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CREEP MODELING IN A ROTATING FGM DISC

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Abstract

Creep behavior in a rotating functionally graded material (FGM) disc is studied. Thickness of the disc and SiC_p content in Al matrix varies as per power law. The effects of variation in reinforcement index (m) on the strain rates are investigated. It is found that the stress (radial) increases throughout for the FGM by decreasing thickness and reinforcement index. But the tangential stress changes significantly by changing reinforcement index (m). The s strain rates in the FGM discs decreases with the decrease in reinforcement index (m).

1. Introduction

FGM's are a new material which are obtained by mixing of two or more components in a proportion that changes along a particular direction [1, 2]. The variation of the property can be predefined to gain the benefits of the properties of the individual components. Rotating discs is very important component used in automobiles, gas turbines, hard disk drives, etc. [3-5]. Shukla [6] studied creep behavior in rotating disc. It is observed that presence of non-homogeneity reduces the stresses in the disc as compared to

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homogeneous disc. Durodola and Attia [7] concluded that the deformations in the FGM disc are significantly different as compared a similar uniform composite disc. Singh and Ray [8] observed creep rates in a variable thickness rotating disc using Norton's power law under constant temperature conditions. It is found that strain rates in the FGM disc decreases as compared composite disc. Çallıoğlu et al., [9] studied the elastic behavior for FG disc under rotation. It is found from that the grading index affects the mechanical response significantly. Khanna et al., [10] calculated creep rates in a rotating composite discs using power law. It is observed that the composite disc with more thickness gradient exhibits lesser distortion. In the present study, stresses and the creep strain rate have been determined in the rotating FGM disc with non-linear variable thickness. The results are compared for the discs having different thickness and reinforcement index.

2. Disc Profile

Suppose a rotating FGM disc with the inner radius $(a = 0.03175\,m)$ and outer radius $(b = .1524\,m)$ under rotation at the speed of 15000 rpm. The thickness of the disc changes as,

$$h(r) = h_a\left(\frac{r}{a}\right)^k \quad h_a = \text{Thickness at } r = a \quad k = \text{Thickess index} \tag{1}$$

The volume of variable thickness disc and constant thickness disc $(t = 0.0254m)$ is same,

$$\int_a^b 2\pi r h(r) dr = \pi(b^2 - a^2)t. \tag{2}$$

Putting the value of $h(r)$ in equation (2), we get,

$$h_a = \frac{(2 + k)t(b^2 a^k - a^{2+k})}{2(b^{2+k} - a^{2+k})}. \tag{3}$$

The percentage of SiC_p in the FGM disc along radius is given by,

$$V(r) = V_a\left(\frac{r}{a}\right)^m. \tag{4}$$

$V_a = SiC_p$ Content at $r = a$, m = Reinforcement index

The density $\rho(r)$ is calculated as [5],

$$\rho(r) = \rho_{Al} + \frac{(\rho_{SiC_P} - \rho_{Al})V_\alpha}{100}\left(\frac{r}{a}\right)^m = A + Br^m. \quad (5)$$

Where

$$A = \rho_{Al} \quad B = \frac{(\rho_{SiC_P}, -\rho_{Al})}{100a^m} \quad \rho_{Al} = 2698.9\text{Kg/m}^3 \quad \rho_{SiC_P} = 3210\text{Kg/m}^3.$$

We also have equal SiC_p content in both the discs,

$$V_{av}[\pi(b^2 - a^2)t] = \int_a^b 2\pi r h(r) V(r) dr. \quad (6)$$

By solving Equation (1) and (4) with Equation (6), we get,

$$V_a = \frac{(2 + k + n)tV_{av}(b^2 - a^2)a^{k+n}}{2h_a[b^{2+k+n} - a^{2+k+n}]}. \quad (7)$$

3. Creep Law. The effective strain rate $(\bar{\varepsilon})$ estimated by the threshold stress-based law [10] as,

$$\dot{\bar{\varepsilon}} = [M(r)\{\bar{\sigma} - \sigma_0(r)\}]^5. \quad (8)$$

Where $M(r)$ and $\sigma_0(r)$ are the creep parameters.

According to Treska yield criteria,

$$\bar{\sigma} = \sigma_\theta. \quad (9)$$

The strain rates $(\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta)$ can be written as, [8, 10]

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{[2x(r) - 1]}{2}[M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n. \quad (10)$$

$$\dot{\varepsilon}_\theta \frac{\dot{u}_r}{r} = \frac{[2 - x(r)]}{2}[M(r)\{\bar{\sigma} - \sigma_0(r)\}]^n \quad (11)$$

where $x(= \sigma_r/\sigma_\theta)$.

The equations (10) and (11) are solved with equation (9). We get,

$$\sigma_\theta = \frac{\left[\sigma_{av}\int_a^b h(r)dr - \int_a^b h(r)\sigma_0(r)dr\right]\psi_1^{1/n}}{M(r)\int_\alpha^b \frac{h(r)\psi^{1/n}(r)}{M(r)}dr} + \sigma_0(r) \tag{12}$$

Where $\psi_1 = \frac{2}{r[2 - x(r)]}\exp\left(\int_a^r \frac{\phi(r)}{r}dr\right).$

The equation for a variable thickness rotating FGM disc [4],

$$\frac{d}{dr}[h(r)r\sigma_r] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0. \tag{13}$$

Integrating equation (13) using free-free condition $[\sigma_r(a) = 0$ and $\sigma_r(b) = 0]$, we get,

$$\sigma_{\theta av} = \frac{\int_a^b \rho(r)\omega^2 r^2 h(r)dr}{\int_a^b h(r)dr} \tag{14}$$

Integrating the equilibrium Equation (13), the radial stress is,

$$\sigma_r = \frac{1}{rh(r)}\left[\int_a^b h(r)\sigma_\theta dr - \omega^2\int_a^r \rho(r)r^2 h(r)dr\right] \tag{15}$$

4. Results and Discussion. Stresses (σ_r and σ_θ) and the strain rates ($\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$) in a rotating FGM at constant temperature $(T = 623K)$ are calculated.

4.1. Effect of Reinforcement index (m) on creep behavior in FGM disc. The impact of changing reinforcement index (m) is observed on creep behavior in FGM disc. The SiC_p content in the matrix of pure Al is varying according to power law. In the present analysis, creep strain rates are calculated for three rotating discs (Table 1).

Table 1. FGM discs with variable thickness $(k = -0.25)$.

Disc Notation	Va	V_b	V_{av}
D1	20	20	20
D2	15.	22.4	20
D3	26.2	17.7	20

The radial stress (Figure 1) for non-linear FGM disc D3 rises but for disc D2 decreases throughout than disc D1. Figure

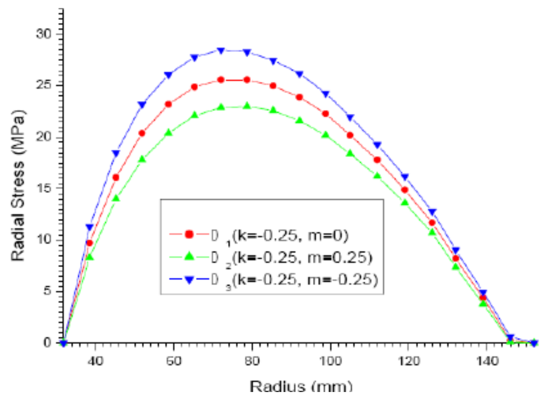

Figure 1. Radial stresses in the FGM discs.

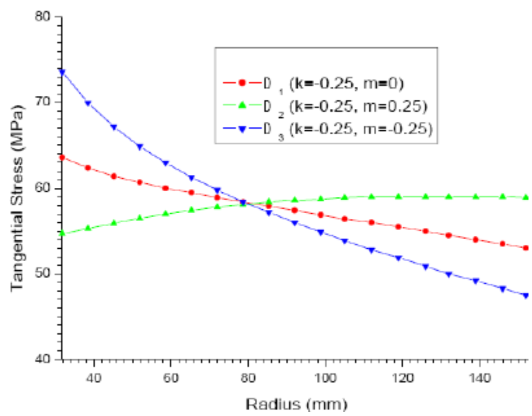

Figure 2. Tangential stresses in the FGM discs.

By imposing SiC_p reinforcement in FGM disc D3, tangential stress (Figure 2) increases in the inner part of the disc but decreases in outer part as compared uniform composite disc D1.

The strain rates (radial and tangential) in the FGM disc D3 decrease over the entire radius. But in case of FGM disc D2, strain rate increases significantly over the entire disc radius when compared with the uniform composite disc D1. The change observed in strain rates for the inner part of the disc is comparatively higher as compared outer portion of the disc. Thus creep behavior of the disc with decreasing SiC_p content (D3) is better to non-linear FGM disc with increasing SiC_p content (D2).

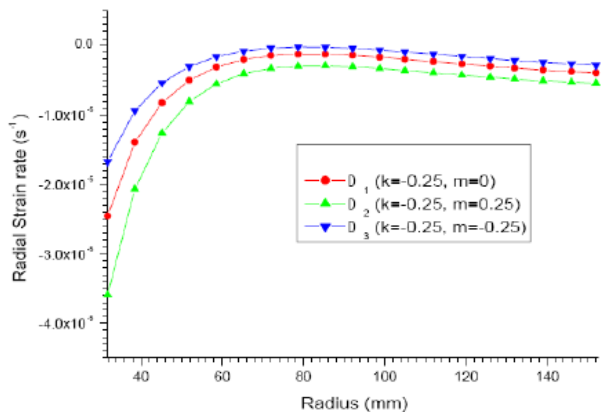

Figure 3. Variation of radial strain rate in the FGM discs.

5. Conclusions

The present study has led to the following conclusions:

(i) Radial stress in the non-linear FGM disc having decreasing reinforcement content increases as compared uniform composite disc.

(ii) By imposing higher SiC_p content in the inner portion of disc as compared outer side, the tangential stress increases in the inner part of the disc but decreases near the outer radius.

(iii) By imposing higher SiC_p content in the inner portion of the disc, strain rates as compared uniform composite disc and FGM disc with higher SiC_p content in the outer portion of the disc.

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