented the concept of neutrosophic θ -quotient function, neutrosophic strongly θ -quotient functions are obtainable in NTS. Some properties are also discussed.

2. Preliminaries

Definition 2.1 [11, 12]. Let be a non-empty fixed set. A neutrosophic set (in a nutshell, NS) Λ is an object so as $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x) \rangle : x \in S_1\}$ wherein $\mu_\Lambda(x)$, $\sigma_\Lambda(x)$ and $\gamma_\Lambda(x)$ which addresses the level of membership function (namely $\mu_\Lambda(x)$), the level of indeterminacy (viz $\sigma_\Lambda(x)$) along with the level of non-membership (namely $\gamma_\Lambda(x)$) separately of every component $x \in S_1$ to the set Λ .

Remark 2.2 [11, 12]. (i) An N -set $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$ can be recognized to an arranged triple $\langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda \rangle$ in $]0^-, 1^+[$ on S_1 .

(ii) We use the symbol $\Lambda = \langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda \rangle$ for the N -set $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$.

Definition 2.3 [11, 12]. Let $S_1 \neq \emptyset$ and the N -sets Λ and Γ be named as $\Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_\Gamma(x), \sigma_\Gamma(x), \Gamma_\Gamma(x) \rangle : x \in S_1\}$. Then I. $\Lambda \subseteq \Gamma$ iff $\mu_\Lambda(x) \leq \mu_\Gamma(x)$, $\sigma_\Lambda(x) \leq \sigma_\Gamma(x)$ and $\Gamma_\Lambda(x) \leq \Gamma_\Gamma(x)$ for all $x \in S_1$,

II. $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;

III. $\bar{\Lambda} = \{\langle x, \Gamma_\Lambda(x), \sigma_\Lambda(x), \mu_\Lambda(x) \rangle : x \in S_1\}$, [Complement of Λ]

IV. $\Lambda \cap \Gamma = \{\langle x, \mu_\Lambda(x) \wedge \mu_\Gamma(x), \sigma_\Lambda(x) \wedge \sigma_\Gamma(x), \Gamma_\Lambda(x) \vee \Gamma_\Gamma(x) \rangle : x \in S_1\}$,

V. $\Lambda \cup \Gamma = \{\langle x, \mu_\Lambda(x) \vee \mu_\Gamma(x), \sigma_\Lambda(x) \vee \sigma_\Gamma(x), \Gamma_\Lambda(x) \wedge \Gamma_\Gamma(x) \rangle : x \in S_1\}$,

VI. $[] \Lambda = \{\langle x, \mu_\Lambda(x), \sigma_\Lambda(x), 1 - \mu_\Lambda(x) \rangle : x \in S_1\}$,

VII. $\langle \rangle \Lambda = \{\langle x, 1 - \Gamma_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1\}$.

Definition 2.4 [12, 13]. Let $\{\Lambda_i : i \in J\}$ be a discretionary family of N -sets in S_1 . Therefore

I. $\cap \Lambda_i = \{\langle p, \mu_{\Lambda_i}(p), \wedge \sigma_{\Lambda_i}(p), \vee \Gamma_{\Lambda_i}(p) \rangle : p \in S_1\}$,

II. $\cup \Lambda_i = \{\langle p, \vee \mu_{\Lambda_i}(p), \vee \sigma_{\Lambda_i}(p), \wedge \Gamma_{\Lambda_i}(p) \rangle : p \in S_1\}$.

The principal subject is to assemble the equipment for creating NTS, so we construct the neutrosophic sets 0_{\aleph} along with 1_{\aleph} in X as follows:

Definition 2.5 [12, 13]. $0_{\aleph} = \{\langle q, 0, 0, 1 \rangle : q \in X\}$ and $1_{\aleph} = \{\langle q, 0, 0, 1 \rangle : q \in X\}$.

Definition 2.6 [10]. A neutrosophic topology (briefly, $\aleph T$) $S_1 \neq \emptyset$ is a family ξ_1 of N -sets in S_1 obeying the axioms given underneath:

I. $0_{\aleph}, 1_{\aleph} \in \xi_1$,

II. $W_1 \cap W_2 \in T$ being $W_1, W_2 \in \xi_1$,

III. $\cup W_i \in \xi_1$ for arbitrary family $\{W_i \mid i \in \Lambda\} \subseteq \xi_1$.

For this circumstances, Ordered pair (S_1, ξ_1) or plainly S_1 is termed as and each NS in ξ_1 is termed as neutrosophic open set (in a nutshell, $\aleph OS$).

The complement of an \aleph -open set Λ in S_1 is known as neutrosophic closed set (\aleph CS, for short) in S_1 .

Definition 2.7 [10, 11]. Consider a NS as \aleph in NTS S_1 . Therefore $\aleph \text{int}(\aleph) = \cup\{W \mid W \text{ is an } \aleph\text{OS in } S_1 \text{ and } W \subseteq \aleph\}$ is called as neutrosophic interior (in a nutshell, $\aleph \text{int}$) of \aleph ; $\aleph cl(\aleph) = \cap\{W \mid W \text{ is an } \aleph \text{CS in } S_1 \text{ and } W \supseteq \aleph\}$ is named as neutrosophic closure (in a nutshell, $\aleph cl$) of \aleph .

Definition 2.8 [4]. Let X be a nonempty set. Whenever r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ at that time the $NSx_{r,t,s}$ is named as neutrosophic point (in a nutshell, NP) in X given by $x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$ for $x_p \in X$ is designated as the support of $x_{r,t,s}$, wherein demonstrates the mf, demonstrates the i and demonstrates then mf of $x_{r,t,s}$.

Definition 2.9 [14]. For an \aleph SD in an NTS (X, T) . We have,

- (i) Neutrosophic semi open set (\aleph SOS) if $D \subseteq \aleph cl(\aleph \text{int}(D))$.
- (ii) Neutrosophic α -open set ($\aleph\alpha$ OS) if $D \subseteq \aleph \text{int}(\aleph cl(\aleph \text{int}(D)))$.

The complement of D is an \aleph SOS, $\aleph\alpha$ OS, is called respectively as \aleph SCS and $\aleph\alpha$ CS.

Definition 2.10 [8]. A NP $x_{(\alpha,\beta,\gamma)}$ is named as neutrosophic θ -cluster point ($\aleph\alpha$ -cluster point, shortly) of a NS K iff considering respective W in $N \in q$ -nbd of $x_{(\alpha,\beta,\gamma)}$ with $Ncl(W)qK$. The collection of all $N\theta$ -cluster points of K is labelled as neutrosophic θ -closure besides represented as $NCl_\theta(K)$.

An NS K will be $N\theta$ -closed set ($N\theta$ CS in precise) iff $K = NCl_\theta(K)$. The complement of a $N\theta$ CS is N -open set (in precise, $N\theta$ OS).

Definition 2.11 [7]. Let $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ be a surjective function. Then η is named as neutrosophic quotient function if η is neutrosophic continuous and $\eta^{-1}(M)$ is NOS in (T_1, Γ) .

3. Neutrosophic θ -Quotient Function

Definition 3.1. Let $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is termed as

(i) Neutrosophic θ -quotient function if η is neutrosophic θ -continuous and $\eta^{-1}(M)$ is NOS in S_1 implies M is a NOS in T_1 .

(ii) Neutrosophic θ^* -quotient function if η is neutrosophic θ -irresolute and $\eta^{-1}(M)$ is N θ OS in implies M is a NOS in T_1 .

(iii) Neutrosophic strongly θ -open function if the image of every N θ OS in S_1 is a NTOS in T_1 .

(iv) Neutrosophic θ -open if the image of every NOS in S_1 is a N θ OS in T_1 .

Theorem 3.2. *If $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is surjective neutrosophic θ -continuous and neutrosophic θ -open, then η is neutrosophic θ -quotient function.*

Proof. Let $\eta^{-1}(M)$ be NOS in S_1 . Thereupon is a NTOS, as is neutrosophic θ -open. Accordingly, N θ OS, as η is surjective along with $(\eta^{-1}(M)) = M$. So, η is neutrosophic θ -quotient function

Theorem 3.3. *If $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic open surjective neutrosophic θ -irresolute and $\kappa : (S_2, \Gamma) \rightarrow (T_2, \Psi)$ be neutrosophic θ -quotient function. Then $(\kappa \circ \eta)$ is neutrosophic θ -quotient.*

Proof. Consider R be any NOS in (T_2, Ψ) . Thereupon $\kappa^{-1}(R)$ is N θ OS as is neutrosophic θ -quotient function along with this η is neutrosophic θ -irresolute, $\eta^{-1}(\kappa^{-1}(R))$ is N θ OS. Henceforth $(\kappa \circ \eta)^{-1}(R)$ is a N θ OS implies $(\kappa \circ \eta)$ is N θ OS. So, $(\kappa \circ \eta)$ is a neutrosophic θ -continuous. Also, suppose $(\kappa \circ \eta)^{-1}(R)$ be NOS in S_1 for $T \subseteq R$, that is $\eta^{-1}(\kappa^{-1}(R))$ N θ OS in S_1 . As η is neutrosophic open, $\eta(\eta^{-1}(\kappa^{-1}(R)))$ is NOS in T_1 . It follows that $\kappa^{-1}(R)$ is NOS in T_1 , η is surjective. Since κ is neutrosophic θ -quotient function, R is N θ OS. Accordingly, $(\kappa \circ \eta)$ is neutrosophic θ -quotient function.

Theorem 3.4. *Let (S_1, τ) and (T_1, Γ) be NTSs. If $\eta : (S_1, \tau^\theta) \rightarrow (T_1, \Gamma^\theta)$ is neutrosophic θ -quotient function then $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic θ -quotient.*

Proof. Consider $W \in \Gamma$ so $W \in \tau^\theta$. As η is neutrosophic θ -quotient function, $\eta^{-1}(W) \in \tau^\theta$. So, it is claimed that when W is a NOS in (T_1, Γ) at that time $\eta^{-1}(W)$ is a N θ OS in (S_1, τ) . Accordingly, η is neutrosophic θ -continuous function. Presume $\eta^{-1}(W)$ is NOS in (S_1, τ) thereupon $\eta^{-1}(W) \in \tau^\theta$. As η is neutrosophic quotient function, $W \in \tau^\theta$ and so W is N θ OS in (T_1, Γ) . Hence $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic θ -quotient.

Theorem 3.5. *If $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ be onto neutrosophic strongly θ -open and a neutrosophic θ -irresolute function. Consider, $\kappa : (S_2, \Psi) \rightarrow (T_2, \Psi)$ be neutrosophic θ^* -quotient function. Then $(\kappa \circ \eta)$ is neutrosophic θ^* -quotient.*

Proof. We prove that $(\kappa \circ \eta)$ is neutrosophic θ -irresolute. Consider R be any N θ OS in (T_2, Ψ) . Then $\kappa^{-1}(R)$ is N θ OS as κ is neutrosophic θ^* -quotient function. Since, η is neutrosophic θ -irresolute, $\eta^{-1}(\kappa^{-1}(R)) = (\kappa \circ \eta)^{-1}(R)$ is N θ OS. So, $(\kappa \circ \eta)$ is a neutrosophic θ -irresolute function. Suppose $(\kappa \circ \eta)^{-1}(R) = \eta^{-1}(\kappa^{-1}(R))$ be N θ OS in S_1 . Since, η is neutrosophic strongly θ -open, $\eta^{-1}(\eta^{-1}(\kappa^{-1}(R)))$ is N θ OS in T_1 . Since, η is an onto function $\eta(\eta^{-1}(\eta^{-1}(\kappa^{-1}(R)))) = \kappa^{-1}(R)$. So $\kappa^{-1}(R)$ is N θ OS in T_1 . This implies that R is NOS in (T_2, Ψ) as κ is a neutrosophic θ^* -quotient function. Therefore, $(\kappa \circ \eta)$ is neutrosophic θ^* -quotient.

Theorem 3.6. *Every neutrosophic quotient function is neutrosophic θ -quotient.*

Proof. Consider J be a NOS in (T_1, Γ) . As, η is neutrosophic quotient function, $\eta^{-1}(J)$ is NOS in (S_1, τ) and so it is N θ OS in (S_1, τ) . So, η is

neutrosophic θ -continuous function. Let $\eta^{-1}(J)$ is NOS in (S_1, τ) and so J is a N θ OS in (T_1, Γ) . Hence is neutrosophic θ -quotient function.

Theorem 3.7. *Every neutrosophic θ^* -quotient function is neutrosophic strongly θ -quotient.*

Proof. Consider $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ be a neutrosophic θ^* -quotient function. Suppose R be NOS in T_1 at that time $\eta^{-1}(R)$ is NOS in S_1 as R is N θ OS in T_1 along with η is neutrosophic θ -irrsolute. As η is neutrosophic θ^* -quotient function $\eta^{-1}(R)$ is N θ OS in S_1 then R is NOS in T_1 . So η is neutrosophic strongly θ -quotient.

Definition 3.8. Let (S_1, τ) and (T_1, Γ) be NTSs. A function $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is named as neutrosophic quasi θ -open if the image of each N θ OS in (S_1, τ) is NOS in (T_1, Γ) .

Theorem 3.9. *Consider two NTSs as (S_1, τ) and (T_1, Γ) . If $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic quasi θ -open then is neutrosophic strongly θ -open.*

Proof. Consider J be a N θ OS in (S_1, τ) . So $J \in \tau^\theta$. Since $\eta : (S_1, \tau^\theta) \rightarrow (T_1, \Gamma^\theta)$ is neutrosophic quasi θ -open, $\eta(J)$ is NOS in (T_1, Γ^θ) . So, $\eta(J)$ is a N θ OS in (T_1, Γ) . Hence it follows that $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic strongly θ -open function.

Definition 3.10. Consider S_1 be a NTS and R is NS in S_1 . Neutrosophic θ -interior of R is noted and defined as $N \text{int}_\theta(R) = Ncl_\theta(R^c)^c$ and we have

$$(i) (Ncl_\theta(R^c)) = (N \text{int}_\theta(R))^c$$

$$(ii) (Ncl_\theta(R))^c = N \text{int}_\theta(R^c)$$

Theorem 3.11. *Taking J and K as NSs in NTS (S_1, τ) , then*

$$(i) N \text{int}_\theta(1_\sim) = 1_\sim$$

- (ii) $N \text{int}_\theta(J) \subseteq J$
- (iii) $J \subseteq K \Rightarrow N \text{int}_\theta(J) \subseteq N \text{int}_\theta(K)$.
- (iv) $N \text{int}_\theta(J \cap K) = N \text{int}_\theta(J) \cap N \text{int}_\theta(K)$.
- (v) $N \text{int}_\theta(J) \cup N \text{int}_\theta(K) \subseteq N \text{int}_\theta(J \cap K)$.

Corollary 3.12. *For a NS R , $N \text{int}_\theta(R) \subseteq N \text{int}(R)$.*

Theorem 3.13. *If $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic quasi θ -open iff each member W of S_1 , $\eta(N \text{int}_\theta(W) \subset N \text{int}(\eta(W)))$.*

Proof. Consider η be neutrosophic quasi θ -open. As $N \text{int}(W) \subset W$ with $N \text{int}_\theta(W)$ is N θ OS. Accordingly, we get $\eta(N \text{int}_\theta(W) \subset \eta(W)$. As, $\eta(N \text{int}_\theta(W))$ is NOS, $\eta(N \text{int}_\theta(W) \subset N \text{int}(\eta(W)))$. Conversely, presume that W is N θ OS in S_1 . Thereupon, $\eta(W) = \eta(N \text{int}_\theta(W) \subset N \text{int}(\eta(W)))$. But, $N \text{int}(\eta(W)) \subset \eta(W)$. Consequently, $\eta(W) = N \text{int}(\eta(W))$ and hence η is neutrosophic quasi θ -open.

Theorem 3.14. *If $\eta : (S_1, \tau) \rightarrow (T_1, \Gamma)$ is neutrosophic quasi θ -open thereupon $N \text{int}_\theta(\eta^{-1}(W)) \subset \eta^{-1}(N \text{int}(W))$ for each member W of T_1 .*

Proof. Consider W be member of T_1 . At that time $N \text{int}_\theta(\eta^{-1}(W))$ is N θ OS in and η is neutrosophic quasi θ -open, so $\eta(N \text{int}_\theta(\eta^{-1}(W))) \subset N \text{int}(\eta(\eta^{-1}(W)))$. Thus, $N \text{int}_\theta(\eta^{-1}(W)) \subset \eta^{-1}(N \text{int}(W))$.

4. Conclusions

We have presented concept of neutrosophic θ -quotient, neutrosophic strongly θ -quotient and neutrosophic quasi θ -open functions in NTS. Some results have been proved to show that how far topological structures are preserved by the new neutrosophic functions defined. Here we have presented the idea; still some more theoretical research is to be carried out to build up an overall frame work for dynamic and to characterize designs for complex organization considering and down to earth application.

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