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FUZZY VARIANT VACATIONS ON SERVER BREAKDOWNS USING PENTAGONAL NUMBERS

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Abstract

We discuss about the deals for analysis of single server variant vacation queueing system with fuzzy parameters. This fuzzy queueing model, researches obtains some performance measure of interest such as the mean system size when the server is on working vocation period, the mean system size when the server is busy period and the mean system size the server is breakdown period. Finally numerical results are presented using pentagonal Fuzzy numbers to show the effects of system parameters.

1. Introduction

Queueing system with general bulks service and vacation have been studied by many researchers because they deal with effective utilization of the servers idle time for secondary jobs such queueing system have a wide range of application in many real-life situations such as production time systems, inventory system, digital communication. Vacation models are useful in systems where the server want to utilize the ideal time for different purposes. In this topic we referred to the surveys of Doshi, Jakasi and Tin and Zang. The server completely stops service during the vacation period in classical vacation queues. However may the server remains active during the vacation period which is called working vacation, it has been happened in various situations.

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The working vacation policy was explained by Serviand Finn [5]. They measured an $M/M/1$ queue with multiple working vacations. Baba discussed a $GI/M/1$ queue with multiple working vacations. The bulk input queue models have wide utilization as in computer information technology and communication systems the units arrive in batches. The batch arrival queues, Xu et al. explored a bulk input $M^X/M/1$ queue with single working vacations and the probability generating function of the stationary system length distribution is concluded with help of the analytic method in matrix form.

The steady state analysis and computation of the $GI^X/M^b/1/L$ queue with multiple working vacation and partial batch rejection is analyzed by Yu et al. and Goswami and Vijaya Laxmi researched the $GI^X/M^b/1/N$ queue with single working vacation and partial batch rejection. A finite buffer $M/M/1$ queue with variant working vacation and balking and reneging has been introduced by Vijaya Laxmi and Jyothsna. They found the steady state probabilities with matrix form solutions.

The fuzzy Markov chains and the Zadeh's extension principle are widely used as a proposed for the queuing system in a fuzzy environment. This research will also give a numerical result so as to more clear the approach in a good manner. Fuzzy queuing models have been explained by such researchers like Kaufmann, Negi and Lee, Li and Lee. Chen has examined fuzzy queues using Zadeh's extension principle and has expanded: $(\infty/FCFS)$ and $(FM/FMk/1):(\infty/FCFS)$ where FM refers fuzzified exponential time based of queuing theory. Usha Madhuri and Chandan explained $FM/FM/1$ queuing model with Pentagon fuzzy numbers using α-cuts.

In this paper, we investigate the $FM^X/FM/1$ queuing system with variant working vacations on batch arrival with reneging and server breakdowns. In section 2, we describe the queue model. In section 3 and 4, we discuss the fuzzy model with the server is on working vacation period, busy period and breakdown period are studied in fuzzy environment respectively. Section 5 includes numerical study about the performance measures.

2. The Crisp Model

We consider an $M^{Y}/M/1$ queuering system with variant working vacation, reneging and server breakdowns during busy period, where in customer arrive in batches according to a poisson process with rate λ. The arrival batch size Y is a random variable with probability mass function $P(Y = l) = d_l$, $l = 1, 2, 3, \ldots$. The services is provided by a single server with exponential service rate μ. The server is subject to breakdowns during busy period with poisson breakdown rate β and repair time γ. At the end of a service, if there is no customer in the system, the server begins a working vacation of a random length with exponentially distributed with parameter φ. During a working vacation service is provided according to a poisson distribution with parameter ζ. During working vacation customer become impatient, whenever a batch of customers arrives during working vacation, an impatience timer T is activated, which is exponentially distributed with parameter α. At time t, let $L_1(t_1)$ be the number of customers in the system and $J_1(t_1)$ denote the status of the server, which is defined as follows

$$J_1(t_1) = \begin{cases} i, \text{ the server is on } (i+1)^{\text{th}} \text{ working vacation at time } t_1 \text{ for } i = (t_i) \\ k, \text{ the server is busy period at time } t_1, \\ b, \text{ the server is in breakdown stse during busy period at } t_1 \end{cases}$$

The process $\{L_1(t_1), J_1(t_1); t \geq 0\}$ defines a continuous time Markov process with state space

$$\Delta = \{(n, i), n \geq 0, i = 0, 1, \ldots, k \text{ and } i = b\}.$$

Let

$$\pi_{n,i} = \lim_{t_1 \to \infty} \pi\{L_1(t_1) = n, J_1(t_1) = i\}, n \geq 0, i = 0, 1, \ldots, k \text{ and } i = b,$$

denote the steady state probabilities of the process.

3. The Model in Fuzzy Environment

The arrival rate, service rate, Breakdown rate, working vacation, service for working vacation, repair time and the parameter of impatience time 'T'

are assign to be fuzzy numbers $\bar{\lambda}$, $\overline{\beta_1}$, $\overline{\beta_2}$, $\overline{\theta_1}$, $\overline{\theta_2}$, $\overline{\gamma_1}$, $\overline{\gamma_2}$ respectively. Now

$$\bar{\lambda} = \{(x, \mu_{\bar{\lambda}}(x)); x \in S(\bar{\lambda})\}$$

$$\overline{\beta_1} = \{(y_1, \mu_{\overline{\beta_1}}(y_1)); y_1 \in S(\overline{\beta_1})\},$$

$$\overline{\beta_2} = \{(y_2, \mu_{\overline{\beta_2}}(y_2)); y_2 \in S(\overline{\beta_2})\},$$

$$\overline{\theta_1} = \{(z_1, \mu_{\overline{\theta_1}}(z_1)); z_1 \in S(\overline{\theta_1})\},$$

$$\overline{\theta_2} = \{(z_2, \mu_{\overline{\theta_2}}(z_2)); z_2 \in S(\overline{\theta_2})\},$$

$$\overline{\gamma_1} = \{(u_1, \mu_{\overline{\gamma_1}}(u_1)); u_1 \in S(\overline{\gamma_1})\},$$

$$\overline{\gamma_2} = \{(u_2, \mu_{\overline{\gamma_2}}(u_2)); u_2 \in S(\overline{\gamma_2})\}.$$

Where $S(\bar{\lambda}), S(\overline{\beta_1})$, $S(\overline{\beta_2}), S(\overline{\theta_1})$, $S(\overline{\theta_2}), S(\overline{\gamma_1})$ and $S(\overline{\gamma_2})$ are the universal sets of the arrival rate, service rate, breakdown rate, working vacation, service for working vacation, repair time and the parameter of impatience time '*T*' respectively. Define $f(x, y_1, y_2, z_1, z_2, u_1, u_2)$ as the system performance measure related to the above defined fuzzy queuing model, which depends on the fuzzy membership function $\bar{\lambda}, \overline{\beta_1}, \overline{\beta_2}, \overline{\theta_1}, \overline{\theta_2}, \overline{\gamma_1}, \overline{\gamma_2}$. Applying Zadeh's extension principle (1978) the membership function of the performance measure $\bar{\lambda}, \overline{\beta_1}, \overline{\beta_2}, \overline{\theta_1}, \overline{\theta_1}, \overline{\gamma_1}, \overline{\gamma_2}$ can be defined as

$$\mu_{f(\bar{\lambda}, \overline{\beta_1}, \overline{\beta_2}, \overline{\theta_1}, \overline{\theta_2}, \overline{\gamma_1}, \overline{\gamma_2})}(H) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\overline{\beta_1}) \\ y_2 \in S(\overline{\beta_2}) \\ z_1 \in S(\overline{\theta_1}) \\ z_2 \in S(\overline{\theta_2}) \\ u_1 \in S(\overline{\gamma_1}) \\ u_2 \in S(\overline{\gamma_2})}} \{\mu_{\bar{\lambda}}(x), \mu_{\overline{\beta_1}}(y_1), \mu_{\overline{\beta_2}}(y_2), \mu_{\overline{\theta_1}}(z_1),$$

$$\mu_{\overline{\theta_2}}(z_2), \mu_{\overline{\gamma_1}}(u_1), \mu_{\overline{\gamma_2}}(u_2) / H = f(x, y_1, y_2, z_1, z_2, u_1, u_2)\} \quad (1)$$

If the α-cuts of $f(\bar{\lambda}, (\overline{\beta_1}, \overline{\beta_2}, \overline{\theta_1}, \overline{\theta_2}, \overline{\gamma_1}, \overline{\gamma_2})$ degenerate to some fixed value, then the system performance is a crisp number, otherwise it is a fuzzy number.

The mean system size when the server is on working vacation period

$$E[L_v] = \frac{[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))][\lambda g - \zeta(1-c)]}{[\lambda a^2(\mu\gamma - \lambda g(\beta+\gamma))] + \left[\begin{array}{l}\frac{\phi\alpha}{1-c^k}[(1-c^k)(\lambda\mu\gamma) \\ -\lambda\zeta(1-c)(\beta+\gamma) + \mu\gamma(\alpha+\phi)c^k(1-c)]\end{array}\right]}.$$

Under the stay state condition

$$\rho = \frac{\lambda g}{\mu} < 1.$$

Here

$$H(k) = \frac{c^k(1-c)}{(1-c^k)}$$

$$g = \frac{1}{q},\ q = 1-p,\ 0 < g < 1$$

$$c = \frac{\phi}{\zeta}(0.5).$$

The mean system size when the server is busy period

$$E(L_k) = \left[\frac{\phi\lambda\alpha q^2[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))][\gamma(\lambda g - (\phi+\zeta+\gamma))] + \lambda^2\alpha^2\phi(\phi+2\alpha)}{[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))]^2(\phi g + 2\alpha)q^2}\right]$$

$$\left[\frac{[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))][\lambda g - \zeta(1-c)]}{[\lambda\alpha^2(\mu\gamma - \lambda g(\beta+\gamma))] + \left[\begin{array}{l}\frac{\phi\alpha}{1-c^k}[(1-c^k)(\lambda\mu\gamma - \lambda\zeta(1-c)(\beta+\gamma)) \\ +\mu\gamma(\alpha+\phi)c^k(1-c)]\end{array}\right]}\right]$$

$$+\left[\frac{\lambda^2(\alpha+\phi)\phi\gamma}{q^2(\phi+2\alpha)(A_1+A_2)}\right] + \left[\frac{\lambda\alpha\mu(\alpha+\phi)(\lambda\mu\gamma^2 + \beta q^2(\lambda g)^3)c^k(1-c)}{q^2 A_1(A_1+A_2)(1-c^k)}\right],$$

Where

$$A_1 = \lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))$$

$$A_2 = \frac{\pi}{1-c^k}[(1-c^k)(\lambda\mu\gamma - \lambda\zeta(1-c)(\beta+\gamma)) + \mu\gamma(\alpha+\pi)c^k(1-c)].$$

The mean system size when the service is breakdown

$$E(L_b) = \frac{\beta}{\gamma}\left[\frac{\phi\lambda\alpha q^2[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))][\gamma(\lambda g - (\phi+\zeta+\gamma))] + \lambda^2\alpha^2\phi(\phi+2\alpha)}{[\lambda\alpha(\mu - \lambda g(\beta+\gamma))]^2(\phi g + 2\alpha)q^2}\right]$$

$$\left[\frac{[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))][\lambda g - \zeta(1-c)]}{[\lambda\alpha^2(\mu\gamma - \lambda g(\beta+\gamma))] + \left[\begin{array}{l}\frac{\phi\alpha}{1-c^k}[(1-c^k)(\lambda\mu\gamma - \lambda\zeta(1-c)(\beta+\gamma)) \\ + \mu\gamma(\alpha+\phi)c^k(1-c)]\end{array}\right]}\right]$$

$$+\left[\frac{\lambda^2(\alpha+\phi)\phi\gamma}{q^2(\phi+2\alpha)(A_1+A_2)}\right] + \left[\frac{\lambda\alpha\mu(\alpha+\phi)(\lambda\mu\gamma^2 + \beta q^2(\lambda g)^3)c^k(1-c)}{q^2A_1(A_1+A_2)(1-c^k)}\right],$$

$$+\left[\frac{\lambda^2 g}{A_1(A_1+A_2)(1-c^k)}\right] + [\alpha\phi(1-c^k)E[L_{wv}](A_1+A_2) + g\phi A_1(\alpha+\phi)c^k(1-c)]$$

Where

$$A_1 = \lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))$$

$$A_2 = \frac{\pi}{1-c^k}[(1-c^k)(\lambda\mu\gamma - \lambda\zeta(1-c)(\beta+\gamma)) + \mu\gamma(\alpha+\pi)c^k(1-c)]$$

$$E[L_v] = \frac{[\lambda\alpha(\mu\gamma - \lambda g(\beta+\gamma))][\lambda g - \zeta(1-c)]}{[\lambda\alpha^2(\mu\gamma - \lambda g(\beta+\gamma))] + \left[\begin{array}{l}\frac{\phi\alpha}{1-c^k}[(1-c^k)(\lambda\mu\gamma - \lambda\zeta(1-c)(\beta+\gamma)) \\ + \mu\gamma(\alpha+\phi)c^k(1-c)]\end{array}\right]}$$

we obtain the membership function some performance measures, namely the mean system size when the server is on working vacation $E[L_v]$, the mean system size when the server is busy period $E(L_k)$ the mean system when the server is breakdown $E[L_b]$. For the system in term of this membership function are, as follows

$$\mu_{\overline{E[L_v]}}(M) = \sup_{\substack{x \in S(\overline{\lambda}) \\ y_1 \in S(\overline{\beta_1}) \\ y_2 \in S(\overline{\beta_2}) \\ z_1 \in S(\overline{\theta_1}) \\ z_2 \in S(\overline{\theta_2}) \\ u_1 \in S(\overline{\gamma_1}) \\ u_2 \in S(\overline{\gamma_2})}} \{\mu_{\overline{\lambda}}(x), \mu_{\overline{\beta_1}}(y_1), \mu_{\overline{\beta_2}}(y_2), \mu_{\overline{\theta_1}}(z_1),$$

$$\mu_{\overline{\theta_2}}(z_2), \mu_{\overline{\gamma_1}}(u_1), \mu_{\overline{\gamma_2}}(u_2) / M = f(x, y_1, y_2, z_1, z_2, u_1, u_2)\} \quad (2)$$

Where

$$M = \frac{[xu_2(y_1u_1 - xg(y_2 + u_2))][xg - z_2(1-c)]}{[xu_2^2(y_1u_1 - xg(y_2 + u_2))] + \frac{z_1u_2}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)(y_2 + u_1) + y_1u_1(y_2 + z_1)c^k(1-c)]}$$

Here, $A_1 = xu_2(y_1u_1 - xg(y_2 + u_2))$

$$A_2 = \frac{z_1}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)(y_2 + u_1) + y_1u_1(u_2 + z_1)c^k(1-c)]$$

and $A_3 = xg - z_2(1-c)$

$$\mu_{\overline{E[L_k]}}(N) = \sup_{\substack{x \in S(\overline{\lambda}) \\ y_1 \in S(\overline{\beta_1}) \\ y_2 \in S(\overline{\beta_2}) \\ z_1 \in S(\overline{\theta_1}) \\ z_2 \in S(\overline{\theta_2}) \\ u_1 \in S(\overline{\gamma_1}) \\ u_2 \in S(\overline{\gamma_2})}} \{\mu_{\overline{\lambda}}(x), \mu_{\overline{\beta_1}}(y_1), \mu_{\overline{\beta_2}}(y_2), \mu_{\overline{\theta_1}}(z_1),$$

$$\mu_{\overline{\theta_2}}(z_2), \mu_{\overline{\gamma_1}}(u_1), \mu_{\overline{\gamma_2}}(u_2) / N = f(x, y_1, y_2, z_1, z_2, u_1, u_2)\} \quad (3)$$

Where

$$N = \left\{ \left[\frac{z_1 x u_2 q^2 [xu_2(y_1u_1 - xg(y_2 + u_2)][u_1[xg - (z_1 + z_2 + u_1)]] + x^2 u_2^2 z_1(z_1 + 2u_2)}{[xu_2(y_1u_1 - xg(y_2 + u_2)]^2 (z_1 + 2u_2) q^2} \right] \right\}$$

$$\left[\frac{[xu_2(y_1u_1 - xg(y_2+u_2))][xg - z_2(1-c)]}{(xu_2^2(y_1u_1 - xg(y_2+u_2)) + \frac{z_1u_1}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)(y_2+u_1) + y_1u_1(u_2+z_1)c^k(1-c)]}\right]$$

$$+\left[\frac{\lambda^2(u_2+z_1)z_1u_1}{q^2(z_1+2u_2)+(A_1+A_2)}\right]+\left[\frac{xu_2y_1(u_2+z_1)(xy_1u_1^2+y_1q^2(xg)^3)c^k(1-c)}{q^2A_1(A_1+A_2)(1-c^k)}\right].$$

Here $A_1 = xu_2(y_1u_1 - xg(y_2+u_2))$

$$A_2 = \frac{z_1}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)(y_2+u_1) + y_1u_1(u_2+z_1)c^k(1-c)]$$

$$\mu_{\overline{E[L_b]}}(O) = \sup_{\substack{x\in S(\overline{\lambda})\\ y_1\in S(\overline{\beta_1})\\ y_2\in S(\overline{\beta_2})\\ z_1\in S(\overline{\theta_1})\\ z_2\in S(\overline{\theta_2})\\ u_1\in S(\overline{\gamma_1})\\ u_2\in S(\overline{\gamma_2})}}\{\mu_{\overline{\lambda}}(x),\ \mu_{\overline{\beta_1}}(y_1),\ \mu_{\overline{\beta_2}}(y_2),\ \mu_{\overline{\theta_1}}(z_1),$$

$$\mu_{\overline{\theta_2}}(z_2),\ \mu_{\overline{\gamma_1}}(u_1),\ \mu_{\overline{\gamma_2}}(u_2)/O = f(x,\ y_1,\ y_2,\ z_1,\ z_2,\ u_1,\ u_2)\} \tag{4}$$

Where

$$O = \frac{y_2}{u_1}\left\{\left[\frac{z_1u_2q^2[xu_2(y_1u_1 - xg(y_2+u_2))][u_1[xg-(z_1+z_2+u_1)]] + x^2u_2^2z_1(z_1+2u_2)}{[xu_2(y_1u_1 - xg(y_2+u_2))]^2(z_1+2u_2)q^2}\right]\right\}$$

$$\left\{\left[\frac{[xu_2(y_1u_1 - xg(y_2+u_2))][xg - z_2(1-c)]}{(xu_2^2(y_1u_1 - xg(y_2+u_2)) + \frac{z_1u_2}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)(y_2+u_1)) + y_1u_1(u_2+z_1)c^k(1-c)]}\right]\right\}$$

$$\left\{ + \left[\frac{x^2 (u_2 + z_1) z_1 u_1}{q^2 (z_1 + 2u_2) + (A_1 + A_2)} \right] + \left[\frac{\begin{matrix} x u_2 y_1 (u_2 + z_1) \\ (x y_1 u_1^2 + y_1 q^2 (xg)^3) c^k (1 - c) \end{matrix}}{q^2 A_1 (A_1 + A_2)(1 - c^k)} \right] \right\}$$

$$\left\{ \left[\frac{x^2 g}{A_1 (A_1 + A_2) + (1 - c^k)} \right] + [u_2 z_1 (1 - c^k) M (A_1 + A_2) + g z_1 A_1 (u_2 + z_1) c^k (1 - c)] \right\}.$$

Here,

$$M = \frac{[x u_2 (y_1 u_1 - xg(y_2 + u_2))][xg - z_2 (1 - c)]}{[x u_2^2 (y_1 u_1 - xg(y_2 + u_2))] + \frac{z_1 u_2}{(1 - c^k)} [(1 - c^k)(x y_1 u_1 - x z_2}$$

$$(1 - c)(y_2 + u_1) + y_1 u_1 (u_2 + z_1) c^k (1 - c)]$$

$$A_1 = x u_2 (y_1 u_1 - xg(y_2 + u_2))$$

$$A_2 = \frac{z_1}{(1 - c^k)} [(1 - c^k)(x y_1 u_1 - x z_2 (1 - c)(y_2 + u_1) + y_1 u_1 (u_2 + z_1) c^k (1 - c)].$$

Using the fuzzy analysis technique, we can find the membership of $\mu_{\overline{E[L_v]}}$, $\mu_{\overline{E[L_k]}}$, $\mu_{\overline{E[L_v]}}$ as a function of parameter α. Thus the α-cut approach can be used to develop the membership function of $\mu_{\overline{E[L_v]}}$, $\mu_{\overline{E[L_k]}}$, $\mu_{\overline{E[L_b]}}$.

4. Performance of Measure

The following performance measures are studied for this model in fuzzy environment.

The mean system size when the server is on working vacation period

Based on Zadeh's extension principle $\mu_{E[L_v]}(M)$ is the supremum of minimum over $\{\mu_{\bar{\lambda}}(x), \mu_{\overline{\beta_1}}(y_1), \mu_{\overline{\beta_2}}(y_2), \mu_{\overline{\theta_1}}(z_1), \mu_{\overline{\theta_2}}(z_2), \mu_{\overline{\gamma_1}}(u_1), \mu_{\overline{\gamma_2}}(u_2)\}$,

$$M = \frac{[xu_2(y_1u_1 - xg(y_2+u_2))][xg - z_2(1-c)]}{[xu_2^2(y_1u_1 - xg(y_2+u_2))] + \frac{z_1u_2}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)) (y_2+u_1) + y_1u_1(u_2+z_1)c^k(1-c)]}$$

to satisfy $\mu_{\overline{E[L_v]}}(M) = \alpha,\ 0 < \alpha \le 1$.

We consider the following seven cases

Case (i) $\mu_\lambda(x) = \alpha,\ \mu_{\beta_1}(y_1) \ge \alpha,\ \mu_{\beta_2}(y_2) \ge \alpha,\ \mu_{\theta_1}(z_1) \ge \alpha,\ \mu_{\theta_2}(z_2) \ge \alpha,\ \mu_{\gamma_1}(u_1) \ge \alpha,\ \mu_{\gamma_2}(u_2) \ge \alpha,$

Case (ii) $\mu_\lambda(x) \ge \alpha,\ \mu_{\beta_1}(y_1) = \alpha,\ \mu_{\beta_2}(y_2) \ge \alpha,\ \mu_{\theta_1}(z_1) \ge \alpha,\ \mu_{\theta_2}(z_2) \ge \alpha,\ \mu_{\gamma_1}(u_1) \ge \alpha,\ \mu_{\gamma_2}(u_2) \ge \alpha,$

Case (iii) $\mu_\lambda(x) \ge \alpha,\ \mu_{\beta_1}(y_1) \ge \alpha,\ \mu_{\beta_2}(y_2) = \alpha,\ \mu_{\theta_1}(z_1) \ge \alpha,\ \mu_{\theta_2}(z_2) \ge \alpha,\ \mu_{\gamma_1}(u_1) \ge \alpha,\ \mu_{\gamma_2}(u_2) \ge \alpha,$

Case (iv) $\mu_\lambda(x) \ge \alpha,\ \mu_{\beta_1}(y_1) \ge \alpha,\ \mu_{\beta_2}(y_2) \ge \alpha,\ \mu_{\theta_1}(z_1) = \alpha,\ \mu_{\theta_2}(z_2) \ge \alpha,\ \mu_{\gamma_1}(u_1) \ge \alpha,\ \mu_{\gamma_2}(u_2) \ge \alpha,$

Case (v) $\mu_\lambda(x) \ge \alpha,\ \mu_{\beta_1}(y_1) \ge \alpha,\ \mu_{\beta_2}(y_2) \ge \alpha,\ \mu_{\theta_1}(z_1) \ge \alpha,\ \mu_{\theta_2}(z_2) = \alpha,\ \mu_{\gamma_1}(u_1) \ge \alpha,\ \mu_{\gamma_2}(u_2) \ge \alpha,$

Case (vi) $\mu_\lambda(x) \ge \alpha,\ \mu_{\beta_1}(y_1) \ge \alpha,\ \mu_{\beta_2}(y_2) \ge \alpha,\ \mu_{\theta_1}(z_1) \ge \alpha,\ \mu_{\theta_2}(z_2) \ge \alpha,\ \mu_{\gamma_1}(u_1) = \alpha,\ \mu_{\gamma_2}(u_2) \ge \alpha,$

Case (vii) $\mu_\lambda(x) \ge \alpha,\ \mu_{\beta_1}(y_1) \ge \alpha,\ \mu_{\beta_2}(y_2) \ge \alpha,\ \mu_{\theta_1}(z_1) \ge \alpha,\ \mu_{\theta_2}(z_2) \ge \alpha,\ \mu_{\gamma_1}(u_1) \ge \alpha,\ \mu_{\gamma_2}(u_2) = \alpha.$

For case (i) the lower and upper bound of α-cuts of $\mu_{\overline{E[L_v]}}$ can be obtained through the corresponding parametric non-linear programs,

$$[E_{[L_v]}]_\alpha^{L_1} = \min_\Omega \{[M]\}$$

and

$$[E_{[L_v]}]_\alpha^{U_1} = \max_\Omega \{[M]\}.$$

Similarly we can calculate the lower and upper bounds of the α-cuts of $[E[L_{wv}]]$ for the case (ii), (iii) and (iv). By considering all the cases simultaneously the lower and upper bounds of the α-cuts of $[E[L_{wv}]]$ can be written as

$$M = \frac{A_1 A_3}{u_2(A_1 + A_2)}.$$

Here,

$$A_1 = xu_2(y_1u_1 - xg(y_2 + u_2))$$

$$A_2 = \frac{z_1}{(1-c^k)}[(1-c^k)(xy_1u_1 - xz_2(1-c)(y_2+u_1) + y_1u_1(u_2+z_1)c^k(1-c)]$$

$$A_3 = xg - z_2(1-c)$$

$$[E[L_v]]_\alpha^L = \min_\Omega\{[M]\} \text{ and } [E[L_v]]_\alpha^U = \max_\Omega\{[M]\}$$

$$x_\alpha^L \le x \le x_\alpha^U,\ y_{1\alpha}^L \le y_1 \le y_{1\alpha}^U,\ y_{2\alpha}^L \le y_2 \le y_{2\alpha}^U,\ z_{1\alpha}^L \le z_1 \le z_{1\alpha}^U,\ z_2 \le z_{2\alpha}^L,$$

$$u_{1\alpha}^L \le u_1 \le u_{1\alpha}^U,\ u_{2\alpha}^L \le u_2 \le u_{2\alpha}^U.$$

If both $[E[L_v]]_\alpha^L$ and $[E[L_v]]_\alpha^U$ are invertible with respect to α, the left and right shape function, $L(M) = [E[L_v]_\alpha^L]^{-1}$ and $R(M) = [E[L_v]_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{E[L_v]}}(M)$ can be constructed as

$$\mu_{\overline{E[L_v]}}(M) = \begin{cases} L(M),\ [E[L_v]_{\alpha=0}^U],\ [E[L_v]_{\alpha=1}^U],\ [E[L_v]]_{\alpha=0}^U \\ 1,\ [E[L_v]]_{\alpha=1}^U \le M \le [E[L_v]]_{\alpha=1}^U \\ R(M),\ [E[L_v]]_{\alpha=0}^U \le M \le [E[L_v]]_{\alpha=0}^U \end{cases}. \quad (5)$$

In the same way we get the following results.

The mean system size when the server is busy period

$$\mu_{\overline{E[L_k]}}(N) = \begin{cases} L(N), [E[L_k]]^U_{\alpha=0}, [E[L_k]]^U_{\alpha=1}, [E[L_k]]^U_{\alpha=0} \\ 1, [E[L_k]]^U_{\alpha=1} \le N \le [E[L_k]]^U_{\alpha=1} \\ R(N), [E[L_k]]^U_{\alpha=0} \le N \le [E[L_k]]^U_{\alpha=0} \end{cases}. \quad (6)$$

The mean system size when the server is breakdown

$$\mu_{\overline{E[L_b]}}(O) = \begin{cases} L(O), [E[L_b]]^U_{\alpha=0}, [E[L_b]]^U_{\alpha=1}, [E[L_b]]^U_{\alpha=0} \\ 1, [E[L_b]]^U_{\alpha=1} \le O \le [E[L_b]]^U_{\alpha=1} \\ R(O), [E[L_b]]^U_{\alpha=0} \le O \le [E[L_b]]^U_{\alpha=0} \end{cases}. \quad (7)$$

5. Numerical Study

The mean system size when the server is on working vacation period. Suppose the fuzzy arrival rate $\overline{\lambda}$, service rate $\overline{\beta_1}$, Breakdown rate $\overline{\beta_2}$, working vacation $\overline{\theta_1}$, service for working vacation $\overline{\theta_2}$, repair time $\overline{\gamma_1}$, and the parameter of impatience time '*T*' is γ_2 are assumed to be pentagonal fuzzy numbers described by

$$\overline{\lambda} = [21, 22, 23, 24, 25], \overline{\beta_1} = [31, 32, 33, 34, 35], \overline{\beta_2} = [71, 72, 73, 74, 75]$$

$$\overline{\theta_1} = [16, 17, 18, 18, 19, 20], \overline{\theta_2} = [11, 12, 13, 14, 15], \overline{\gamma_1} = [36, 37, 38, 39, 40],$$

$$\overline{\gamma_2} = [26, 27, 28, 29, 30]$$

per mins respectively.

Then

$$\lambda(\alpha) = \min_{x \in S(\lambda)} \{x \in s(\overline{\lambda}), G(x) \ge \alpha\}, \max_{x \in S(\lambda)} \{x \in s(\overline{\lambda}), G(x) \ge \alpha\},$$

Where

$$G(x) = \begin{cases} 0, \text{ if } x \le a_1 \\ 1 - (1-r)\dfrac{x - a_2}{a_3 - a_2}, \text{ if } a_2 \le x \le a_3 \\ 1, \text{ if } x = a_3 \\ 1 - (1-r)\dfrac{a_4 - x}{a_4 - a_3}, \text{ if } a_3 \le x \le a_4 \\ r\dfrac{a_5 - x}{a_5 - a_4}, \text{ if } a_4 \le x \le a_5 \\ 0, \text{ if } x \ge a_5 \end{cases}$$

That is,

$\lambda(\alpha) = [21 + \alpha, 25 - \alpha]$, $\beta_1(\alpha) = [31 + \alpha, 35 - \alpha]$, $\beta_2(\alpha) = [71 + \alpha, 75 - \alpha]$,

$\theta_1(\alpha) = [16 + \alpha, 20 - \alpha]$, $\theta_2(\alpha) = [11 + \alpha, 15 - \alpha]$, $\gamma_1(\alpha) = [36 + \alpha, 40 - \alpha]$,

$\gamma_2(\alpha) = [26 + \alpha, 30 - \alpha]$.

It is clear that, when $x = x_\alpha^U$, $y_1 = y_{1\alpha}^U$, $y_2 = y_{2\alpha}^U$, $z_1 = z_{1\alpha}^U$, $z_2 = z_{2\alpha}^U$, $u_1 = u_{1\alpha}^U$, and $u_2 = u_{2\alpha}^U$, M attains its maximum value and when $x = x_\alpha^L$, $y_1 = y_{1\alpha}^L$, $y_2 = y_{2\alpha}^L$, $z_1 = z_{1\alpha}^L$, $z_2 = z_{2\alpha}^L u_1 = u_{1\alpha}^L$, and $u_2 = u_{2\alpha}^L$, M attains its minimum value.

From the generated for the given input values of $\overline{\lambda}$, $\overline{\beta_1}$, $\overline{\beta_2}$, $\overline{\theta_1}$, $\overline{\theta_2}$, $\overline{\gamma_1}$ and $\overline{\gamma_2}$,

(i) For fixed values of x, y_1, y_2, z_1, z_2 and u_1, M decreases as u_2 increases

(ii) For fixed values of y_1, y_2, z_1, z_2, u_1 and u_2, M decreases as x increases

(iii) For fixed values of y_2, z_1, z_2, u_1, u_2 and x, M decreases as y_1 increases

(iv) For fixed values of z_1, z_2, u_1, u_2, x and y_1, M decreases as y_2 increases

(v) For fixed values of z_2, u_1, u_2, x, y_1 and y_2, M decreases as z_1 increases

(vi) For fixed values of u_1, u_2, x, y_1, y_2 and z_1, M decreases as z_2 increases

(vii) For fixed values of u_2, x, y_1, y_2, z_1 and z_2, M decreases as u_1 increases

The smallest value of occurs when x-takes its lower bound, i.e., $x = 21 + \alpha$ and y_1, y_2, z_1, z_2, u_1; and u_2 take their upper bounds given by $y_1 = 35 - \alpha$, $y_2 = 75 - \alpha$, $z_1 = 20 - \alpha$, $z_2 = 15 - \alpha$, $u_1 = 40 - \alpha$, $u_2 = 30 - \alpha$ respectively.

And maximum value of $E[L_v]$ occurs when $x = 25 - \alpha$, $y_1 = 31 + \alpha$, $y_2 = 71 + \alpha$, $z_1 = 16 + \alpha$, $z_2 = 11 + \alpha$, $u_1 = 36 + \alpha$ and $u_2 = 26 + \alpha$.

If both $[E[L_v]]_\alpha^L$ and $[E[L_v]]_\alpha^U$ are invertible with respect to 'α', then the left shape function $L(M) = [E[L_v]_\alpha^L]^{-1}$ and right shape function $R(M) = [E[L_v]_\alpha^U]^{-1}$ can be obtained and from which the member function $\mu_{\overline{E[L_v]}}(M)$ can be constructed as

$$\mu_{\overline{E[L_v]}}(M) = \begin{cases} 0, \text{ if } M \le M \\ 0.5(x-2), \text{ if } M_1 \le M \le M, \\ 0.5(4-x), \text{ if } M_2 \le M \le M \\ 0.5(5-x), \text{ if } M_3 \le M \le M \\ 0, \text{ if } M \le M_5. \end{cases} \tag{8}$$

The values of M_1, M_2, M_3, M_4 and M_5 as obtained from (8) are

$$\mu_{\overline{E[L_p]}}(M) = \begin{cases} 0, \quad \text{if } M \le 0.0000 \\ 0.5(x-2) \text{ if } 0.0000 \le M \le 1.4204, \\ 1, \text{ if } x = 1 \\ 0.5(4-x), \text{ if } 1.4204 \le M \le 3.1731 \\ 0.5(5-x), \text{ if } 3.1731 \le M \le 1.4102 \\ 0, \quad \text{if } M \ge 0.0000. \end{cases}$$

In the same way we get the following results.

The mean system size when the server is busy period

$$\mu_{\overline{E[L_k]}}(N) = \begin{cases} 0, \quad \text{if } N \le N_1 \\ 0.6(x-2), \quad \text{if } N_1 \le N \le N_2, \\ 0.6(4-x), \quad \text{if } N_2 \le N \le N_3, \\ 0.6(5-x), \quad \text{if } N_3 \le N \le N_4, \\ 0, \quad \text{if } N \ge N_5. \end{cases} \tag{9}$$

The values of N_1, N_2, N_3, N_4 and N_5 as obtained from (9) are

$$\mu_{\overline{E[L_k]}}(N) = \begin{cases} 0, \quad \text{if } N \le 0.0000 \\ 0.6(2-x), \quad \text{if } 0.0000 \le N \le 2.1054, \\ 0.6(4-x), \quad \text{if } 2.1054 \le N \le 5.2757, \\ 0.6(5-x), \quad \text{if } 5.2757 \le N \le 2.0023, \\ 0, \quad \text{if } N \ge 0.0000. \end{cases}$$

The mean system size when the server is breakdown

$$\mu_{\overline{E[L_b]}}(O) = \begin{cases} 0, & \text{if } O \le O_1 \\ -48(x-2), & \text{if } O_1 \le O \le O_2, \\ -48(4-x), & \text{if } O_2 \le O \le O_3, \\ 50(5-x), & \text{if } O_3 \le O \le O_4, \\ 0, & \text{if } O \ge O_5. \end{cases} \tag{10}$$

The values of O_1, O_2, O_3, O_4 and O_5 as obtained from (10) are

$$\mu_{\overline{E[L_b]}}(O) = \begin{cases} 0, & \text{if } O \le 0.0000 \\ -48(2-x), & \text{if } 0.0000 \le O \le 42.0621 \\ -48(4-x), & \text{if } 42.621 \le O \le 86.2749, \\ 50(5-x), & \text{if } 86.2749 \le O \le 48.1721, \\ 0, & \text{if } O \ge 0.0000. \end{cases}$$

Conclusion

In this research paper we have studied the analysis of the $M^Y/M/1$ queue with variant vacation on server breakdowns using Pentagonal Fuzzy numbers. We have obtained the performance measure such as the mean system size when the server is on working vacation period, the server is busy period and the server is breakdown period. We have obtained numerical results to all the performance measures for this fuzzy queues. The application of this fuzzy queues, there are situations particularly customers arrivals in super markets, restaurants and voice calls in communication centers where the service provided is a group that, a group of customers can be served simultaneously, batch servicing in this process.

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