



APPLICATION OF PYTHAGOREAN FUZZY ROUGH SET BASED ON NEW SIMILARITY MEASURES IN AGRICULTURAL PRODUCTION PLANNING PROBLEM

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Abstract

Present paper is an application study of Pythagorean fuzzy rough set based on new similarity measures in agricultural production planning problem particularly a case of small holder farmer.

1. Introduction

Pythagorean fuzzy set was introduced by yager [5]. It deals about the both membership and non-membership grade with the condition $0 \leq (\mu_{PyF})^2 + (\lambda_{PyF})^2 \leq 1$. Distance and similarity measures for Pythagorean fuzzy (PyF) sets was given by Paul Augustine Ejegwa [3] deals about the MCDM Problems. In this paper, deals an application study of Pythagorean fuzzy rough set based on new similarity measures in agricultural production planning problem particularly a case of small holder farmer.

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2. Preliminaries

Definition 2.1. Let Z is a universal set. An Pythagorean fuzzy (PyF) set G in Z of the form $G = \{ \langle n : \mu_{PyF}(m); \lambda_{PyF}(m) \rangle | n \in Z \}$ where the degree of membership function and non-membership function is defined as is defined as $\mu_{PyF} : Z \rightarrow [0, 1]$, $\lambda_{PyF} : Z \rightarrow [0, 1]$ of an object $z \in Z$ with the condition $0 \leq (\mu_{PyF})^2 + (\lambda_{PyF})^2 \leq 1$ and its denoted by $PyFR(Z)$ is $= \langle \mu_{PyF}, \lambda_{PyF} \rangle$.

Definition 2.2. Let non-empty set Z be finite universe. PyF equivalence relation defined on $Z \times Z$. The pair $(Z; PyFR)$ is defined to be PyF approximation space. For any $G \in PyF(Z)$, the lower $PyFR(G)$ and upper approximation $PyFR(G)$ respect to $(Z; PyFR)$ defined as:

$$PyFR(G) = \{v, \mu_{PyFR(G)}(v), \lambda_{PyFR(G)}(v) | v \in Z\}, \quad (2.1)$$

$$PyFR(G) = \{v, \mu_{PyFR(G)}(v), \lambda_{PyFR(G)}(v) | v \in Z\}, \quad (2.2)$$

Membership function is defined by

$$\mu_{PyFR(G)} = \bigwedge_{z \in Z} [(1 - PyFR(v, z)) \vee \mu_G(z)], \quad (2.3)$$

$$\mu_{PyFR(G)} = \bigvee_{z \in Z} [(1 - PyFR(v, z)) \vee \mu_G(z)], \quad (2.4)$$

Non- membership function is defined by

$$\lambda_{PyFR(G)}(v) = \bigwedge_{z \in Z} [(1 - PyFR(v, z)) \vee \lambda_G(z)], \quad (2.5)$$

$$\lambda_{PyFR(G)}(v) = \bigvee_{z \in Z} [(PyFR(v, z)) \wedge \lambda_G(z)], \quad (2.6)$$

The pair $PyFR(G) = [PyFR(G), PyFR(G)]$ is said to be $PyFR$ Pythagorean fuzzy rough set of G with respect to $(Z, PyFR)$.

Definition 2.3 [4]. Let K be non-empty set and $X, Y, Z \in PyFRS(K)$. Similarity measure ‘ Sm ’ between X and Y is a function $Sm : PyFRS \times PyFRS \rightarrow [0, 1]$ satisfies

1. $0 \leq s_m(X, Y) \leq 1$

- 2. $Sm(X, Y) = 0$ iff $X = Y$
- 3. $Sm(X, Y) = Sm(Y, X)$
- 4. $Sm(X, Z) + Sm(Y, Z) \geq Sm(X, Y)$

Distance measure for $PyFRSs$ is a dual concept of similarity measure for $PyFRSs$

Proposition 2.4 [4]. *Let $X, Y \in PyFRS(A)$. If $d(X, Y)$ is a distance measure between $PyFRSs$ X and Y , then $S(X, Y) = 1 - d(X, Y)$ is a similarity measure of X and Y .*

New similarity measures of Pythagorean Fuzzy Rough Set ($PyFRS$)

By using the Definition 2.3 and Proposition 2.4

$$S1(J, S) = 1 - \frac{1}{2n} \sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| \right\} \tag{2.7}$$

$$S2(J, S) = 1 -$$

$$\sqrt{\frac{1}{2n} \sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right|^2 + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right|^2 \right\}} \tag{2.8}$$

$$S3(J, S) = 1 - \frac{1}{2n}$$

$$\sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right|^2 + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right|^2 \right\} \tag{2.9}$$

3. Results

3.1 Application of Pythagorean fuzzy rough set based on new similarity measures in agricultural production planning problem.

Consider a yield arranging issues where a farmer can develop food crops millets like ragi, kakum, jowar, bajar etc., in sornavari (Chithiraipattam) sowing during month of April to May and harvesting during month of August to September. The land accessible in 3.25 acre being a landholder farmer need at least 15.52 quintal of food crops to meet his yearly food grains necessity as his fundamental need. Presently the issues of farmer is to design an appropriate mix model based on his criteria for period of sornavari for his territory to get greatest benefit also good level of rupees 1,80,000 is set to meet his other yearly family necessity. The work accessibility for each season is given to be 100 to 110 days. The goal of the issues are to amplify the benefit and limit the investment and to give his base food grain prerequisite. The computational calculation created in segment 3 are being executed bit by bit to discover a ideal arrangement of the above farmer cro demonstrating issues.

Algorithm:

Construct the criteria set $CT = \{CT1, CT2, CT3\}$ for MCDMP.

Case (1). Compute PyF set $PyF(J)$ over U into $PyFRS$ using definition [2.3] and find $\mu_{PyFR(J)}(x)$, $\mu_{PyFR(J)}(x)$ Values and $\lambda_{PyFR(J)}(x)$, $\lambda_{PyFR(J)}(x)$ values based on previous record data to the specific problem. Similarly compute $PyFRS$ for R and S .

2. Compute the new similarity measures $S1(J, S)$ for $PyFR(J)$ and $PyFR(R)$

$$S1(J, S) = 1 - \frac{1}{2n} \sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| \right\} \quad (3.1)$$

3. Compute the new similarity measures $S2(J, S)$ for $PyFR(J)$ and

$PyFR(R)$

$$S2(J, S) = 1 -$$

$$\sqrt{\frac{1}{2n} \sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 \right\}}$$

(3.2)

4. Compute the new similarity measures $S3(J, S)$ for $PyFR(J)$ and $PyFR(R)$

$$S3(J, S) = 1 - \frac{1}{2n}$$

$$\sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 \right\}$$

(3.3)

5. Case (2) Repeat step 3, 4 and 5 for ‘J and S’
6. Case (3) Repeat step 3, 4 and 5 for ‘R and S’
7. Checking the similarity measure (definition 2.5)
8. Compare the values of J, R, S and ranking alternatives based on similarity measure and conclude.

Step 1. Let the criteria set be $CT = \{CT1, CT2, CT3\}$

Where, $CT1 =$ High Nutrition, $CT2 =$ Production cost less, $CT3 =$ Less water farming respectively.

Step 2. Consider, $U = \{X1, X2, X3\}$

Table 1. $PyFRS$ values for Jowar, Ragi and Sajra.

	Jowar(J)	Ragi(R)	Sajra(S)
X1	(0.4,0.6)(0.7,0.4)	(0.5,0.6)(0.5,0.6)	(0.4,0.6)(0.5,0.6)
X2	(0.4,0.7)(0.4,0.4)	(0.4,0.6)(0.5,0.6)	(0.6,0.4)(0.5,0.4)

X3	(0.6,0.6)(0.6,0.4)	(0.4,0)(0.4,0.6)	(0.4,0.4)(0.4,0.6)
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Step 3.

Case 1. Compute the new similarity measures $S1(J, S)$ for $PyFR(J)$ and $PyFR(R)$

$$\begin{aligned}
 S1(J, S) &= 1 - \frac{1}{2n} \\
 &\sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| + \right. \\
 &\left. \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| \right\} \quad (3.4) \\
 &= 1 - \frac{1}{2(3)} \sum_{k=1}^3 \left\{ \begin{array}{l} |0.4 - 0.5| + |0.6 - 0.6| + |0.7 - 0.5| + |0.4 - 0.6| + \\ |0.4 - 0.4| + |0.7 - 0.6| + |0.4 - 0.5| + |0.4 - 0.4| + \\ |0.6 - 0.4| + |0.6 - 0| + |0.6 - 0.4| + |0.4 - 0.6| \end{array} \right\} \\
 &= 1 - \frac{1}{6} \sum_{k=1}^3 \{0.1 + 0 + 0.2 + 0.2 + 0 + 0.1 + 0.1 + 0 + 0.2 + 0.6 + 0.2 + 0.2\} \\
 &= 1 - \frac{1.9}{6} = 1 - 0.316
 \end{aligned}$$

$$S1(J, S) = 0.684$$

Step 4. Compute the new similarity measures $S2(J, S)$ for $PyFR(R)$ and $PyFR(R)$

$$\begin{aligned}
 S2(J, S) &= 1 - \\
 &\sqrt{\frac{1}{2n} \sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 + \right. \\
 &\left. \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 \right\}} \quad (3.5)
 \end{aligned}$$

$$= 1 - \sqrt{\frac{1}{2(3)} \sum_{k=1}^3 \left\{ \begin{array}{l} |0.4 - 0.5|^2 + |0.6 - 0.6|^2 + |0.7 - 0.5|^2 + |0.4 - 0.6|^2 + \\ |0.4 - 0.5|^2 + |0.7 - 0.6|^2 + |0.4 - 0.5|^2 + |0.4 - 0.4|^2 + \\ |0.6 - 0.4|^2 + |0.6 - 0|^2 + |0.6 - 0.4|^2 + |0.4 - 0.6|^2 \end{array} \right\}}$$

$$= 1 - \sqrt{\frac{1}{6}(0.59)} = 1 - 0.3135$$

$$S2(J, S) = 0.6865$$

Step 5. Compute the new similarity measures $S3(J, S)$ for $PyFR(J)$ and $PyFR(R)$

$$S3(J, S) = 1 - \frac{1}{2n} \sum_{k=1}^n \left\{ \begin{aligned} & \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 + \\ & \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 \end{aligned} \right\} \tag{3.6}$$

$$= 1 - \frac{1}{6} \sum_{k=1}^3 \left\{ \begin{aligned} & \left| 0.4 - 0.5 \right|^2 + \left| 0.6 - 0.6 \right|^2 + \left| 0.7 - 0.5 \right|^2 + \left| 0.4 - 0.6 \right|^2 + \\ & \left| 0.4 - 0.4 \right|^2 + \left| 0.7 - 0.6 \right|^2 + \left| 0.4 - 0.5 \right|^2 + \left| 0.4 - 0.4 \right|^2 + \\ & \left| 0.6 - 0.4 \right|^2 + \left| 0.6 - 0 \right|^2 + \left| 0.6 - 0.4 \right|^2 + \left| 0.4 - 0.6 \right|^2 \end{aligned} \right\}$$

$$= 1 - \frac{1}{6} \sum_{k=1}^3 \{0.01 + 0.04 + 0.04 + 0.01 + 0.01 + 0.04 + 0.36 + 0.04 + 0.04\}$$

$$= 1 - \frac{0.59}{6} = 1 - 0.098$$

Step 6. Case 2

Compute the new similarity measures $S1(J, S)$ for $PyFR(J)$ and $PyFR(S)$

$$S1(J, S) = 1 - \frac{1}{2n} \sum_{k=1}^n \left\{ \begin{aligned} & \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| + \\ & \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right| + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right| \end{aligned} \right\} \tag{3.7}$$

$$= 1 - \frac{1}{2(3)} \sum_{k=1}^3 \left\{ \begin{aligned} & \left| 0.4 - 0.4 \right| + \left| 0.6 - 0.6 \right| + \left| 0.7 - 0.5 \right| + \left| 0.4 - 0.6 \right| + \\ & \left| 0.4 - 0.6 \right| + \left| 0.7 - 0.4 \right| + \left| 0.4 - 0.5 \right| + \left| 0.4 - 0.4 \right| + \\ & \left| 0.6 - 0.4 \right| + \left| 0.6 - 0.4 \right| + \left| 0.6 - 0.4 \right| + \left| 0.4 - 0.6 \right| \end{aligned} \right\}$$

$$1 - \frac{1}{6} \sum_{k=1}^3 \{0 + 0 + 0.2 + 0.2 + 0.2 + 0.3 + 0.1 + 0 + 0.2 + 0.2 + 0.2 + 0.2\}$$

$$= 1 - \frac{1.8}{6} = 1 - 0.3$$

$$S1(J, S) = 0.7$$

Compute the new similarity measures $S2(J, S)$ for $PyFR(J)$ and $PyFR(S)$

$$S2(J, S) = 1 -$$

$$\sqrt{\frac{1}{2n} \sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right|^2 \right\}}$$

(3.8)

$$= 1 - \sqrt{\frac{1}{2(3)} \sum_{k=1}^3 \left\{ \left| 0.4 - 0.4 \right|^2 + \left| 0.6 - 0.6 \right|^2 + \left| 0.7 - 0.5 \right|^2 + \left| 0.4 - 0.6 \right|^2 + \left| 0.4 - 0.6 \right|^2 + \left| 0.7 - 0.3 \right|^2 + \left| 0.4 - 0.5 \right|^2 + \left| 0.4 - 0.4 \right|^2 + \left| 0.6 - 0.4 \right|^2 + \left| 0.6 - 0.4 \right|^2 + \left| 0.6 - 0.4 \right|^2 + \left| 0.4 - 0.6 \right|^2 \right\}}$$

$$= 1 - \sqrt{\frac{1}{6} \sum_{k=1}^3 \{0.04 + 0.04 + 0.04 + 0.16 + 0.01 + 0.04 + 0.04 + 0.04 + 0.04\}}$$

$$= 1 - \sqrt{\frac{1}{6} (0.49)}$$

$$S2(J, S) = 0.726$$

Compute the new similarity measures $S3(J, S)$ for $PyFR(J)$ and $PyFR(S)$

$$S3(J, S) = 1 - \frac{1}{2n} =$$

$$\sum_{k=1}^n \left\{ \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(R)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(R)}(x_k) \right|^2 + \left| \mu_{PyFR(J)}(x_k) - \mu_{PyFR(S)}(x_k) \right|^2 + \left| \lambda_{PyFR(J)}(x_k) - \lambda_{PyFR(S)}(x_k) \right|^2 \right\}$$

(3.9)

$$1 - \frac{1}{6} \sum_{k=1}^3 \left\{ \begin{array}{l} |0.4 - 0.4|^2 + |0.6 - 0.6|^2 + |0.7 - 0.5|^2 + |0.4 - 0.6|^2 + \\ |0.4 - 0.6|^2 + |0.7 - 0.3|^2 + |0.4 - 0.5|^2 + |0.4 - 0.4|^2 + \\ |0.6 - 0.4|^2 + |0.6 - 0.4|^2 + |0.6 - 0.4|^2 + |0.4 - 0.6|^2 \end{array} \right\}$$

$$= 1 - \frac{1}{6}$$

$$\sum_{k=1}^3 \{0.04 + 0.04 + 0.04 + 0.04 + 0.16 + 0.01 + 0.04 + 0.04 + 0.04 + 0.04\}$$

$$S3(J, S) = 0.9184$$

Step 7. case3

Similarly, compute the new similarity measures $S1(R, S)$, $S2(R, S)$, $S3(R, S)$ for $PFR(R)$ and $PFR(S)$

$$S1(R, S) = 1 - \frac{1}{2n} (1.3) = 1 - \frac{1.3}{6} = 1 - 0.216 = 0.784$$

$$S2(R, S) = 1 - \sqrt{\frac{1}{6}} (0.01 + 0.04 + 0.04 + 0.16 + 0.04 + 0.04)$$

$$= 1 - \sqrt{\frac{1}{6}} (0.33) = 1 - \sqrt{0.055} = 1 - 0.2345 = 0.7655$$

$$S3(R, S) = 1 - \frac{1}{2n} (0.33) = 1 - \frac{0.33}{6} = 1 - 0.055$$

$$S3(R, S) = 0.945$$

Table 2. Similarity measure values for JR, JS and RS.

Similarity measure	Jowarand Ragi (J, R)	Jowar and Sajra (J, S)	Ragi and Sajar (R, S)
S1	0.684	0.7	0.784
S2	0.6865	0.726	0.7655

S3	0.902	0.9184	0.945
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Step 8. It follows from definition 2.3 that it satisfy all the below condition

1. $0 \leq Sm(J, R) \leq 1$ holds, since $d(J, R), d(J, S), d(R, S) \in [0, 1]$
2. $Sm(J, S) = 0$ iff $J = R$ holds, its straight forward
3. $Sm(J, S) = Sm(R, J)$ holds, use of absolute and square values.
4. $Sm(J, S) + Sm(R, S) \geq Sm(J, R)$ holds for all four proposed distances

Step 9. Similarity measure and alternatives of ranking.

Table 3. Ranking similarity measures for JR, JS and RS.

Similarity measure	Jowar and Ragi (J, R)	Jowarand Sajra (J, S)	Ragiand Sajar (R, S)
S1	0.684	0.7	0.784
S2	0.6865	0.736	0.7655
S3	0.902	0.93	0.945
Rank	3	2	1

4. Conclusions

In this paper, we introduce similarity measures on Pythagorean fuzzy rough sets and an application study of Pythagorean fuzzy rough set based on new similarity measures in agricultural production planning problem particularly a case of small holder farmer. We conclude that the Ragi and Sajar (R, S) is the best mix model for period of sornavari for his territory to get greatest benefit. Compare to other food crops Jowar and Ragi (J, R), Jowar and Sajra (J, R).

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