Advances and Applications in Mathematical Sciences
Volume 18, Issue 9, July 2019, Pages 851-866

STUDY OF SOME BEAMFORMING ALGORITHMS

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Abstract

In the paper, we discuss about Beam forming Technique. Beam forming is a type of spatial filtering to filter out the interference signal and get the desired result. It is a signal processing technique to form a beam in a particular direction for transmitting and receiving of a signal. In most of the application signals which is used are non-stationary, so with the help of some Adaptive Beam forming Algorithms like Least Minimum square (LMS) and Frost's Algorithm. We obtain optimum weights of the antenna Array, with the help of optimum weight we can adjust the main lobe to focus on the arriving direction of the desired signal, as well as suppress the interfering signal. By these ways, the antenna can receives the desired signal efficiently. It has many applications such as radar, seismology, sonar and wireless communications. Basically the speed of convergence and complexity are the main factors when choosing an Adaptive Beam forming algorithms. In this paper we study and compare some Beam forming algorithms using various Beam forming techniques. A quick overview is present of some Adaptive algorithms such as LMS and detailed discussion is present for Adaptive Linearly Constrained Minimum Variance (LCMV) beam former with the help of Frost's algorithm for LCMV beam forming. The simulation and results of Beam forming are obtained using MATLAB.

Motivation and Main Results

I. Introduction: The directional signal transmission and reception employing sensor arrays for the purpose, is a common sight in the modern communication. They heavily use beam forming as an array processing technique [1, 2]. In beam forming the elements of antenna array are tuned in such a way that the angles of interest have constructive interference but

2010 Mathematics Subject Classification: 11Y16, 65D15.

Keywords: adaptive beam forming; least mean squares; recursive least square; linearly constrained minimum variance.

Received January 2, 2019; Accepted January 28, 2019

other angles face destructive interference. Further the classification of narrowband and wideband beam forming is done based on the bandwidth of the signal. The narrowband beam forming is relatively simple as it depends only on the instantaneous linear combination of the received array signals but wideband requires additional processing for effective operation i.e. tapped delay-lines in filters. Wherever the requirement of interference suppression and signal enhancement comes into picture the wideband adaptive beam froming finds its application such as: communications, microphone arrays, radar, sonar, radio astronomy, seismology, medical diagnosis and treatment. A satisfactory output signal to-interference-plus-noise ratio (SINR) can be achieved using beam forming. Factors upon which such algorithms differ as: convergence, robustness and the computational cost.

So in this paper an attempt has been made to study the basics of beam forming algorithms. Rest of the paper is organized as section II introduces briefly the narrowband and wideband beam forming. The adaptive beam forming is then presented in section III. Specific LCMV beam forming with Frost's algorithm is presented in section IV. Finally the experiments and results are shown in section V followed by conclusions.

II. Beam forming: Beam forming is an arrangement to direct the system towards a particular direction for better transmission or reception of beams. It tries to minimize the involvement of hardware and mechanical interaction in the whole process. For better understanding the study of beam forming is divided into two parts as narrowband and wideband beam forming. In the following suspection these are explained.

(A). Narrowb and beamforming: The scheme of narrowband beam forming is shown in Figure 1. Sensors are distributed at different spatial positions which then sample the propagating waves in space. These spatial samples are then processed to null out the interfering signals and spatially extract the desired signal resulting beams pointing to the desired signals. In this scheme when the bandwidth of the impinging signal is such that the signals received by the opposite end of the arrays are correlated then the arrangement is called narrowband beam forming.

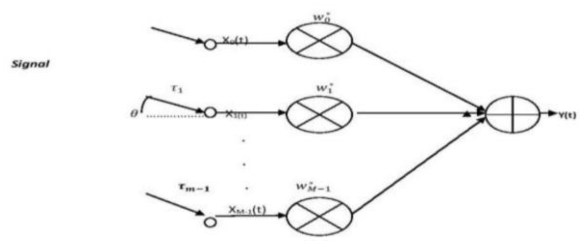


Figure 1. A general structure For Narrowband Beam forming.

Let there be N sensors. These sensors samples the wave field spatially. The output $z(t)$ is given by an instantaneous linear combination of the spatial samples $x_n(t)$ as shown in (1).

$$z(t) = \sum_{n=0}^{N-1} x_n(t) w_m^*. \tag{1}$$

The first sensor receives the signal

$$x_0(t) = e^{j\omega t}. \tag{2}$$

Similarly the signal received by n-th sensor is

$$x_n(t) = e^{j\omega(t-\tau_n)}, \tag{3}$$

where τ_n is the propagation delay for the signal from sensor 0 to sensor N.

The output of this narrowband beam former is given by

$$z(t) = e^{j\omega(t)} P(\omega, \theta). \tag{4}$$

The response of the beam former is denoted by $P(\omega, \theta)$ and is given by

$$P(\omega, \theta) = \mathbf{w}^H S(\omega, \theta), \tag{5}$$

where the vector $\mathbf{w}$ holds the N complex conjugate coefficients of the sensors as:

$$\mathbf{w} = [w_0^* \; w_1^* \ldots w_{N-1}^*] \tag{6}$$

and the steering vector

$$\mathbf{s}(\omega, \theta) = [1 \; e^{j\omega\tau_1} \ldots e^{j\omega\tau_{N-1}}]. \tag{7}$$

Thus the uniformly spaced narrowband beam former gives a final response as:

$$P(\omega,\ \theta) = \sum_{n=0}^{N-1} e^{jn\pi \sin\theta} w_n^*. \tag{8}$$

B. Wideband Beam forming: The narrowband structure of beam forming is modified slightly to cater for the wide band need of the system [1, 2]. The modified structure for wideband beam forming is presented in Figure 2.

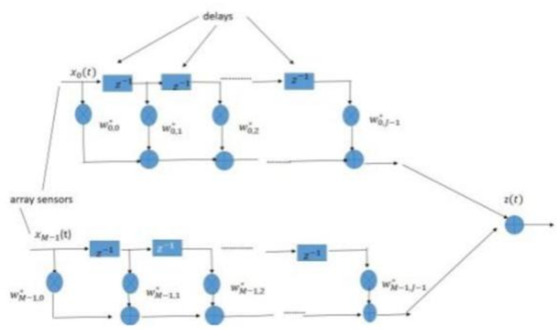


Figure 2. A general structure of Wideband Beam forming.

Now the steering vector in this schemes can be written as:

$$\mathbf{s}_n(\omega,\ \theta) = [1\ e^{-j\omega\tau_1\theta_n}\ \ldots\ldots e^{-j\omega\tau_N\theta_n}]^T \tag{9}$$

and the weight vectors are present now as

$$\mathbf{w}(\omega) = [w_0(\omega),\ w_1(\omega)\ldots\ldots w_{N-1}(\omega)]^T. \tag{10}$$

In the wideband an infinite number of frequency componets are possible. To handle them a number of weight values are used here. Different frequencies are handled by different weights. These weights are arranged in the tapped delay fashion in FIR/IIR filters [5]. The output of the wideband beam former is given by the expression:

$$z(t) = \sum_{n=0}^{N-1} \sum_{i=0}^{I-1} x_m(t - iT_s) w_{n,i}^*. \tag{11}$$

The delay component is presented by I which are associated with each of the N sensor channels. In matrix form (11) can be rewritten as:

$$z(t) = \mathbf{w}^H \mathbf{x}(t). \tag{12}$$

Here the weight vector W holds all the (NxI) sensor coefficients with:

$$\mathbf{w} = [\mathbf{w}_0, \mathbf{w}_1, \ldots\ldots \mathbf{w}_{I-1}]^T. \tag{13}$$

The input data is also taken in the vector form as

$$\mathbf{x} = [\mathbf{x}_0(t)\ \mathbf{x}_1(t - T_s), \ldots\ldots, \mathbf{x}_{I-1}(t - (I-1)T_s]^T, \tag{14}$$

where

$$\mathbf{x}(t - iT_s) = [\mathbf{x}_0(t - iT_s), \mathbf{x}_1(t - iT_s) \ldots \mathbf{x}_{N-1}(t - iT_s)]^T. \tag{15}$$

Assuming the phase of the signal is zero at the first sensor. So the signal at the first sensor is given by the expression:

$$x_0(t) = e^{j\omega t}\ \ x_0\, x_n\, x_n(t - iT_s) = e^{j\omega(t-(\tau_m + iT_s)}, \tag{16}$$

where $n = 0, 1, \ldots, N-1$, $i = 0, 1, \ldots, I-1$.

The Beam former array output is given by the expression:

$$z(t) = e^{j\omega t} \sum_{n=0}^{N-1} \sum_{i=1}^{I-1} e^{-j\omega(\tau_n + iT_s)} w_{n,i}^* w_{n,i}^* = e^{j\omega t} \times P(\theta, \omega), \tag{17}$$

where $P(\theta, \omega)$ is expressed in vector form as:

$$P(\theta, \omega) = \mathbf{w}^H S(\theta, \omega), \tag{18}$$

where $\mathbf{s}(\theta, \omega)$ is steering vector of Wideband Beamforming.

III. Adaptive Wideband Beamforming

In various beamforming applications depending on the scenario we often need to update the vector according to the received array data to achieve the optimum solution at the receiver [7]. For the updating of weights, all kinds of adaptive algorithms are used either in block by block or in continuous manner.

The wideband beamformer further can be divided into two types-the referenced signal based adaptive beamformers and blind beam formers. Being simple only referenced signal based adaptive beam former is discussed in the following section.

A. Reference Signal-Based Beamformer

The major task of a beam former is to estimate the direction from where the signal is arriving. They require continuous updation of filter weights to keep a good track of the signal. To update the filter weights if a reference signal is available then the scheme is called as the reference signal based beam former. The beam-forming can be performed with the help of Multiple Channel Adaptive Filter (MCAF) as shown in below Figure 3.

In Multiple-Channel Adaptive Filter, the N received sensors signal are fed into this adaptive filter and the filter coefficients are adjusted by minimizing the cost function which is based on the error signal $e[n]$ calculated by taking the difference between reference signal $r[n]$ and the MCAF output $y[n]$ [8, 9]. If the number of channels in adaptive filter is N and the length of each channel is I then total number of adaptive coefficients are (NxI). For input signal vector x and weight vector W, the

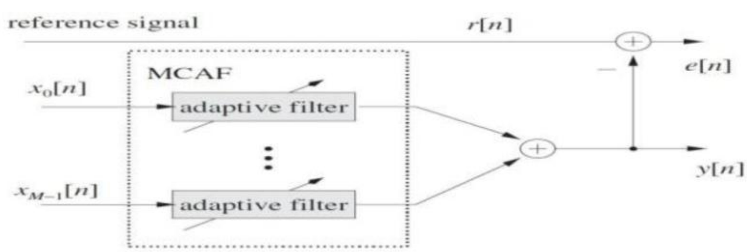


Figure 3. The reference signal based Wideband Beam former error signal is.

$$e[n] = r[n] - y[n] = r[n] - \mathbf{w}^H \mathbf{x}[n].$$

This error is used to adjust the filter weight values w and method is called adaptation of weights. The adaptation follow some criterion. A popular criterion is to minimize the error in a mean square sense.

(1) LMS Algorithm: The LMS Algorithm is a stochastic gradient descent method [8, 9]. The cost function involved is popularly known as MMSE

(Minimum Mean Square Error). The minimum point of the cost function can be founded by an update of the successive correction of the weight vector w[n] from an initial vector in the direction of the negative gradient of the MMSE [3]. The filter weight updation is done by

$$w[n+1] = w[n] - \mu\nabla\zeta[n]. \tag{19}$$

Here $\nabla\zeta[n] = -E[e[n]X^*[n]]$.

$$E[e[n]x[n]] = \frac{1}{L}\sum_{l=0}^{L-1} e[n-l]x^*[n-l]-l].$$

Here we suppose ensemble average is equal to time average for $L = 1$. The variable which denotes the value of the gradient vector at time n, which can be evaluated as:

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \mu(p - R_{xx}\mathbf{w}[n]). \quad (20).$$

Each of the expectation values can be replaced by an instantaneous single sample estimate based on the input $x[n]$ and the desired signal $d[n]$.

$$R_{xx}^*[n] = E[x[n]x^H[n]] \text{ and } P^* = E[x[n]d^*[n]]. \tag{21}$$

Thus the weight updation in (20) is simplified as

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \mu(d^*[n] - \mathbf{x}^H[n])\mathbf{w}[n])x[n]. \tag{22}$$

The expression in (22) is the popular algorithm of LMS. The stability of LMS algorithm depends upon the step size, which should satisfy the condition:

$$0 < \mu < \frac{2}{\lambda_{\max}}, \tag{23}$$

where $\lambda_{\max}$ is maximum eigenvalue of the convariance matrix R_{xx}. It is given by the inequality:

$$\lambda_{\max} \le \sum_{i=0}^{p}\lambda_i = trace(R_{xx}) = (p+1)r_x(0), \tag{24}$$

where $r_x(0) = E[|x(n)|^2]$.

Using equation (24), equation (23) becomes

$$\mu < \frac{2}{(p+1)E[|x[n]|^2]}, \tag{25}$$

when exceeding this limit the LMS algorithm becomes unstable.

IV. Linearly Constrained Minimum Variance Beam forming

The major task is to minimise the power of the beam former output. This minimisation can become easy if the direction of arrival (DOA), angle of the signal of interest and their bandwidth range are known a prior. When the adaptive algorithm, that minimises the signal variance, has to work under a few constraints then the whole system is better known as Linearly Constrained Minimum Variance (LCMV) Beamformer [4, 6]. The constraints are formed such as they will help in preserving the signals coming on the array from the desired direction and the effect of interfering signals from other directions are suppressed.

A simple constraints setup can be given as $T_0[10]$

$$\mathbf{w}^H \mathbf{s}(\theta, \omega) = T_0, \tag{26}$$

where ω and θ are the frequency and angle representation respectively. So the output power or variance is

$$E\,|\,y[n]^2 = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}, \tag{27}$$

where,

$$\mathbf{R}_{xx} = E(xx^H) \tag{28}$$

$\mathbf{R}_{xx}$ is the autocorrelation matrix of observed array data. The LCMV beam forming problem is defined as

$$\mathbf{w} = \arg\min \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}.$$

If there are $r < (N \times I)$ number of linearly independent constraints endured on W, the constraint matrix can be given by the equation:

$$\mathbf{K}^H \mathbf{w} = \mathbf{g}, \tag{29}$$

where $\mathbf{K}$ is the constraint matrix having size $(NI \times r)$ and $\mathbf{g}$ is the response vector of size $(r \times 1)$.

A. Solution to the LCMV Problem : The solution to the LCMV problem of (30) can be solved by using the method of Lagrangian Multipliers. To apply the Lagrangian Multipliers method, a Lagrangian function [7] is formed using (29).

$$\zeta(w, \lambda) = \mathbf{w}^H \mathbf{R}_{xx}\mathbf{w} + (\lambda^H \mathbf{K}^H \mathbf{w} - \mathbf{g}) + (\lambda^T \mathbf{K}^T \mathbf{w}^* - \mathbf{g}^*). \tag{30}$$

Differentiating (30) with respect to w, and equating to zero the optimal weight vector w_{opt} is obtained as

$$\mathbf{w}^H = \lambda \mathbf{K}\mathbf{R}_{xx}^{-1} \quad \mathbf{w}_{opt} = -\mathbf{R}_{xx}^{-1}\mathbf{K}\lambda. \tag{31}$$

Since the optimal weight vector w_{opt} must satisfy the following equation:

$$g = \mathbf{K}^H \mathbf{w}. \tag{32}$$

Using equation (31) we can write:

$$\mathbf{g} = -\mathbf{K}^H \mathbf{R}_{xx}^{-1}\mathbf{K}\lambda. \tag{33}$$

Solving the above equation (35) and using equation (33) we get:

$$\lambda = -\{\mathbf{K}^H \mathbf{R}_{xx}^{-1}\mathbf{K}\}^{-1}\mathbf{g}. \tag{34}$$

The above equation (37) represents the solution to the LCMV optimization problem.

B. Frost's Algorithm for LCMV Beamforming. The calculations of $\mathbf{R}_{xx}$ involves the second order statistics of the array data which are either unknown or time-dependent. Thus instead of directly calculating them coefficients of weight vector w is determined by constrained adaptive algorithms. Frost's algorithm is one such polar algorithm for this purpose.

Initially, the weight vector is set as $w(0) = \mathbf{K}\mathbf{K}^H \mathbf{K}_{\mathbf{g}}^{-1}$.

It satisfies the constraint of (32). At each iteration, the vector w is updated as

$$\mathbf{w}(n+1) = \mathbf{w}[n] - \mu(\mathbf{R}_{xx}\mathbf{w}[n] + \mathbf{K}\lambda[n]). \tag{35}$$

Since equation (38) must satisfy the constraint in equation (32), the Lagrange multipliers $\lambda[n]$ is calculated with the help of (35) and (32). This solved values of $\lambda[n]$ is the replaced into (35) to get

$$\mathbf{w}[n+1] = \mathbf{K}(\mathbf{K}^H\mathbf{K})^{-1}\mathbf{g} + \mathbf{P}(w[n] - \mu\mathbf{R}_{xx}\mathbf{w}[n]). \tag{36}$$

Where, $P = I - \mathbf{K}(\mathbf{K}^H\mathbf{K})^{-1}\mathbf{K}^H$.

As the exact second order statistics $\mathbf{R}_{xx}$ are unknown thus they are replaced by their simple approximations $\mathbf{R}_{xx} = \mathbf{x}\mathbf{x}^H$. The weight update expression is then given by

$$\mathbf{w}[n+1] = \mathbf{K}(\mathbf{K}^H\mathbf{K})^{-1} + \mathbf{P}(\mathbf{w}[n] - \mu e^*[n]x[n]), \tag{37}$$

which is obtained after minimisation of instantaneous squared error. This is know as the popular Frost's Algorithm.

V. Simulation and Result

The narrowband and wideband beam formers were simulated on MATLAB. Results of these simulations are briefly discussed in the following sections.

A. Narrow band Beamforming: The narrowband expression of (8) simulated in Figure 4. The number of sensors used for Narrowband Beam forming is 10, Beam response pointing in the direction of $\theta = [-30^\circ, 30^\circ]$ and suppressing signal after from the direction of $\theta = [-90^\circ, 45^\circ]$ and $[45^\circ, 90^\circ]$ Weight Vector $[\mathbf{w}^H] = [0.0332, 0.0302, -0.1201, 0.0530, 0.4122, 0.4122, 0.0530, -0.1201, 0.0302, 0.0332]$, Beam pattern (BP) is in db, which is defined as:

$$BP = 20\left(\log 10 \frac{|P(\theta, \omega)|}{\max|P(\theta, \omega)|}\right).$$

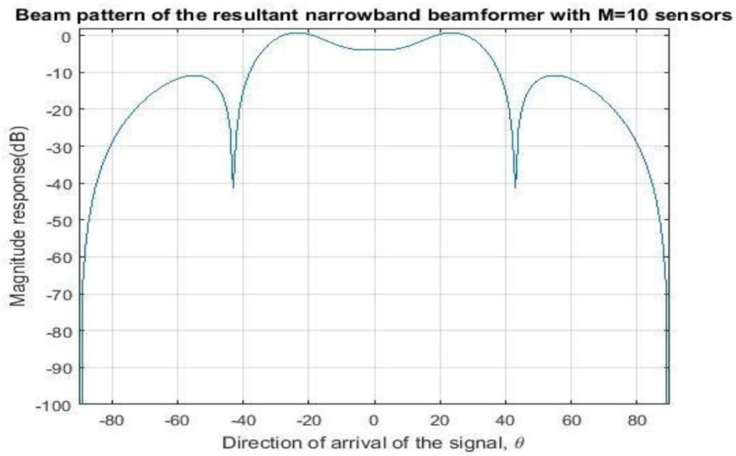


Figure 4. Beam pattern before steering.

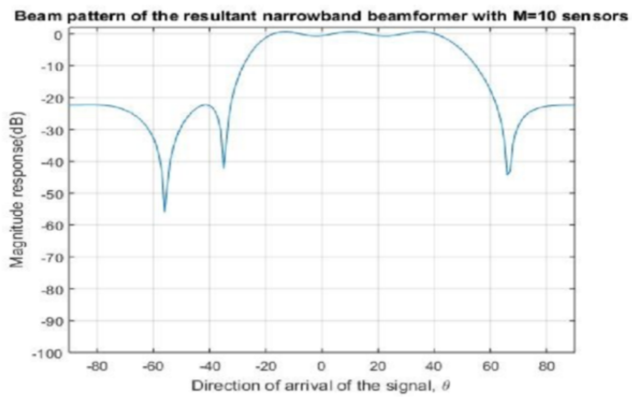


Figure 5. Previous figure from Figure 4 is steered to +10°.

When the shifting is applied then beam starts shifting in the desired direction. Figure 5 shows the beam pattern after shifting right by adding appropriate steering delays or phase shifts as shown in Figure 5.

B. Wide band Beam forming: The wideband beam former was also simulated and presented here. In this total number of sensors used are 4 and tapped delay length is 3. Thus 12 weight values are used in weight vector $(\mathbf{w}^H) = [0, 0, 0, 0, 0.3, 0.3, 0.3, 0.3, 0, 0, 0, 0]^T$. The beam patterns presented in Figure 6 (without shifting) and Figure 7 (with +10° shift).

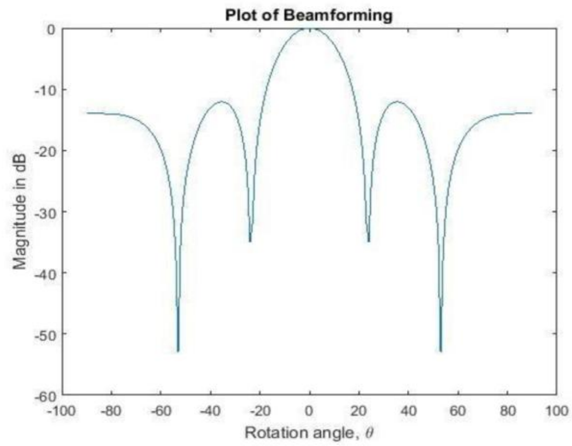


Figure 6. Wideband Beam pattern without steering.

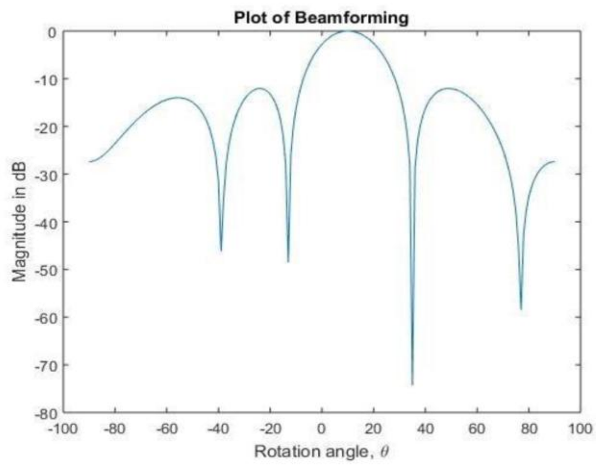


Figure 7. Wideband Beam pattern with steering 10°.

The main lobe of Figure 6 is shifted right to the direction $\theta = 10^{\circ}$. When the beam pattern depends on frequencies then a 3-dimensional result can clear some confusions. Such 3 dimensional are presented in Figures 8-9.

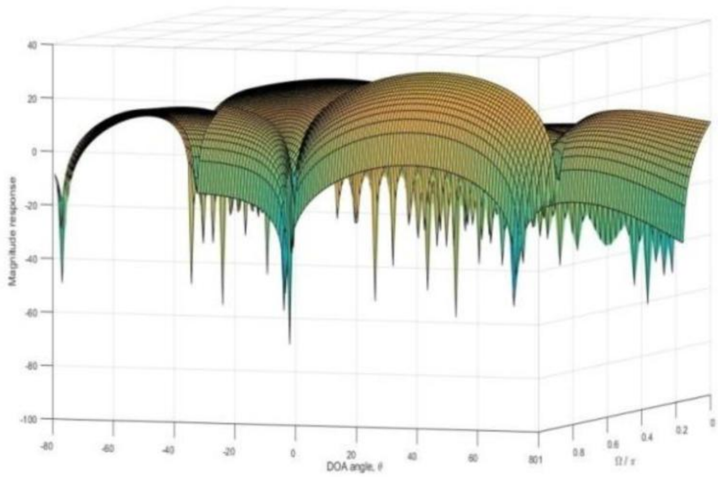


Figure 8. 3-Dimensional presentation of wideband beam form.

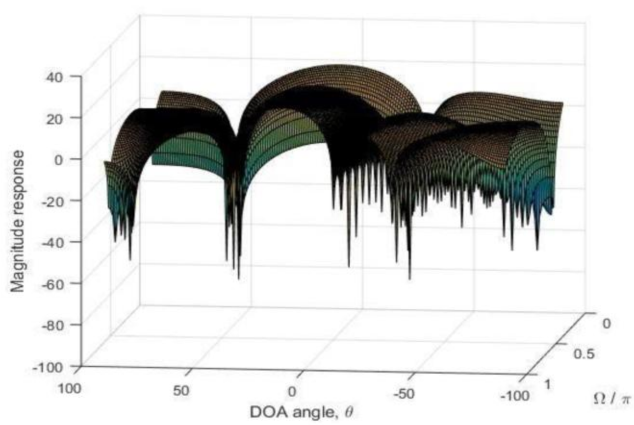


Figure 9. The wide band beam is shifted to 10^0.

Figure 8 presents a 3-Dimensional wideband Beam pattern based on an equally spaced liner array with $M = 4$, $J = 3$ and $\mu = 1$. Here the values of the weight coefficient are fixed and the resultant Beam form will maintain a fixed response independent of the signal or interference. Similarly a broadside main beam for a linear wideband array with $M = 21$ sensors and $J = 25$ coefficients for each of the attached FIR filter is shown in Figure 9. It is resulted after the broadside main beam is steered to an offbroadside direction 10^o.

The width of main lobe and number of side lobes are highly dependent upon the number of sensors used. In the Figures of 10 and 11 different number of sensors were used. It can be seen that a higher number of sensors although increase the accuracy of direction but number of side lobes are also increased resulting in power loss. Thus a tradeoff is required here.

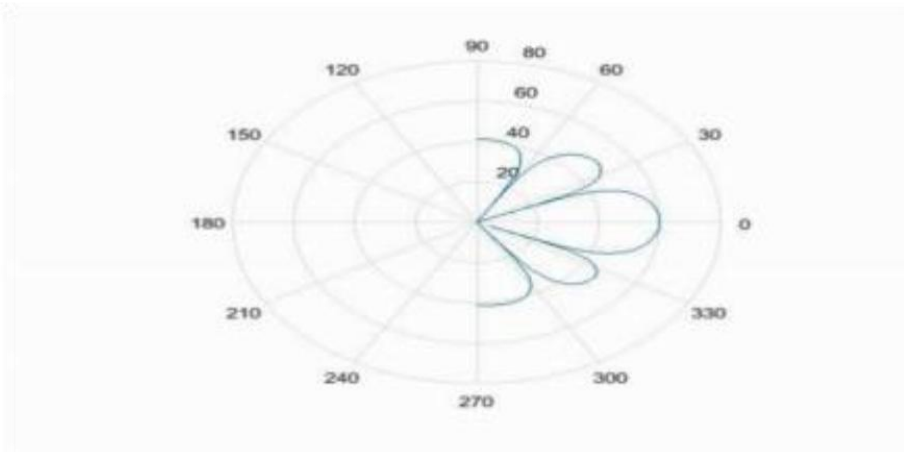


Figure 10. Polar plot of Beam pattern, $M = 5$ and direction of arrival is 0°.

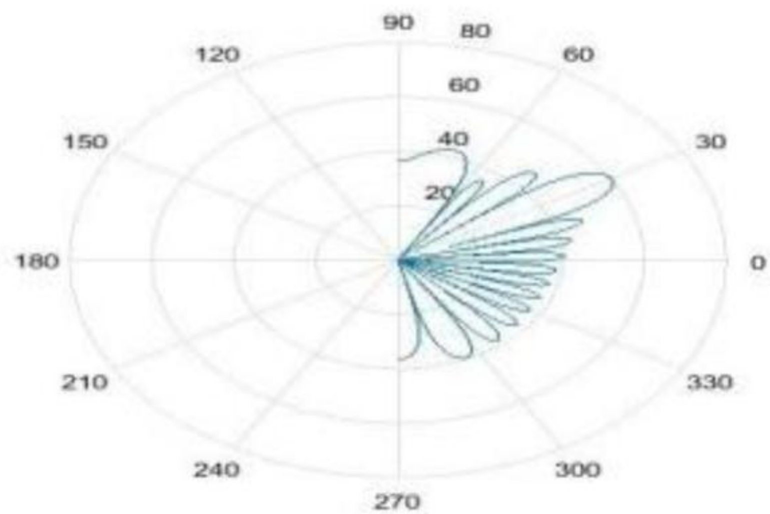
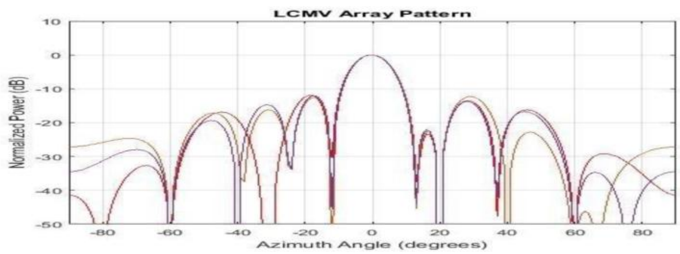


Figure 11. Polar plot of Beam pattern, $M = 15$ and direction of arrival is 30°.

C. LCMV Beam pattern (using Frost's Algorithm): The LCMV algorithm is simulated and result is presented in Figure 12. The number of sensors used are $N = 10$ and the FIR filter length I for each sensors is $I = 50$. The objective is to receive a signal of interest from the broadside $\theta = 0^\circ$ and suppress two wide band interfering signals with a normalized frequency bandwidth $\Omega = [0.4\Pi, \Pi]$, arriving from angles $\theta = -30^\circ$ and -40° respectively. The SIR for each interfering signal is 21dB and the SNR is 21dB. Step size used is $4 * 10^{-6}$ in the adaption.



VI. Conclusions

In this paper a study for beamforming with few words the beamforming is introduced. Basic difference in narrowband and wideband beamformers were discussed. The popular wideband adaptive beam former Frost's Beam former, is presented and all these were also simulated. The results presented help to understand various complex expressions involved. The future works can be the development of new updation algorithms having better performance than the conventional one.

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