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FIXED POINT THEOREM WITH A PAIR (α, ρ) CONTRACTION IN *F*-METRIC SPACE

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Abstract

The neutral delay differential equations with unbounded delay are used in modeling of networks involving lossless transmission lines. The solutions of these differential equations play very important role in fixing the problem. Here in this paper we prove fixed point theorem with a pair of specialized type of contractions such as (α, φ) in F-metric space.

1. Introduction

Metric space was given by Frechet [8] as follows:

Consider a non-empty set W. A metric on that set is a distance function $d_1 : W \times W \to [0, +\infty)$ fulfilling the following postulates:

(i) $d_1(\bar{o}, g) = 0 \Leftrightarrow \bar{o} = g$,

(ii) $d_1(\bar{o}, g) = d_1(g, \bar{o})$, and

(iii) $d_1(\bar{o}, g) \leq d_1(\bar{o}, l) + d_1(l, g)$,

for all $\bar{o}, g, l$ belongs to W. The pair of (W, d_1) is known as metric space. Matthews [14] Branciari [2] and Czerwik [7] used of partial metric spaces, generalized metric spaces and b-metric spaces respectively to prove the

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results. In recent times, Jleli et al. [10] proposed an interesting conclusion of a metric space given as follow.

Let F be a set of functions $h : (0, +\infty) \to R$, fulfilling the following conditions:

(1F) h is non decreasing, and

(2F) For every sequence (seq.) $\{\alpha_m\} \subseteq R^+$, $\lim_{n\to\infty}(\alpha_m) = 0$ iff $\lim_{m\to\infty} h(\alpha_m) = -\infty$.

Definition 1.1 [3]**.** Let $W \neq \phi$. Consider $d_{f1} : W \times W \to [0, +\infty)$ be a given function Let $(h, j) \in F \times [0, +\infty)$ such that

(1D) $(\bar{o}, g) \in W \times W$, $d_{f_1}(\bar{o}, g) = 0 \Leftrightarrow \bar{o} = g$.

(2D) $d_{f_1}(\bar{o}, g) = d_{f1}(g, \bar{o})$, for all $(\bar{o}, g) \in W \times W$

(3D) For each $(\bar{o}, g)$ belongs to $W \times W$, $\forall m$ belongs to $N(m \geq 2)$ and for each $(r_i)_1^m \subset W$ such that $(r_1, r_m) = (\bar{o}, g)$,

we have $d_{f1}(\bar{o}, g) > 0 \Rightarrow h(d_{f_1}(\bar{o}, g)) \leq h(\sum_{i=1}^{m-1} d_{f1}(\bar{o}_i, \bar{o}_{i+1})) + j$. Then, d_{f1} is known as F-metric on W and (W, d_{f1}) is known as F-metric space.

Hussain et al. [9], Som et al. [15], Mitrovic et al. [12], Chauhan [6], Asif et al. [4] and Ashis et al. [5] used F-metric space and some of its extension for proving their required results.

Definition 1.2 [10]**.** Suppose that (W, d_{f1}) is an F-metric space.

(i) A seq. $\{\bar{o}_m\}$ in W is known as F-covergent (cgt.) to any $\bar{o} \in W$ if $\{\bar{o}_m\}$ is cgt. to $\bar{o}$, w.r.t. d_{f1}.

(ii) $\{\bar{o}_m\}$ known as F-Cauchy,

if $\lim_{m,n\to\infty} d_{f1}(\bar{o}_m, g_n) = 0$.

(iii) If every F-Cauchy seq. in W is F-cgt., then (W, d_{f1}) is F-complete

Theorem 1.1 [10]. *Consider* (W, d_{f1}) *is F-metric space and* $A : W \to W$. *Suppose the following postulates are satisfied*:

(i) (A, d_{f1}) *is F-complete, and*

(ii) $\exists l \in (0, 1)$ s.t. $d_{f1}(A(\bar{o}), A(g)) \leq l d_{f1}(\bar{o}, g)$.

Then A has a only fixed point $\bar{o}^* \in W$. Moreover, $\forall \bar{o}_o \in W$, the sequence $\{\bar{o}_m\} \subset W$ given by $\bar{o}_{m+1} = y(\bar{o}_m)$, $\epsilon \to N$, is F-cgt. to $\bar{o}^*$.

Moreover, Lateef and Ahmad [11] and Alnaser et al. [1] purposed the fixed point theorems of Dass and Gupta and the relation-theoretic contraction results, owing to the approach of F-metric spaces.

2. Preliminaries

Definition 2.1 [13]. Let (α, φ) contractive and α-admissible mappings.

Suppose ψ be the P family of nondecreasing functions $\varphi : [0, +\infty) \to [0, +\infty)$ s.t. $\sum_{i=1}^{+\infty} \varphi^i(k) < +\infty$ for each $k > 0$, where φ^i be i-th iteration of φ.

Definition 2.2 [13]. If φ belongs to Ψ, after that we get the following:

(a) $(\varphi_i(k))_{i \in N}$ converges to 0 as $i \to \infty \forall k \in (0, +\infty)$;

(b) $\varphi(k) < k \forall k > 0$

(c) $\varphi(k) = 0$ if and only if $k = 0$.

Definition 2.3 [13]. Suppose $A : W \to W$ and $\alpha : W \times W \to [0, +\infty)$. Then, W is known as α-admissible if $\bar{o}, g \in W, \alpha(\bar{o}, g) \geq 1 \Rightarrow \alpha(A\bar{o}, Ag) \geq 1$.

Theorem 2.1 [13]. *Suppose a complete metric space* (W, d_1) *and A be* α-*admissible mapping. Suppose that* $\alpha(\bar{o}, g) d_1(A\bar{o}, Ag) \leq \varphi(d_1(\bar{o}, g)) \forall \bar{o}$, $g \in W \forall \varphi \in \Psi$. *Also, suppose that*

(i) $\exists \bar{o}_o \in W$ *s.t.* $\alpha(\bar{o}_o, A\bar{o}_o) \geq 1$

(ii) *either* A *is continuous or for any seq.* $\{\bar{o}_m\}$ *in* W *s.t.* $(\bar{o}_m, \bar{o}_{m+1}) \geq 1 \forall m \in N$ *and* $\bar{o}_m \to \bar{o}$ *as* $m \to +\infty$, *we have* $\alpha(\bar{o}_m, \bar{o}) \geq 1 \forall m \in N$. *Then, A has a fixed point.*

3. Results

Now, we define (α, φ) rational contraction for two mappings as given below:

Definition 3.1. Suppose (W, d_{f1}) be an F-metric space. The mappings $A, B : W \to W$ are known as (α, φ) rational contraction if $\exists$ two mappings $\varphi \in \Psi$ and $\alpha : W \times W \to [0, +\infty)$, s.t.

$$\alpha(\bar{o}, g) d_{f1}(A(\bar{o}), B(g)) \leq \varphi(T(\bar{o}, g)),$$

where

$$T(\bar{o}, g) = \max\left\{d_{f1}(\bar{o}, g), \frac{d_{f1}(\bar{o}, A(\bar{o}))d_{f1}(g, B(g))}{1 + d_{f1}(\bar{o}, g)}\right\}$$

for $\bar{o}, g \in W$.

Theorem 3.1. *Suppose* (W, d_{f1}) *be an F-metric space and* $B, A : W \to W$ *be both* α*-admissible and* (α, φ) *rational contraction. Let us consider that following conditions are fulfilled*:

(i) (W, d_{f1}) *is F-complete,*

(ii) $\exists \bar{o}_o \in W$ *s.t.* $\alpha(\bar{o}_o, A(\bar{o}_o)) \geq 1$, $\alpha(\bar{o}_o, B(\bar{o}_o)) \geq 1$

(iii) (a) *A and B is continuous*

(b) *if* $\{\bar{o}_m\}$ *is a seq. in* W, *such that* $\alpha(\bar{o}_m, \bar{o}_{m+1}) \geq 1 \forall m$ *and* $\bar{o}_m \to \bar{o}^* \in W$ *as* $m \to \infty$ *after that* $\alpha(\bar{o}_m, \bar{o}^*) \geq 1 \forall m \in N$.

(iv) $d_{f1}(\bar{o}_{2m}, w_{2m+1}) d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2}) \leq d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+1})$.

After that A and B have common fixed point $\bar{o}^* \in W$.

Proof. Let $\bar{o}_o \in W$ s.t. $\alpha(\bar{o}_o, A(\bar{o}_o)) \geq 1$. Set a seq. $\{\bar{o}_m\}$ in W by $A(\bar{o}_{2m}) = \bar{o}_{2m+1}$, $B(\bar{o}_{2m+1}) = \bar{o}_{2m+2}$ for $m = 0, 1, 2, 3, \ldots$.

If $\bar{o}_{n_0} = \bar{o}_{n_0+1}$ for some $n_0 \in N$, then proving that A and B have same fixed become easy.

As (A, B) is the pair of α-admissible

$$\alpha(\bar{o}_1, \bar{o}_2) = \alpha(A(\bar{o}_o), B(\bar{o}_1)) \geq 1[\because \alpha(\bar{o}_{2m}\bar{o}_{2m+1} \geq 1)]$$

$$\alpha(\bar{o}_2, \bar{o}_1) = \alpha(A(\bar{o}_1), A(\bar{o}_o)) \geq 1.$$

By induction we get, $\alpha(\bar{o}_m, \bar{o}m+1) \geq 1$. Also $\alpha(\bar{o}_{2m}, \bar{o}_{2m+1}) \geq 1$. By property with $\bar{o} = \bar{o}_{2m}$ and $g = \bar{o}_{2m+1}$ we have

$$\begin{aligned} d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2}) &= d_{f1}(A(\bar{o}_{2m}), B(\bar{o}_{2m+1})) \\ &\leq \alpha(\bar{o}_{2m}, \bar{o}_{2m+1}) d_{f1}(A(\bar{o}_{2m}), B(\bar{o}_{2m+1}))[\because \alpha(\bar{o}_{2m}, \bar{o}_{2m+1} \geq 1)] \\ &\leq \varphi T(\bar{o}_{2m}, \bar{o}_{2m+1}) \end{aligned}$$

$$\begin{aligned} T(\bar{o}_{2m}, \bar{o}_{2m+1}) &= \max\{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}_{2m}, A(\bar{o}_{2m}))d_{f1}(\bar{o}_{2m+1}, B(\bar{o}_{2m+1}))}{1 + d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}\} \\ &= \max\{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}\} \end{aligned}$$

if $\max\{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}\}$

$$= \frac{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}$$

then from above we obtain

$$\begin{aligned} d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2}) &\leq \varphi\left(\frac{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}\right) \\ &< \frac{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{\bar{o}}(\bar{o}_{2m}, \bar{o}_{2m+1})} \\ &\leq d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2}) \end{aligned}$$

which is a contradiction.

So

$$\max\{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1+d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}\} = d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})$$

$\therefore$ above equation becomes

$$d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2}) \leq \varphi d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})$$

for $m \in N$. Assume we have $(h, j) \in F \times [0, 1)$

$$d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2}) \leq \varphi^{m} d_{f1}(\bar{o}_{o}, \bar{o}_{1})$$

so that property (3D) from definition 1.1 is settled and, set $\varepsilon > 0$ from (2F) we have

$$h\left(\sum_{l=m}^{k-1} d_{f1}(\bar{o}_{1}, \bar{o}_{l+1})\right) \leq h\left(\sum_{l=m}^{k-1} \varphi^{l} d_{f1}(\bar{o}_{0}, \bar{o}_{1})\right)$$

$$\leq h\left(\sum_{m \geq m(\varepsilon)} \varphi^{l} d_{f1}(\bar{o}_{o}, \bar{o}_{1})\right)$$

$$\leq h(\epsilon) - j$$

for $k > m \geq m(\epsilon)$ by property (3D) and above equation we get

$$d_{f1}(\bar{o}_{m}, \bar{o}_{k}) > 0, \; k > m \geq m(\epsilon)$$

and

$$d_{f1}(\bar{o}_{m}, \bar{o}_{k}) \leq h\left(\sum_{p=m}^{k-1} d_{f1}(\bar{o}_{p}, \bar{o}_{p+1})\right) < h(\epsilon)$$

by (1F) $d_{f1}(\bar{o}_{m}, \bar{o}_{k}) < \epsilon, \; k > m \geq m(\epsilon)$ which gives $\{\bar{o}_{m}\}$ is F-Cauchy

Since (W, d_{f1}) is F-complete, $\exists \bar{o}^{*} \in W$. Such that $\bar{o}_{m} \to \bar{o}^{*}$ as $m \to \infty$ we too have $\bar{o}_{2m+2} \to \bar{o}^{*}$ and $\bar{o}_{2m+1} \to \bar{o}^{*}$. As A and B are continuous mappings, we get

$$\bar{o}^* = \lim_{m\to\infty} \bar{o}_{2m+2} = \lim_{m\to\infty} B\bar{o}_{2m+1} = B \lim_{m\to\infty} \bar{o}_{2m+1} = B\bar{o}^*$$

$$\bar{o}^* = \lim_{m\to\infty} \bar{o}_{2m+1} = \lim_{m\to\infty} A\bar{o}_{2m} = A \lim_{m\to\infty} \bar{o}_{2m} = A\bar{o}^*$$

As (W, d_{f1}) is F-complete there exist $\bar{o}^* \in W$ so that $\bar{o}_{2m} \to \bar{o}^*$ as $m \to \infty$ $\lim_{m\to\infty} d_{f1}(\bar{o}_{2m}, \bar{o}^*) = 0$. Suppose that $d_{f1}(Bw^*, \bar{o}^*) > 0$ by (1F) and (3D), we obtain

$$h(d_{f1}(B(\bar{o}^*), \bar{o}^*)) \le h(d_{f1}(B(\bar{o}^*), A(\bar{o}^*_{2m})) + d_{f1}(A(\bar{o}_{2m}), \bar{o}^*) + j$$

$$h(d_{f1}(B(\bar{o}^*), \bar{o}^*)) \le h(\alpha(\bar{o}^*, \bar{o}_{2m})d_{f1}(B(\bar{o}^*), A(\bar{o}_{2m})) + d_{f1}(A(\bar{o}_{2m}), \bar{o}^*) + j$$

$$\le h(\varphi(\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m}), \frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, A(\bar{o}_{2m}))}{1+d_{f1}(\bar{o}^*, \bar{o}_{2m})}\}) + d_{f1}(A(\bar{o}_{2m}), \bar{o}^*)) + j$$

$$< h(\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m}), \frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}{1+d_{f1}(\bar{o}^*, \bar{o}_{2m})}\} + d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) + j \quad \text{if}$$

$m \in N$

if $\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m}), \frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}{1+d_{f1}(\bar{o}^*, \bar{o}_{2m})}\}) = d_{f1}(\bar{o}^*, \bar{o}_{2m})$

then $h(d_{f1}(B(\bar{o}^*), \bar{o}^*)) \le h(d_{f1}(\bar{o}^*, \bar{o}_{2m}) + d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) + j)$.

Take the limit as $m \to \infty$ using (1F) and $\lim_{m\to\infty} d_{f1}(\bar{o}_{2m}, \bar{o}^*) = 0$

$$\therefore h(d_{f1}(\bar{o}_{2m}, \bar{o}^*)) = -\infty \text{ (by (2F))}$$

$$\lim_{m\to\infty} h(d_{f1}(B(\bar{o}^*), \bar{o}^*)) \le \lim_{m\to\infty} d_{f1}(\bar{o}^*, \bar{o}_{2m}) + d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) + j = -\infty$$

which proves $d_{f1}(B(\bar{o}^*), \bar{o}^*) = 0$ which leads to contradiction if

$$\max\left\{\left\{d_{f1}(\bar{o}^*, \bar{o}_{2m}), \frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m})}\right\}\right.$$

$$= \frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m})}$$

then

$$h(d_{f1}(B(\bar{o}^*), \bar{o}^*)) \leq h\left(\frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m})}\right) + d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) + j.$$

Take the limit as $m \to \infty$, using (1F) and $\lim_{m\to\infty} d_{f1}(\bar{o}_{2m}, \bar{o}^*) = 0$. We get

$$\lim_{m\to\infty} h(d_{f1}(B(\bar{o}^*), \bar{o}^*)) \leq \lim_{m\to\infty} h\left(\frac{d_{f1}(\bar{o}^*, B(\bar{o}^*))d_{f1}(\bar{o}_{2m}, \bar{o}_{2m+1})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m})}\right)$$

$$+ d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) + j = -\infty$$

which proves that $d_{f1}(B(\bar{o}^*), \bar{o}^*) = 0$ which is a contradiction

$\therefore$ we have

$$d_{f1}(B(\bar{o}^*), \bar{o}^*) = 0 \text{ i.o } B(\bar{o}^*) = \bar{o}^*.$$

Now we have to prove $A(\bar{o}^*) = \bar{o}^*$.

Now again assume that

$d_{f1}(\bar{o}^*, A\bar{o}^*) > 0$ by (1F) and (3D), we have

$$h(d_{f1}(\bar{o}^*, A(\bar{o}^*)) \leq h(d_{f1}(\bar{o}^*), B(\bar{o}_{2m+1})) + d_{f1}(B(\bar{o}_{2m+1}), A\bar{o}^*)) + j$$

$$h(d_{f1}(\bar{o}^*), A(\bar{o}^*)) \leq h(\alpha(\bar{o}^*, \bar{o}_{2m+1})d_{f1}(A(\bar{o}^*), B(\bar{o}_{2m+1})) + d_{f1}(B(\bar{o}_{2m+1}), \bar{o}^*)) + j$$

$$\leq h(\varphi(\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, B(\bar{o}_{2m+1}))}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})}\})$$

$$+ d_{f1}((\bar{o}_{2m+1}), \bar{o}^*)) + j$$

$$< h(\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})}\} + d_{f1}(\bar{o}_{2m+2}, \bar{o}^*) + j$$

if $m \in N$

if $\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})}\}) = d_{f1}(\bar{o}^*, \bar{o}_{2m+1})$

then $h(d_{f1}(\bar{o}^*, A(\bar{o}^*))) \leq h(d_{f1}(\bar{o}^*, \bar{o}_{2m+1}) + d_{f1}(\bar{o}_{2m+2}, \bar{o}^*)) + j.$

Take the limit as $m \to \infty$, using (1F) and $\lim_{m\to\infty} d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) = 0$

$$\lim_{m\to\infty} h(d_{f1}(\bar{o}^*, A(\bar{o}^*))) \leq \lim_{m\to\infty} d_{f1}(\bar{o}^*, \bar{o}_{2m+1}) + d_{f1}(\bar{o}_{2m+2}, \bar{o}^*) + j = -\infty \text{ (by (2F))}$$

which shows $d_{f1}(\bar{o}^*, A(\bar{o}^*)) = 0$ which leads to contradiction if

$$\max\{d_{f1}(\bar{o}^*, \bar{o}_{2m+1}), \frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, B(\bar{o}_{2m+2}))}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})}\}$$
$$= \frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})}.$$

Then

$$h(d_{f1}(\bar{o}^*), A(\bar{o}^*)) \leq h\left(\frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})} + d_{f1}(\bar{o}_{2m+1}, \bar{o}^*)\right) + j.$$

Take the limit as $m \to \infty$, using (1F) and $\lim_{m\to\infty} d_{f1}(\bar{o}_{2m+1}, \bar{o}^*) = 0$ we get

$$\lim_{m\to\infty} h(d_{f1}(\bar{o}^*), A(\bar{o}^*)) \leq \lim_{m\to\infty} h\left(\frac{d_{f1}(\bar{o}^*, A(\bar{o}^*))d_{f1}(\bar{o}_{2m+1}, \bar{o}_{2m+2})}{1 + d_{f1}(\bar{o}^*, \bar{o}_{2m+1})}\right)$$
$$+ d_{f1}(\bar{o}_{2m+2}, \bar{o}^*) + j = -\infty \text{ (by(2F))}$$

which shows that $d_{f1}(\bar{o}^*, A(\bar{o}^*)) = 0,$ a contradiction.

$\therefore$ we have $d_{f1}(\bar{o}^*), A(\bar{o}^*) = 0$ i.o $A(\bar{o}^*) = \bar{o}^*$

hence we deduced that $A(\bar{o}^*) = \bar{o}^*$ and $B(\bar{o}^*) = \bar{o}^*$.

Hence we concluded that A and B have common fixed point.

4. Conclusion

In this paper we have defined (α, φ) rational contraction for two mappings then we prove a fixed point theorem on F-metric space.

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