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MAGIC AND ANTIMAGIC LABELINGS ON CAYLEY DIGRAPHS OVER THE GROUPS $Zp^n \oplus Zp^n$ AND $Z_m \oplus Z_n$

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Abstract

In this paper we show the existence of super vertex (a, d)- antimagic labeling and vertex magic total labeling for Cayley digraphs arising from the groups $Zp^n \oplus Zp^n$ and $Z_m \oplus Z_n$.

1. Introduction

By a digraph $G = (V, E)$ we mean a finite digraph without self loops and multiple arcs and is defined by a set V of vertices and a set E of arcs or directed edges. The set E is a subset of elements (u, v) of $V \times V$. The out-degree (or in-degree) of a vertex u of a digraph G is the number of arcs (u, v) (or (v, u)) of G and is denoted by $d^+(u)$ (or $d^-(u)$). A digraph G is said to be regular of out-degree d if $d^+(u) = d^-(u)$ for every vertex u of G. Let $| V | = p$ and $| E | = q$. The concept of graph labeling was introduced by Rosa in [1].

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A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Labeled graphs serve as useful models for a broad range of applications such as coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management [12]. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, mean labeling, arithmetic labeling etc., have been studied. Almost all of the labelings mentioned in Gallian's dynamic survey [2] deal with labelings of undirected graphs. Bloom et al. [3] defined magic labelings for directed graphs. MacDougal et al. [4] introduced the notion of vertex-magic total labeling. For a graph G with p vertices and q edges, a vertex-magic total labeling is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ such that for every vertex $u \in V(G)$, $f(u) + f(uv) = k$ for some constant k. This constant k is called the magic constant of the vertex-magic total labeling. A vertex-magic total labeling is super if f $f(V(G)) = \{1, 2, 3, \ldots, p\}$, we call it as V-super vertex-magic labeling. A graph G is called V-super vertex-magic if it admits a V-super vertex-magic labeling. Swaminathan and Jeyanthi [5] called a vertex-magic total labeling as a super vertex-magic total labeling if $f(E(G)) = \{1, 2, \ldots, q\}$. Thamizharasi and Rajeswari [7] studied magic labelings of Cayley digraphs and its line digraphs.

An antimagic labeling of a graph with p vertices q edges is a bijection $f : E(G) \rightarrow \{1, 2, \ldots, q\}$ such that the values at the vertices are distinct, where the value of v is the sum of the labels on edges incident to v. In [13], Hartsfield and Ringel made a conjecture on vertex-antimagic labeling and Martin Baca proposed a conjecture about edge-antimagic vertex labeling [14]. Thirusangu et al. [6] studied super vertex (a, d) antimagic labeling and vertex magic total labeling of certain classes of Cayley digraphs.

The Cayley digraph of a group provides a method of visualizing the group and its properties. The idea of representing a group in such a manner was originated by Cayley in 1878. The Cayley graphs and Cayley digraphs are excellent models for interconnection networks [8, 9, 10]. Many well-known interconnection networks are Cayley digraphs.

2. Preliminaries

In this section we give the basic notions relevant to this paper. Let $G = G(V, E)$ be a finite, simple, and undirected graph with v vertices and e edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers). Here, we deal with labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labeling the vertex labeling or the edge labeling or the total labeling, respectively.

Definition 2.1. The vertex-weight of a vertex v in G under an edge labeling to be the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels corresponding to all the edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or vertex-magic total labeling, respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling, respectively.

Definition 2.2. The edge-weight of an edge e under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with e. Under a total labeling, we also add the label of e. Using edge-weight; we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling.

Definition 2.3. A digraph $G = (V, E)$ is defined by a set V of vertices and a set E of arcs or directed edges. The set E is a subset of elements (u, v) of $V \times V$. The out-degree (or in-degree) of a vertex u of a digraph G is the number of arcs (u, v) (or (v, u)) of G and is denoted by $d^{+}(u)$ (or $d^{-}(u)$). A digraph G is said to be regular of out-degree d if $d^{+}(u) = d^{-}(u)$ for every vertex u of G. Let $|V| = p$ and $|E| = q$.

Definition 2.4. A vertex-magic total labeling of a digraph is an assignment of integers $1, 2, \ldots, p + q$ to the vertices and the edges of G, so that at each vertex, the vertex label and the labels of its outgoing edges incident at that vertex, add to a fixed constant, called the magic constant of G. Such a labeling is V-super vertex-magic total if $f(V(G)) = \{1, 2, \ldots, p\}$ and is E-super vertex-magic total if $f(E(G)) = \{1, 2, \ldots, q\}$. A digraph that admits a V-super vertex-magic total labelling is called V-super vertex-magic. Similarly, a digraph that admits an E-super vertex magic total labeling is called E-super vertex-magic.

Definition 2.5. By an (a, d)- edge-antimagic vertex labeling we mean a one-to-one mapping f from V onto $\{1, 2, \ldots, p\}$ such that the set of edge-weights of all edges in G is $\{a, a + d, \ldots, a + (q - 1)d\}$, where a and d are two fixed positive integers. An (a, d)- edge-antimagic total labeling is defined as a one-to-one mapping f from $E \cup V$ onto the set $\{1, 2, \ldots, p + q\}$, so the set of edge-weights of all edges in G is equal to $\{a, a + d, \ldots, a + (q - 1)d\}$, for two positive integers a and d.

Definition 2.6. A (p, q)- digraph $G = (V, E)$ is said to have a super-vertex (a, d)- antimagic labeling if there exists a function $f : \{V \cup E\} \to \{1, 2, \ldots, p + q\}$ such that $f(v) = \{1, 2, \ldots, p\}$ and, for any vertex v_i, the sums of the labels of the outgoing edges of vi and its own label are distinct. Moreover the set of all such distinct elements associated with V of G is equal to $\{a, a + d, \ldots, a + (p - 1)d\}$, where a and d are any two positive integers.

Definition. 2.7. Let G be a finite group, and let S be a generating subset of G. The Cayley digraph Cay (G, S) is the digraph whose vertices are the elements of G, and there is an edge from g to gs whenever $g \in G$ and $s \in S$. If $S = S^{-1}$, then there is an arc from g to gs if and only if there is an arc from gs to g.

3. Main Results

3.1. Super vertex $-(a, d)$ antimagic labeling for a Cayley digraph associated with a group $Zp^n \oplus Zp^n$

Algorithm 3.1.

Input. The group $Zp^n \oplus Zp^n$ with the generating set $\{(0, 1), (1, 0)\}$.

Step 1. Construct the Cayley digraph Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$.

Step 2. Denote the vertex set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$. $V = \{u_{00}, u_{01}, u_{02}, \ldots, u_0, p^n, u_{1,0}, u_{1,1}, \ldots, u_{p-1}^n, u_{p-1}^n\}$ as $V = \{v_1, v_2, v_3, \ldots, v_m\}$. Here $m = (p^n)^2$.

Step 3. Denote {the edge set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ as $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2m}\}$, where,

E_a = the set of all outgoing arcs from v_i generated by $(1, 0)$;

E_b = the set of all outgoing arcs from v_i generated by $(0, 1)$.

Step 4. Define f such that $f(v_i) = i$, for $1 \leq i \leq m$.

Step 5. Define θ_a such that $\theta_a(v_i) = 3m + 1 - f(v_i)$.

Step 6. Define θ_b such that $\theta_b(v_i) = f(v_i) + m$.

Output: Labeled Cayley digraph.

Theorem 3.1. *The Cayley digraph associated with* $Zp^n \oplus Zp^n$ *group admits super-vertex* (a, d)*- antimagic labeling*.

Proof. From the construction of the Cayley digraph for the dihedral group $Zp^n \oplus Zp^n$, we have that Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ has m vertices and $2m$ arcs. Let us denote the vertex set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ as $V = \{v_1, v_2, v_3, \ldots, v_m\}$. To prove Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ admits super-vertex (a, d)- antimagic labeling we have to show that for any vertex v_i, the sums of the labels of vertex v_i and the label of its outgoing arcs are distinct and the set of all such distinct elements corresponds to V being equal to $\{a, a + d, \ldots, a + (n-1)d\}$, where a

and d are any two positive integers. Consider an arbitrary vertex $v_i \in V$ of the Cayley digraph. Note that each vertex has exactly two outgoing arcs out of which one arc is from the set E_a and another is from the set E_b where E_a and E_b are as defined in the above algorithm. Now define maps f, θ_a and θ_b as defined in steps 4, 5 and 6 of the above algorithm.

Thus for any vertex $v_i \in V$, $f(v_i) + \theta_a(v_i) + \theta_b(v_i) = f(v_i) + 3m + 1 - f(v_i) + f(v_i) + m = f(v_i) + 4m + 1 = (4m + 1 + i)$. Moreover for any two integers i, j such that $i \neq j$, $f(v_i) \neq f(v_j)$ and the sums of the labels are also distinct. Also for any integer i, $f(v_{i+1}) - f(v_i) = 1 = d$ (say). The initial value of the label is $a = 4m + 2$ which proves that the vertex sums forms an arithmetic progression $\{a + a + d, , a + 2d, \ldots, a + (n-1)d\}$. Hence the Cayley digraph associated with $Zp^n \oplus Zp^n$ group admits super-vertex (a, d)- antimagic labeling. Hence the theorem.

Example 3.1. Antimagic labeling for a Cayley digraph associated with a group $Zp^n \oplus Zp^n$, is shown in figure 3.1

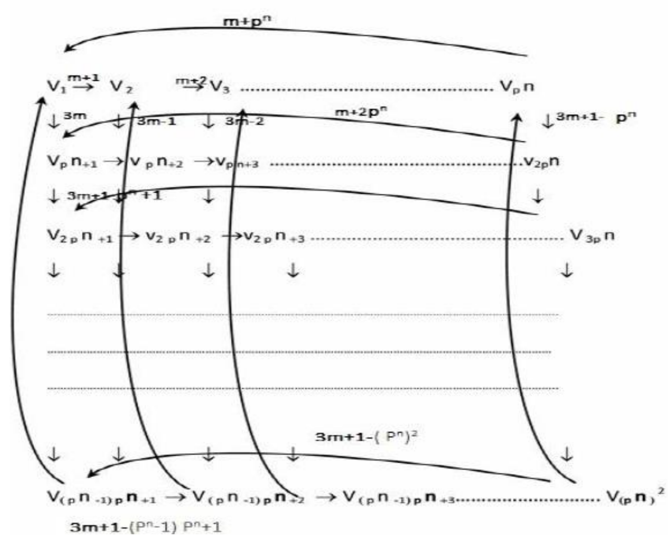

Figure 3.1. Antimagic labeling for a Cayley digraph associated with a group $Zp^n \oplus Zp^n$.

3.2. Super vertex $-(a, d)$ antimagic labeling for a Cayley digraph associated with a group $Z_m \oplus Z_n(m, n) = 1$

Algorithm 3.2.

Input: The group $Z_m \oplus Z_n(m, n) = 1$ with the generating set $\{(0, 1), (1, 0)\}$.

Step 1. Construct Cayley digraph Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$

Step 2. Denote the vertex set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ as $V = \{v_1, v_2, v_3, \ldots, v_{mn}\}$.

Step 3. Denote the edge set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ as $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2mn}\}$, where: $E_a =$ the set of all outgoing arcs from v_i generated by (1, 0);

$E_b =$ the set of all outgoing arcs from v_i generated by (0, 1).

Step 4. Define f such that $f(v_i) = i/2 + 1$, for $0 \leq i \leq (2m - 2)$, $i = 0, 2, 4, \ldots, (2m - 2)$.

Step 5. Define ψ_a such that $\psi_a(v_i) = 2m + 2 - f(v_i)$

Step 6. Define ψ_b such that $\psi_b(v_i) = (mn + 2) - f(v_i)$.

Output: Labeled Cayley digraph.

Theorem 3.2. *The Cayley digraph associated with Cay $Z_m \oplus Z_n, (m, n) = 1$ group admits super-vertex (a, d)- antimagic labeling.*

Proof. From the construction of the Cayley digraph for the group $Z_m \oplus Z_n, (m, n) = 1$, we have that Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ has m vertices and $2m$ arcs. Let us denote the vertex set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ as $V = \{u_{00}, u_{01}, u_{02}, \ldots, u_{0,n-1}, \ldots, u_{1,0}, u_{1,1}, \ldots, u_{m-1,n-1}\}$ where $V = \{v_1, v_2, v_3, \ldots, v_{mn}\}$.

To prove Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ admits super-vertex

(a, d)- antimagic labeling we have to show that for any vertex v_i the sums of the labels of vertex v_i and the label of its outgoing arcs are distinct and the set of all such distinct elements corresponds to V being equal to $\{a, a + d, \ldots, a + (n - 1)d\}$, where a and d are any two positive integers. Consider an arbitrary vertex $v_i \in V$ of the Cayley digraph Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$. By construction of the Cayley digraph, we have that each vertex has exactly two outgoing arcs out of which one arc is from the set E_a and another is from the set E_b where E_a and E_b are as defined in the above algorithm. Now define maps f, ψ_a and ψ_b as defined in steps 4, 5 and 6 of the above algorithm.

Thus for any vertex $v_i \in V, f(v_i) + \psi_a(v_i) + \psi_b(v_i) = f(v_i) + 2mn$ $+2 - f(v_i) + (mn + 2) - f(v_i) = (3mn + 3 - i/2)$. Moreover for any two integers i, j such that $i \neq j, f(v_i) \neq f(v_i)$ and the sums of the labels are also distinct. Also for any integer $i, f(v_{i+1}) - f(v_i) = 1 = d$ (say). The initial value of the label is $a = 3m + 2$ which proves that the vertex sums form an arithmetic progression $\{a, a + d, a + 2d, \ldots, a + (n - 1)d\}$. Hence the Cayley digraph associated with group admits super-vertex (a, d)- antimagic labeling. Hence the theorem.

Example 3.2. Super-vertex (a, d)- antimagic labeling for a Cayley digraph associated with a group $Z_m \oplus Z_n, (m, n) = 1$ is given in figure 3.2.

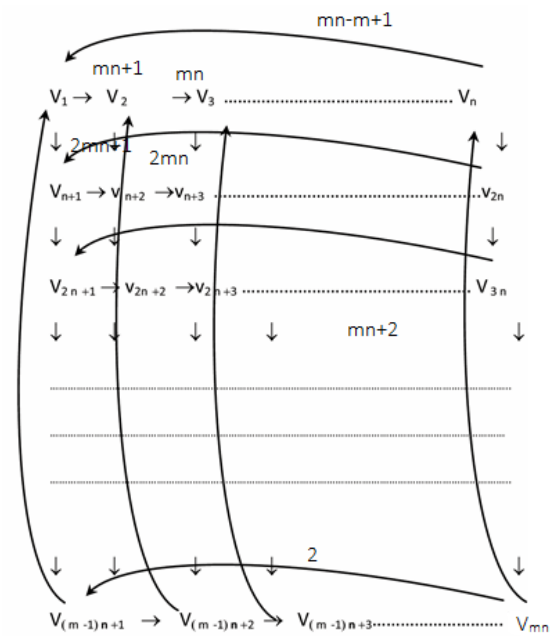

Figure 3.2. Super-vertex (a, d)- antimagic labeling for a Cayley digraph associated with a group $Z_m \oplus Z_n$, $(m, n) = 1$.

3.3. Magic labeling for a Cayley digraph associated with group $Zp^n \oplus Zp^n$

In this section we obtain magic labeling for Cayley digraphs associated with group $Zp^n \oplus Zp^n$

Algorithm 3.3.

Input: The group $Zp^n \oplus Zp^n$ with the generating set $\{(0, 1), (1, 0)\}$

Step 1. Construct the Cayley digraph Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$

Step 2. Let $V = (v_1, v_2, v_3, \ldots, v_m\}$ where $m = (p^n)^2$ denotes the vertex set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$.

Step 3. Let $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2m}\}$ denotes the edge set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$

E_a = the set of all outgoing arcs from v_i generated by (1, 0);

E_b = the set of all outgoing arcs from v_i generated by (0, 1);

Step 4. Define *f* such that $f(v_i) = i - 1,$ for $2 \leq i \leq m + 1$.

Step 5. Define φ_a such that $\varphi_a(v_i) = (m + 1) + f(v_i)$ for $2 \leq i \leq m + 1$.

Step 6. Define φ_b such that $\varphi_b(v_i) = 2m + 1 - 2f(v_i)$.

Output: Labeled Cayley digraph Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$.

Theorem 3.3. *The Cayley digraph associated with the group* $Zp^n \oplus Zp^n$ *with generating set* $\{(0, 1), (1, 0)\}$ *admits magic labeling*.

Proof. From the construction of the Cayley digraph for the group $Zp^n \oplus Zp^n$ we have that Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ has m vertices and $2m$ arcs. Let us denote the vertex set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ as $V = \{u_{00}, u_{01}, u_{02}, \ldots, u_{0, p^n}, u_{1,0}, u_{1,1}, \ldots, u_{p^n - 1, p^n - 1}\}$ as $V = \{v_1, v_2, v_3, \ldots, v_m\}$. Where $m = (p^n)^2$ as $V = \{v_0, v_1, v_2, \ldots, v_{m+1}\}$ corresponding to the elements $\{0, 1, 2, 3, \ldots, m + 1\}$ of the group respectively and the edge set of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ as $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2m}\}$.

To prove Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ admits magic labeling we have to show that for any vertex v_i the sums of the label of vertex v_i and the label if its outgoing arcs are constant. Define the edges of Cay $(Zp^n \oplus Zp^n, \{(0, 1), (1, 0)\})$ as defined in step 3 of above algorithm. It is evident that every vertex has exactly two outgoing arcs one each from the sets E_a, E_b. Now define maps f, φ_a, φ_b as defined in steps 4, 5 and 6 of the above algorithm.

Thus for any vertex $v_i \in V, f(v_i) + \varphi_a(v_i) + \varphi_b(v_i) = (i - 1) + (m + 1) + (i - 1) + (2m + 1) - 2(i - 1) = 3m + 2,$ which is a constant as m is constant. Hence the Cayley digraph associated with group $Zp^n \oplus Zp^n$ admits magic labeling.

Example 3.3. Magic labeling for a Cayley digraph associated with group $Zp^n \oplus Zp^n$ is given in figure 3.3.

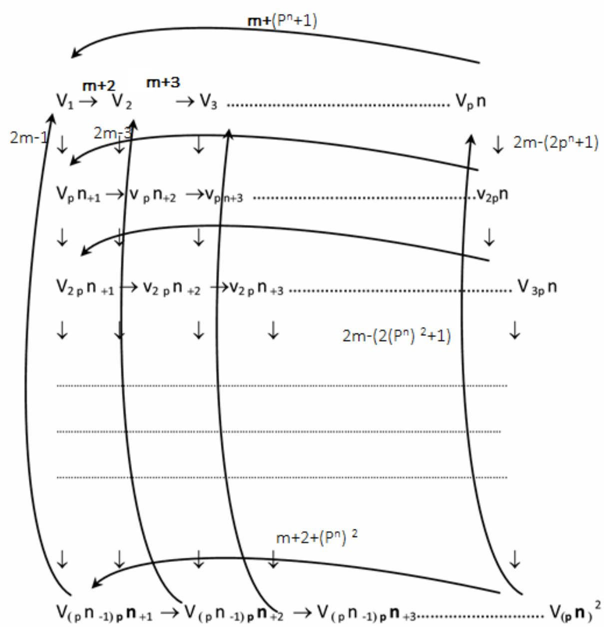

Figure 3.3. Magic labeling for a Cayley digraph associated with group $Zp^n \oplus Zp^n$.

3.4. Magic labeling for a Cayley digraph associated with group $Z_m \oplus Z_n$, $(m, n) = 1$

In this section we obtain magic labeling Cayley digraphs associated with group $Z_m \oplus Z_n$, $(m, n) = 1$.

Algorithm 3.4.

Input: The group $Z_m \oplus Z_n, (m, n) = 1.$ with the generating set $\{(0, 1), (1, 0)\}$

Step 1. Construct Cayley digraph Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$.

Step 2. Let $V = \{v_1, v_2, v_3, \ldots, v_{mn}\}$. denote the vertex set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$.

Step 3. Denote the edge set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$. as $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2mn}\}$ where E_a = the set of all outgoing arcs from v_i generated by (1, 0);

E_b = the set of all outgoing arcs from v_i generated by (0, 1).

Step 4. Define f such that $f(v_i) = i + 1$, for $0 \leq i \leq mn - 1$.

Step 5. Define φ_a such that $\varphi_a(v_i) = mn + f(v_i)$, for $0 \leq i \leq mn - 1$.

Step 6. Define φ_b such that $\varphi_b(v_i) = 3mn + 1 - 2f(v_i)$.

Output. Labeled Cayley digraph.

Theorem 3.4. *The Cayley digraph associated with the group* $Z_m \oplus Z_n, (m, n) = 1$ *with generating set* $\{(0, 1), (1, 0)\}$ *admits magic labeling*.

Proof. From the construction of the Cayley digraph for the group $Z_m \oplus Z_n$ we have Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ has mn vertices and $2mn$ arcs. Let us denote the vertex set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$, $V = \{u_{00}, u_{01}, u_{02}, \ldots, u_{0, n-1}, \ldots, u_{1, 0}, u_{1,1}, \ldots, u_{m-1, n-1}\}$ as $V = \{v_0, v_1, v_2, \ldots, v_{2mn-2}\}$ corresponding to the elements $\{0, 1, 2, 3, \ldots, 2mn-2\}$ of the group $Z_m \oplus Z_n, (m, n) = 1$ respectively and the edge set of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ as $E(E_a, E_b) = \{e_1, e_2, e_3, \ldots, e_{2mn}\}$.

To prove $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ admits magic labeling we have to show that for any vertex v_i, the sums of the label of vertex v_i and the label if its outgoing arcs are constant. Define the edges of Cay $(Z_m \oplus Z_n, (m, n) = 1, \{(0, 1), (1, 0)\})$ as given in step 3 of the above algorithm. Clearly each vertex has exactly two outgoing arcs one each are from the sets E_a and E_b Now define maps f, φ_a, φ_b as defined in steps 4, 5 and 6 of the above algorithm.

Thus for any vertex $v_i \in V, f(v_i) + \varphi_a(v_i) + \varphi_b(v_i) = (i+1) + (mn+i+1) + 3mn + 1 - 2i - 2 = 4mn + 1$, which is a constant as m is constant. Hence the Cayley digraph associated with group $Z_m \oplus Z_n, (m, n) = 1$ admits magic labeling.

Example 3.4. Magic labeling for a Cayley digraph associated with $Z_m \oplus Z_n$, $(m, n) = 1$ is given in figure 3.4.

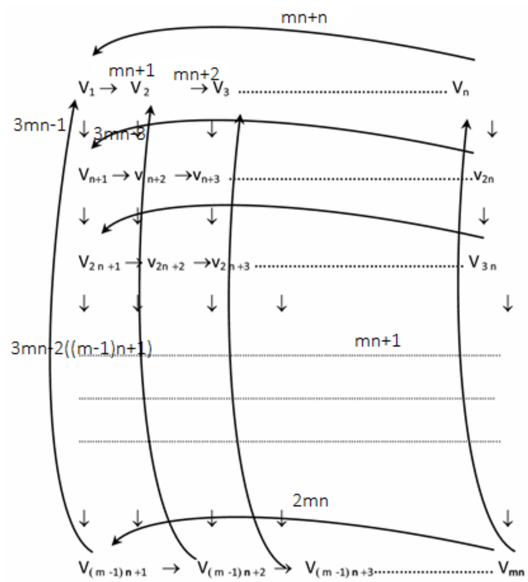

Figure 3.4. Magic labeling for a Cayley digraph associated with $Z_m \oplus Z_n$, $(m, n) = 1$.

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