Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4551-4561

EVALUATION OF INTERVAL SEQUENCING PROBLEM APPLICATION IN WATER POLLUTION CONTROL MACHINE USING FUZZY DECISION MAKING

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Abstract

The sequencing problem deals with '*n*' job on single machine, '*n*' job on two machine, '*n*' job on three machines, '*n*' job on '*m*' machine. The main goal of the sequencing problems is to obtaining a minimum duration of work in water pollution control machine. The sequencing problem is design to obtaining an optimal sequence and also for determining the minimum duration taken to complete all the work.

1. Introduction

Mathematics-specifically applied mathematics-is an extension of operation research. McClosky and Trefthen invented the term in 1940. Operational research can also take a scientific approach to analyzing and resolving management issues. Decision making, game theory, and optimization are only a few of the fields of operation research. The sequencing challenge of the 'War baby' is an important component of operation research. The term "sequencing" refers to the order in which a number of jobs or tasks are to be completed.

In the majority of sequencing problems, the processing times are precisely calculated. Processing times during task performance, on the other hand, are

2020 Mathematics Subject Classification: 05A15, 11B68, 34A05.
Keywords: Sequencing problems, Optimal sequencing, Idle time and Minimum duration.
Received March 24, 2022; Accepted April 2, 2022

considered as chaotic and unpredictable. We employ fuzzy intervals and fuzzy integers to deal with these uncertainties. Here, we'll look at intervals. Dwyer [2] was the first to propose interval computation. Zadeh [3] developed the concept of fuzzy sets. Interval parameters were used by Radhakrishnan et al. [4] to answer difficulties in game theory. Radhakrishnan et al. [5] examined and solved problems with intervals in the Critical Path Method and the Program Evaluation Review Technique, as well as the conversion of fuzzy parameters (triangular and trapezoidal) into intervals using -cuts.

2. Basic Preliminaries

Definition 1 (Sequencing Problem)**.** The selection of an appropriate order for finite number of different jobs to be done on a finite number of machines is called a sequencing problem.

Definition 2 (Interval Number)**.** An interval number A is defined as $A = [x_1, x_2] = \{x : x_1 \leq x \leq x_2, x € R\}$. Here $x_1, x_2 € R$ are the lower and upper bounds of the interval.

Arithmetic Operation of Interval:

Let $X = [x^L + x^R]$, and $Y = [y^L + y^R]$

Addition: $X + Y = [x^L + y^L, x^R + y^R]$

Subtraction: $X - Y = [x^L - y^L, x^R - y^R]$

Multiplication: $X * Y = [x^L * y^L, x^R * y^R]$ if $x^L \geq 0, y^L \geq 0$

Division: $X/Y = [x^L/y^L, x^R/y^R]$ if $x^L \geq 0, y^L \geq 0$

Types of Interval

Let $X = [x_1, x_2]$ and $Y = [y_1 + y_2]$ be the two intervals. These can be classified into three types as follows.

Type 1. Non-Overlapping Intervals

If two intervals are disjoint then they are known as non-overlapping intervals.

Type 2. Partially Overlapping Intervals.

If one interval contains the other interval partially then they are known as partially overlapping intervals.

Type 3. Completely Overlapping Intervals

Totally overlapping intervals are those in which one interval is completely enclosed within the other.

These are the tree types of intervals

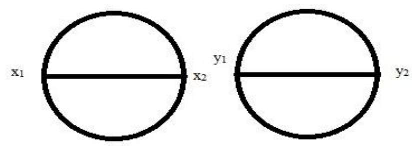

Figure 1. Non overlapping.

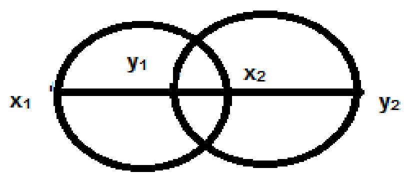

Figure 2. Partially overlapping.

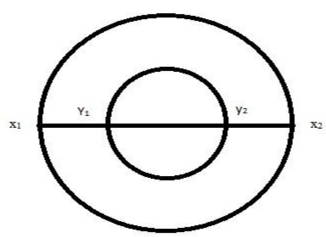

Figure 3. Complete overlapping.

Principle Assumption for Sequencing Problem

Using the following assumptions solve the sequence problem.

1. The processing times on different machines are not dependent of the order of the job in which they are to be performed.

2. No machine can handle more than one job at the same time.

3. The job transferring from one machine to another machine the time required negligible.

4. A job once started must be completed.

5. Processing times are known and predetermined.

6. The task must be completed before starting the next task.

3. Basic Terminologies

(a) Number of Machine

Indicates the number of service objects that an operation must go through before it is considered complete.

(b) Processing Order

It refers to the time required by each job on each machine.

(c) Processing Order

This refer to the order (sequence) in which particular machine are required for completing the job.

(d) Total Elapsed Time

This is the time interval between the start of the first task and the completion of the last task in the sequence.

(e) Idle Time on a Machine

A machine's idle time is the time the machine has no work to process.

i.e.) Idle time is defined as the period between the end of a job (i-1) and the commencement of a new job (i-2).

(f) No Passing Rule

It is refer to the rule of each of the n job is to be processed through three machine M1, M2 and M3 in the order M1M2M3 then it means that each job will go to machine M1 first then to M2 and finally to M3.

Algorithm for Solving Interval Sequencing Problem

There is no standard procedure to obtain an optimal sequence for '*n*' job three machine problem. We have to convert the three-machine problem into two machine problem by satisfying any one or both of the following condition.

(i) Minimum time on machine $M1$ is $\geq$ Maximum time on machine $M2$

i.e.) $Min(Ai) \geq Max(Bi)$ for $i = 1, 2, 3, \ldots, n$

(ii) Minimum time on machine $M3$ is $\geq$ Maximum time on machine $M2$

i.e.) $Min(Ci) \geq Max(Bi)$ for $i = 1, 2, 3, \ldots, n$

If one of the above conditions are satisfied, we introduce the G and H are two fictitious machine, there are

$G = Ai + Bi$ for $i = 1, 2, 3, \ldots, n$

$H = Bi + Ci$ for $i = 1, 2, \ldots, n$.

Total Elapsed Time

Total elapsed time means that the amount of time, that time passes from the start of an even to its finish.

(i) Idle Time for Machine 1

Total elapsed time is the time it takes for the last job in a series to complete on $M1$.

(ii) Idle Time for Machine 2

The time the first task in the sequence completed $M1 + \Sigma j = 2n$ { (j^{th} job start time on machine 2) – ($(j-1)^{th}$ finish time on machine 2)

(iii) Idle Time for Machine 3

Time at which the first job in the sequence ends at $M + \Sigma j = 2n$ {(Time at which the j^{th} job starts on machine 3) - (Time at which (j-1) ends on machine 3)}

4. Numerical Example

1. A water pollution control equipment processes 5 job on 3 machine A, B and C. Processing time on A and B, C are given below. Find the optimal sequence for each machine, as well as the total minimum elapsed time and idle time.

Job	A	B	C	D	E
M_1	3	12	5	2	9
M_2	8	6	4	6	3
M_3	13	14	9	12	8

Solution:

Here,

$Min(M1) = 2$

$Max(M2) = 8$

$Min(M3) = 8$

Since, $Min(M3) = Max(M2)$

The equivalent problem involving 5 jobs and 2 fictitious operation

Job	G_i = Screening + Grid removal (Minutes)	H_i =Grid removal + Digester(Minutes)
1	11	21
2	18	20
3	9	13
4	8	18
5	12	11

Examine the columns G_i and H_i, we find that the smallest value is 8 under operation G_i in row.

4. As a result, job 4 is scheduled first, as indicated below.

4				

The reducing set of processing times becomes

Job	G_i	H_i
1	11	21
2	18	20
3	9	13
5	12	11

The next smallest value in 9 under column G_i for job 3

Here, we schedule job as shown below

4	3			

The reducing set of processing times becomes

Job	G_i	H_i
1	11	21
2	18	20
5	12	11

There are 2 equal minimal values, processing time of 11 under column G_i job for 1 and processing time of 11 under column H_i for job 5.

According to the rules,

Job 1 is scheduling next to job 3 and job 5 is scheduled last as shown below.

4	3	1		5

The reducing set of processing times becomes,

Job	G_i	H_i
2	18	20

Hence we schedule job 2 next to job 1 and the optimal sequence.

4	3	1	2	5

(ii) Elapsed Time

Using the individual processing times given the problem.

"The minimum elapsed time from the start of the first screening to the completion of the last digester corresponding to the optimal sequence is computed.

Job	Screening		Grid Removal		Digester	
	Time In	Time Out	Time In	Time Out	Time In	Time Out
4	0	2	2	8	8	20
3	2	7	8	12	20	29
1	7	10	12	20	29	42
2	10	22	20	26	42	56
5	22	31	26	29	56	64

Minimum Elapsed Time: 64

Idle time for machine 1: 33 minutes

Idle time for machine 2: 37 minutes

Idle time for machine 3: 8 minutes.

2. A water pollution control equipment processes 5 job on 3 machines M_1, M_2 and M_3. Processing time on M_1, M_2 and M_3 are given below. Find the optimal sequence for each machine, as well as the total minimum elapsed time and idle time. Fuzzy intervals are used.

Machines	Job				
	1	2	3	4	5
M_1	[1, 5]	[7, 9]	[6, 8]	[2, 6]	[8, 16]
M_2	[2, 6]	[1, 5]	[1, 3]	[3, 7]	[0, 2]

M_3	[4, 8]	[6, 8]	[3, 7]	[10, 12]	[3, 7]

Solution:

Since the problem is a 3 machine problem, we convert it into a 2 machine problem for that it has to satisfy one of the following condition

$Min(M_1) \geq Max(M_2)$

$Min(M_1) = [1, 5], Max[M2] = [3, 7]$

$Min(M_1) \ngeq Max(M_2)$

And

$Min(M3) \geq Max(M2)$

$Min(M3) = [3, 7]$

$Max(M2) = [3, 7]$

$Min(M3) = Max(M2)$

Therefore, the second condition is satisfied,

We convert this problem into 2 machines as G and H

$$G = M_1 + M_2$$

$$H = M_2 + M_3$$

The processing time of the 2 machine G and H for 5 are as following

Machine	1	2	3	4	5
G	[3, 11]	[8, 14]	[7, 11]	[5, 13]	[8, 12]
H	[6, 4]	[7, 13]	[4, 10]	[13, 19]	[3, 9]

Order of cancellation: E- A-C-D-B

Optimal Sequence: A-D-B-C-E

JOB	Gi =Screening + Grid removal (minutes)	Hi = Grid removal +Digester(minute)
1	[3,11]	[6,14]

2	[8,4]	[7,13]
3	[7,11]	[4,10]
4	[5,13]	[13,19]
5	[8,14]	[7,13]

Optimal Sequence:

A-D-B-C-E

Total minimum elapsed time

JOB	MACHINE 1		MACHINE 2		MACHINE 3	
	Time In	Time Out	Time In	Time Out	Time In	Time Out
A	[0,0]	[1,5]	[1,5]	[3,11]	[3,11]	[7,19]
B	[1,5]	[3,11]	[3,11]	[6,18]	[7,19]	[17,31]
C	[3,11]	[10,20]	[6,18]	[7,23]	[17,31]	[23,39]
D	[10,20]	[16,28]	[7,23]	[8,26]	[23,39]	[26,46]
E	[16,28]	[24,38]	[8,26]	[8,28]	[26,46]	[29,53]

Minimum total elapsed time $= [29, 53]$

Idle time on $M_1 = [29, 53] - [24, 38]$

$= [-9, 29]$

Idle time on $M_2 := [44, 4]$

Idle time on $M_3 = [3, 11]$

5. Conclusion

The intention of this paper has been to offer you a top level view of sequencing trouble. In this phase we mentioned sequencing trouble for '*n*' jobs 3 machine. The numerical illustrations are greater green to attain an finest sequencing, minimal elapsed time, and idle time to manner all jobs via machine.

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